63. False. $f(x)$ is not nonnegative on $[0, 2]$.

64. True.

Using Technology
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1. 6.1787	**2.** 3.3279	**3.** 0.7873	**4.** 0.2024	**5.** −0.5888
6. 737,038.44	**7.** 2.7044	**8.** 0.4251	**9.** 3.9973	**10.** 0.4182
11. 46%, 24%	**12.** 149.14 million	**13.** 60,156	**14.** 3,761,490	

6.5 Evaluating Definite Integrals

Concept Questions
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1. *Approach I:* We first find the indefinite integral. Let $u = x^3 + 1$, so that $du = 3x^2 dx$ and or $x^2 dx = \frac{1}{3} du$. Then $\int x^2 (x^3 + 1)^2 dx = \frac{1}{3} \int u^2 du = \frac{1}{9} u^3 + C = \frac{1}{9} (x^3 + 1)^3 + C$. Therefore, $\int_0^1 x^2 (x^3 + 1)^2 dx = \frac{1}{9} \left(x^3 + 1^3\right)\Big|_0^1 = \frac{1}{9} (8 - 1) = \frac{7}{9}$.

Approach II: Transform the definite integral in x into an integral in u: Let $u = x^3 + 1$, so that $du = 3x^2\, dx$ and $x^2\, dx = \frac{1}{3}\, du$. Next, find the limits of integration with respect to u. If $x = 0$, then $u = 0^3 + 1 = 1$ and if $x = 1$, then $u = 1^3 + 1 = 2$. Therefore, $\int_0^1 x^2 (x^3 + 1)^2 dx = \frac{1}{3} \int_1^2 u^2\, du = \frac{1}{9} u^3 \Big|_1^2 = \frac{1}{9} (8 - 1) = \frac{7}{9}$.

2. See the definition and interpretation on pages 447 and 448 of the text.

Exercises
page 463

1. Let $u = x^2 - 1$, so $du = 2x\, dx$ and $x\, dx = \frac{1}{2}\, du$. If $x = 0$, then $u = -1$ and if $x = 2$, then $u = 3$, so $\int_0^2 x (x^2 - 1)^3 dx = \frac{1}{2} \int_{-1}^3 u^3\, du = \frac{1}{8} u^4 \Big|_{-1}^3 = \frac{1}{8} (81) - \frac{1}{8} (1) = 10$.

2. Let $u = 2x^3 - 1$, so $du = 6x^2\, dx$ and $x^2\, dx = \frac{1}{6}\, du$. If $x = 0$, then $u = -1$ and if $x = 1$, then $u = 1$, so $\int_0^1 x^2 (2x^3 - 1)^4 dx = \frac{1}{6} \int_{-1}^1 u^4\, du = \frac{1}{30} u^5 \Big|_{-1}^1 = \frac{1}{30} - \left(-\frac{1}{30}\right) = \frac{1}{15}$.

3. Let $u = 5x^2 + 4$, so $du = 10x\, dx$ and $x\, dx = \frac{1}{10}\, du$. If $x = 0$, then $u = 4$ and if $x = 1$, then $u = 9$, so $\int_0^1 x\sqrt{5x^2 + 4}\, dx = \frac{1}{10} \int_4^9 u^{1/2}\, du = \frac{1}{15} u^{3/2} \Big|_4^9 = \frac{1}{15} (27) - \frac{1}{15} (8) = \frac{19}{15}$.

4. Let $u = 3x^2 - 2$, so $du = 6x\, dx$ and $x\, dx = \frac{1}{6}\, du$. If $x = 1$, then $u = 1$ and if $x = 3$, then $u = 25$, so $\int_1^3 x\sqrt{3x^2 - 2}\, dx = \frac{1}{6} \int_1^{25} u^{1/2}\, du = \frac{1}{9} u^{3/2} \Big|_1^{25} = \frac{1}{9} (125) - \frac{1}{9} (1) = \frac{124}{9}$.

5. Let $u = x^3 + 1$, so $du = 3x^2\, dx$ and $x^2\, dx = \frac{1}{3}\, du$. If $x = 0$, then $u = 1$ and if $x = 2$, then $u = 9$, so $\int_0^2 x^2 (x^3 + 1)^{3/2} dx = \frac{1}{3} \int_1^9 u^{3/2}\, du = \frac{2}{15} u^{5/2} \Big|_1^9 = \frac{2}{15} (243) - \frac{2}{15} (1) = \frac{484}{15}$.

6. Let $u = 2x - 1$, so $du = 2\,dx$ and $dx = \frac{1}{2}\,du$. If $x = 1$, then $u = 1$ and if $x = 5$ then $u = 9$, so $\int_1^5 (2x-1)^{5/2}\,dx = \frac{1}{2}\int_1^9 u^{5/2}\,du = \frac{1}{7}u^{7/2}\Big|_1^9 = \frac{1}{7}(2187) - \frac{1}{7}(1) = \frac{2186}{7}$.

