76. $\int_0^3 (1+x^3)\,dx = \left(x+\frac{1}{4}x^4\right)\Big|_0^3 = 3+\frac{81}{4} = \frac{93}{4}$ and

$\int_0^1 (1+x^3)\,dx + \int_1^2 (1+x^3)\,dx + \int_2^3 (1+x^3)\,dx = \left(x+\frac{1}{4}x^4\right)\Big|_0^1 + \left(x+\frac{1}{4}x^4\right)\Big|_1^2 + \left(x+\frac{1}{4}x^4\right)\Big|_2^3$

$= \left(1+\frac{1}{4}\right) + (2+4) - \left(1+\frac{1}{4}\right) + \left(3+\frac{81}{4}\right) - (2+4) = \frac{93}{4}.$

77. $\int_3^3 \left(1+\sqrt{x}\right)e^{-x}\,dx = 0$ by Property 1 of the definite integral.

78. $\int_3^0 f(x)\,dx = -\int_0^3 f(x)\,dx = -4$ by Property 2 of the definite integral.

79. **a.** $\int_{-1}^2 [2f(x)+g(x)]\,dx = 2\int_{-1}^2 f(x)\,dx + \int_{-1}^2 g(x)\,dx = 2(-2)+3 = -1.$

b. $\int_{-1}^2 [g(x)-f(x)]\,dx = \int_{-1}^2 g(x)\,dx - \int_{-1}^2 f(x)\,dx = 3-(-2) = 5.$

c. $\int_{-1}^2 [2f(x)-3g(x)]\,dx = 2\int_{-1}^2 f(x)\,dx - 3\int_{-1}^2 g(x)\,dx = 2(-2)-3(3) = -13.$

80. **a.** $\int_{-1}^0 f(x)\,dx = \int_{-1}^2 f(x) - \int_0^2 f(x)\,dx = 2-3 = -1.$

b. $\int_0^2 f(x)\,dx - \int_{-1}^0 f(x)\,dx = 3-(-1) = 4.$

81. True. This follows from Property 1 of the definite integral.

82. False. The integrand $f(x) = \dfrac{1}{x-2}$ is not defined at $x=2$.

83. False. Only a constant can be "moved out" of the integral.

84. True. This follows from the fundamental theorem of calculus.

85. True. This follows from Properties 3 and 4 of the definite integral.

86. True. We have $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, and so $\int_c^b f(x)\,dx = \int_a^b f(x)\,dx - \int_a^c f(x)\,dx$, $-\int_c^b f(x)\,dx = \int_c^b f(x)\,dx = \int_a^b f(x)\,dx - \int_a^c f(x)\,dx$, and $\int_c^b f(x)\,dx = \int_a^c f(x)\,dx - \int_a^b f(x)\,dx$.

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1. 7.716667 **2.** 1.153100 **3.** 17.564865 **4.** −14.333333

5. 159/bean stem **6.** 60.5 mg/day **7.** 0.48 g/cm^3/day

6.6 Area between Two Curves

Concept Questions

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1. $\int_a^b [f(x)-g(x)]\,dx$

2. $\int_a^b [f(x)-g(x)]\,dx + \int_b^c [g(x)-f(x)]\,dx + \int_c^d [f(x)-g(x)]\,dx$

Exercises

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1. $-\int_0^6 (x^3-6x^2)\,dx = \left(-\frac{1}{4}x^4+2x^3\right)\Big|_0^6 = -\frac{1}{4}(6)^4+2(6)^3 = 108.$

2. $-\int_0^2 \left(x^4-2x^3\right) dx = \left(\frac{1}{2}x^4 - \frac{1}{5}x^5\right)\Big|_0^2 = 8 - \frac{32}{5} = \frac{8}{5}.$

3. $A = -\int_{-1}^0 x\sqrt{1-x^2}\, dx + \int_0^1 x\sqrt{1-x^2} dx = 2\int_0^1 x\left(1-x^2\right)^{1/2} dx$ by symmetry. Let $u = 1-x^2$, so $du = -2x\, dx$ and $x\, dx = -\frac{1}{2}\, du$. If $x = 0$, then $u = 1$ and if $x = 1$, then $u = 0$, so $A = (2)\left(-\frac{1}{2}\right) \int_0^1 u^{1/2}\, du = -\frac{2}{3}u^{3/2}\Big|_1^0 = \frac{2}{3}.$

4. $A = -\int_{-2}^0 \frac{2x}{x^2+4}\, dx + \int_0^2 \frac{2x}{x^2+4}\, dx = 2\int_0^2 \frac{2x}{x^2+4}\, dx = 2\ln\left(x^2+4\right)\Big|_0^2 = (\ln 8 - \ln 4)\, 2 = \ln 4.$

5. $A = -\int_0^4 \left(x - 2\sqrt{x}\right) dx = \int_0^4 \left(-x + 2x^{1/2}\right) dx = \left(-\frac{1}{2}x^2 + \frac{4}{3}x^{3/2}\right)\Big|_0^4 = 8 + \frac{32}{3} = \frac{8}{3}.$

6. $A = \int_0^4 \left[\sqrt{x} - (x-2)\right] dx = \int_0^4 \left(x^{1/2} - x + 2\right) dx = \left(\frac{2}{3}x^{3/2} - \frac{1}{2}x^2 + 2x\right)\Big|_0^4 = \frac{16}{3} - 8 + 8 = \frac{16}{3}.$

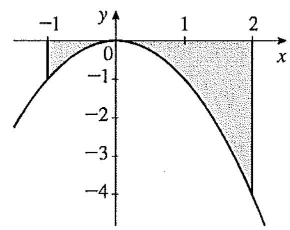
7. The required area is given by

$\int_{-1}^0 \left(x^2 - x^{1/3}\right) dx + \int_0^1 \left(x^{1/3} - x^2\right) dx = \left(\frac{1}{3}x^3 - \frac{3}{4}x^{4/3}\right)\Big|_{-1}^0 + \left(\frac{3}{4}x^{4/3} - \frac{1}{3}x^3\right)\Big|_0^1 = -\left(-\frac{1}{3} - \frac{3}{4}\right) + \left(\frac{3}{4} - \frac{1}{3}\right) = \frac{3}{2}.$

8. $A = \int_{-4}^0 \left[(x+6) - \left(-\frac{1}{2}x\right)\right] dx + \int_0^2 \left[(x+6) - x^3\right] dx = \int_{-4}^0 \left(\frac{3}{2}x + 6\right) dx + \int_0^2 \left[(x+6) - x^3\right] dx$

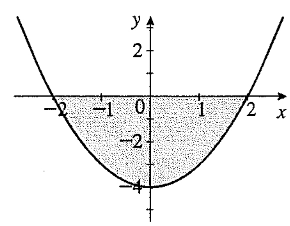
$= \left(\frac{3}{4}x^2 + 6x\right)\Big|_{-4}^0 + \left(\frac{1}{2}x^2 + 6x - \frac{1}{4}x^4\right)\Big|_0^2 = -(12-24) + (2+12-4) = 22.$

9. The required area is given by $-\int_{-1}^2 -x^2 dx = \frac{1}{3}x^3\Big|_{-1}^2 = \frac{8}{3} + \frac{1}{3} = 3.$



10. $A = -\int_{-2}^2 \left(x^2 - 4\right) dx = -2\int_0^2 \left(x^2 - 4\right) dx$

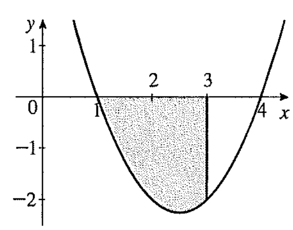
$= 2\left(-\frac{1}{3}x^3 + 4x\right)\Big|_0^2 = 2\left(-\frac{8}{3} + 8\right) = \frac{32}{3}.$



11. $y = x^2 - 5x + 4 = (x-4)(x-1) = 0$ if $x = 1$ or 4, the x-intercepts of the graph of f. Thus,