7. Let $u = 2x + 1$, so $du = 2\,dx$ and $dx = \frac{1}{2}\,du$. If $x = 0$, then $u = 1$ and if $x = 1$ then $u = 3$, so
$$\int_0^1 \frac{1}{\sqrt{2x+1}}\,dx = \frac{1}{2}\int_1^3 \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_1^3 u^{-1/2}\,du = u^{1/2}\big|_1^3 = \sqrt{3} - 1.$$

8. Let $u = x^2 + 5$, so $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. If $x = 0$, then $u = 5$ and if $x = 2$, then $u = 9$, so
$$\int_0^2 \frac{x}{\sqrt{x^2+5}}\,dx = \frac{1}{2}\int_5^9 \frac{du}{\sqrt{u}} = u^{1/2}\big|_5^9 = 3 - \sqrt{5}.$$

9. Let $u = 2x - 1$, so $du = 2\,dx$ and $dx = \frac{1}{2}\,du$. If $x = 1$, then $u = 1$ and if $x = 3$, then $u = 5$, so $\int_1^3 (2x-1)^4\,dx = \frac{1}{2}\int_1^5 u^4\,du = \frac{1}{10}u^5\Big|_1^5 = \frac{1}{10}(3125 - 1) = \frac{1562}{5}$.

10. Let $u = x^2 + 4x - 8$, so $du = (2x+4)\,dx$. If $x = 1$ then $u = -3$ and if $x = 2$, then $u = 4$, so $\int_1^2 (2x+4)\left(x^2+4x-8\right)^3 dx = \int_{-3}^4 u^3\,du = \frac{1}{4}u^4\Big|_{-3}^4 = \frac{1}{4}(256) - \frac{1}{4}(81) = \frac{175}{4}$.

11. Let $u = x^3 + 1$, so $du = 3x^2\,dx$ and $x^2\,dx = \frac{1}{3}\,du$. If $x = -1$, then $u = 0$ and if $x = 1$, then $u = 2$, so $\int_{-1}^1 x^2\left(x^3+1\right)^4 dx = \frac{1}{3}\int_0^2 u^4\,du = \frac{1}{15}u^5\Big|_0^2 = \frac{32}{15}$.

12. Let $u = x^4 + 3x$, so $du = \left(4x^3+3\right)dx = 4\left(x^3+\frac{3}{4}\right)dx$ and $dx = \frac{1}{4}\left(x^3+\frac{3}{4}\right)^{-1} du$. If $x = 1$, then $u = 4$ and if $x = 2$, then $u = 22$, so
$$\int_1^2 \left(x^3+\frac{3}{4}\right)\left(x^4+3x\right)^{-2} dx = \frac{1}{4}\int_4^{22} u^{-2}\,du = -\frac{1}{4u}\Big|_4^{22} = -\frac{1}{88} + \frac{1}{16} = \frac{-2+11}{176} = \frac{9}{176}.$$

13. Let $u = x - 1$, so $du = dx$. If $x = 1$, then $u = 0$ and if $x = 5$, then $u = 4$, so $\int_1^5 x\sqrt{x-1}\,dx = \int_0^4 (u+1)\,u^{1/2}\,du = \int_0^4 \left(u^{3/2}+u^{1/2}\right)du = \left(\frac{2}{5}u^{5/2}+\frac{2}{3}u^{3/2}\right)\Big|_0^4 = \frac{2}{5}(32) + \frac{2}{3}(8) = \frac{272}{15}$.

14. Let $u = x + 1$, so $x = u - 1$ and $du = dx$. If $x = 1$, then $u = 2$ and if $x = 4$, then $u = 5$, so $\int_1^4 x\sqrt{x+1}\,dx = \int_2^5 (u-1)\sqrt{u}\,du = \int_2^5 \left(u^{3/2}-u^{1/2}\right)du = \left(\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right)\Big|_2^5 = \left[\frac{2}{15}u^{3/2}(3u-5)\right]_2^5$
$= \frac{2}{15}\left(50\sqrt{5} - 2\sqrt{2}\right)$.

15. Let $u = x^2$, so $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. If $x = 0$, then $u = 0$ and if $x = 2$, then $u = 4$, so $\int_0^2 2xe^{x^2}\,dx = \int_0^4 e^u\,du = e^u\big|_0^4 = e^4 - 1$.

16. Let $u = -x$, so $du = -dx$ and $dx = -du$. If $x = 0$, then $u = 0$ and if $x = 1$, then $u = -1$, so $\int_0^1 e^{-x}\,dx = -\int_0^{-1} e^u\,du = -e^u\big|_0^{-1} = -e^{-1} + 1 = 1 - \frac{1}{e}$.