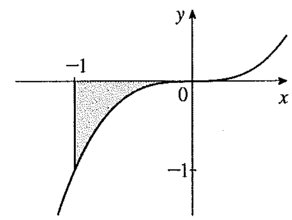
$A = -\int_1^3 \left(x^2 - 5x + 4\right) dx = \left(-\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x\right)\Big|_1^3$

$= \left(-9 + \frac{45}{2} - 12\right) - \left(-\frac{1}{3} + \frac{5}{2} - 4\right) = \frac{10}{3}.$



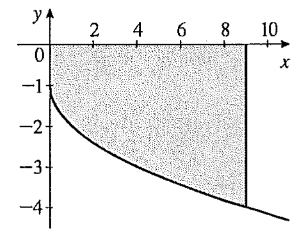
12. The required area is given by

$$-\int_{-1}^{0} x^3\,dx = -\tfrac{1}{4}x^4\Big|_{-1}^{0} = -\tfrac{1}{4}(0) + \tfrac{1}{4}(1) = \tfrac{1}{4}.$$



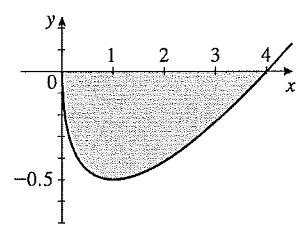
13. The required area is given by

$$-\int_{0}^{9} -\left(1+\sqrt{x}\right)dx = \left(x + \tfrac{2}{3}x^{3/2}\right)\Big|_{0}^{9} = 9 + 18 = 27.$$

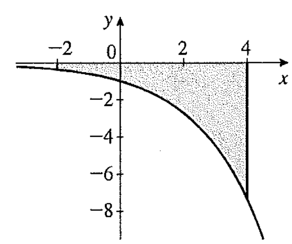


14. $A = -\int_{0}^{4}\left(\tfrac{1}{2}x - x^{1/2}\right)dx = \left(-\tfrac{1}{4}x^2 + \tfrac{2}{3}x^{3/2}\right)\Big|_{0}^{4}$

$= \left(-4 + \tfrac{16}{3}\right) = \tfrac{4}{3}.$

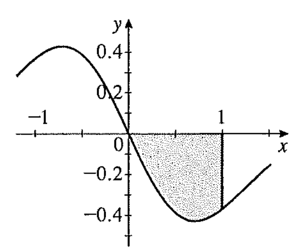


15. $-\int_{-2}^{4}\left(-e^{x/2}\right)dx = 2e^{x/2}\Big|_{-2}^{4} = 2\left(e^2 - e^{-1}\right).$



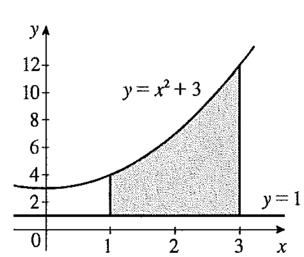
16. $A = -\int_{0}^{1}\left(-xe^{-x^2}\right)dx = \int_{0}^{1} xe^{-x^2}\,dx$. Let $u = -x^2$, so $du = -2x\,dx$ and $x\,dx = -\tfrac{1}{2}\,du$. If $x = 0$, then $u = 0$ and if $x = 1$ then $u = -1$, so

$$A = -\tfrac{1}{2}\int_{0}^{-1} e^u\,du = -\tfrac{1}{2}e^u\Big|_{0}^{-1} = -\tfrac{1}{2}e^{-1} + \tfrac{1}{2} = \tfrac{1}{2}\left(1 - e^{-1}\right).$$



17. $A = \int_{1}^{3}\left[\left(x^2+3\right) - 1\right]dx = \int_{1}^{3}\left(x^2+2\right)dx = \left(\tfrac{1}{3}x^3 + 2x\right)\Big|_{1}^{3}$

$= (9+6) - \left(\tfrac{1}{3} + 2\right) = \tfrac{38}{3}.$



56. False. The area is given by $\int_0^2 [g(x) - f(x)]\,dx$ because $g(x) \geq f(x)$ on $[0, 2]$.

57. False. Take $f(x) = x$ and $g(x) = 0$ on $[0, 1]$. Then the area bounded by the graphs of f and g on $[0, 1]$ is $A = \int_0^1 (x - 0)\,dx = \frac{1}{2}x^2\Big|_0^1 = \frac{1}{2}$ and so $A^2 = \frac{1}{4}$. However, $\int_0^1 [f(x) - g(x)]^2\,dx = \int_0^1 x^2\,dx = \frac{1}{3}$.

58. False. Take $f(t) = t^2$ and $g(t) = 1$ on $[0, 2]$. Then $\int_0^2 [f(t) - g(t)]\,dt = \int_0^2 (t^2 - 1)\,dt = \left(\frac{1}{3}t^3 - t\right)\Big|_0^2 = \frac{8}{3} - 2 = \frac{2}{3} > 0$, but $f(t)$ is not greater than or equal to $g(t)$ for all t in $[0, 2]$. For instance, $f\left(\frac{1}{2}\right) = \frac{1}{4} < 1 = g\left(\frac{1}{2}\right)$.

59. The area of R' is $A = \int_a^b \{[f(x) + C] - [g(x) + C]\}\,dx = \int_a^b [f(x) + C - g(x) - C]\,dx = \int_a^b [f(x) - g(x)]\,dx$.

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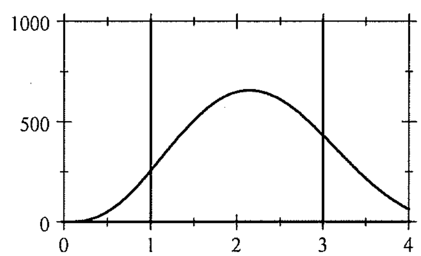
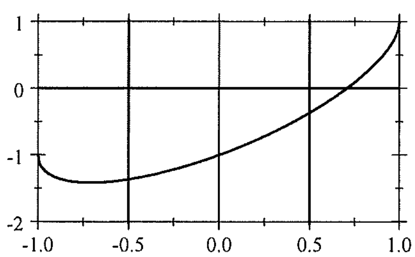
1. a.



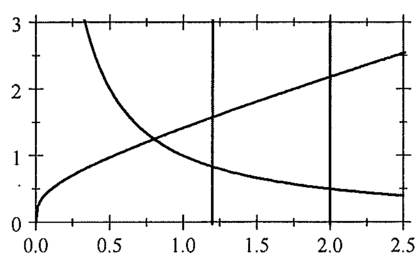
b. $A \approx 1074.2857$.

2. a.



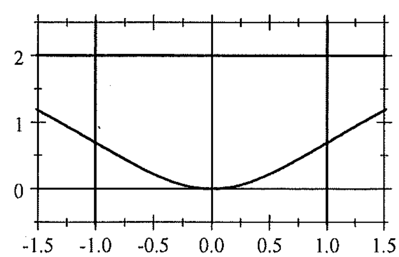
b. $A \approx 0.9566$.

3. a.



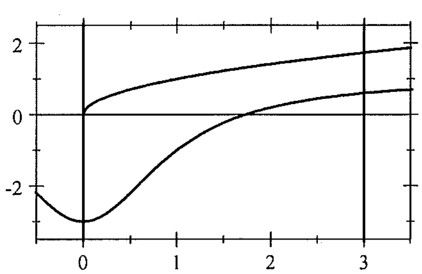
b. $A \approx 0.9961$.

4. a.



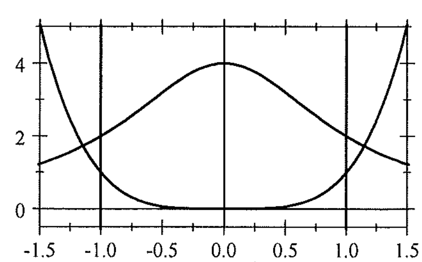
b. $A \approx 3.4721$.

5. a.



b. $A \approx 5.4603$.

6. a.



b. $A \approx 5.8832$.