17. $\int_0^1 \left(e^{2x}+x^2+1\right)dx = \left(\frac{1}{2}e^{2x}+\frac{1}{3}x^3+x\right)\Big|_0^1 = \left(\frac{1}{2}e^2+\frac{1}{3}+1\right) - \frac{1}{2} = \frac{1}{2}e^2 + \frac{5}{6}$.

18. $\int_0^2 \left(e^t - e^{-t}\right)dt = \left(e^t + e^{-t}\right)\big|_0^2 = \left(e^2+e^{-2}\right) - (1+1) = e^2 + e^{-2} - 2$.

19. Put $u = x^2 + 1$, so $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. Then $\int_{-1}^{1} xe^{x^2+1}\,dx = \frac{1}{2}\int_{2}^{2} e^u\,du = \frac{1}{2}e^u\Big|_2^2 = 0$ because the upper and lower limits are equal.

20. Let $u = \sqrt{x}$, so $du = \dfrac{1}{2\sqrt{x}}\,dx$. If $x = 1$, then $u = 1$, and if $x = 4$, then $u = 2$, so
$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int_1^2 e^u\,du = 2e^u\big|_1^2 = 2\left(e^2 - e\right) = 2e\,(e-1).$$

21. Let $u = x - 2$, so $du = dx$. If $x = 3$, then $u = 1$ and if $x = 6$, then $u = 4$, so
$$\int_3^6 \frac{1}{x-2}\,dx = \int_1^4 \frac{du}{u} = \ln|u|\big|_1^4 = \ln 4.$$

22. Let $u = 1 + 2x^2$, so $du = 4x\,dx$ and $x\,dx = \frac{1}{4}\,du$. If $x = 0$, then $u = 1$ and if $x = 1$, then $u = 3$, so
$$\int_0^1 \frac{x}{1+2x^2}\,dx = \frac{1}{4}\int_1^3 \frac{du}{u} = \frac{1}{4}\ln|u|\Big|_1^3 = \frac{1}{4}\ln 3.$$

23. Let $u = x^3 + 3x^2 - 1$, so $du = \left(3x^2 + 6x\right)dx = 3\left(x^2 + 2x\right)dx$. If $x = 1$, then $u = 3$, and if $x = 2$, then $u = 19$, so $\displaystyle\int_1^2 \frac{x^2+2x}{x^3+3x^2-1}\,dx = \frac{1}{3}\int_3^{19}\frac{du}{u} = \frac{1}{3}\ln u\Big|_3^{19} = \frac{1}{3}\left(\ln 19 - \ln 3\right).$

24. $\displaystyle\int_0^1 \frac{e^x}{1+e^x}\,dx = \ln\left(1+e^x\right)\big|_0^1 = \ln(1+e) - \ln 2 = \ln\frac{1+e}{2}.$

25. $\displaystyle\int_1^2 \left(4e^{2u} - \frac{1}{u}\right)du = 2e^{2u} - \ln u\big|_1^2 = \left(2e^4 - \ln 2\right) - \left(2e^2 - 0\right) = 2e^4 - 2e^2 - \ln 2.$

26. $\displaystyle\int_1^2 \left(1 + \frac{1}{x} + e^x\right)dx = \left(x + \ln x + e^x\right)\big|_1^2 = \left(2 + \ln 2 + e^2\right) - (1+e) = 1 + \ln 2 + e^2 - e.$

27. $\displaystyle\int_1^2 \left(2e^{-4x} - x^{-2}\right)dx = \left(-\tfrac{1}{2}e^{-4x} + \tfrac{1}{x}\right)\Big|_1^2 = \left(-\tfrac{1}{2}e^{-8} + \tfrac{1}{2}\right) - \left(-\tfrac{1}{2}e^{-4} + 1\right) = -\tfrac{1}{2}e^{-8} + \tfrac{1}{2}e^{-4} - \tfrac{1}{2}$
$\displaystyle = \tfrac{1}{2}\left(e^{-4} - e^{-8} - 1\right).$

28. Let $u = \ln x$, so $du = \frac{1}{x}\,dx$. If $x = 1$, then $u = 0$ and if $x = 2$, then $u = \ln 2$, so
$$\int_1^2 \frac{\ln x}{x}\,dx = \int_0^{\ln 2} u\,du = \frac{1}{2}u^2\Big|_0^{\ln 2} = \frac{1}{2}(\ln 2)^2.$$

29. $A = \int_{-1}^{2}\left(x^2 - 2x + 2\right)dx = \left(\frac{1}{3}x^3 - x^2 + 2x\right)\Big|_{-1}^{2} = \left(\frac{8}{3} - 4 + 4\right) - \left(-\frac{1}{3} - 1 - 2\right) = 6.$

30. $A = \int_0^1 \left(x^3 + x\right)dx = \left(\frac{1}{4}x^4 + \frac{1}{2}x^2\right)\Big|_0^1 = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}.$

31. $\displaystyle A = \int_1^2 \frac{dx}{x^2} = \int_1^2 x^{-2}\,dx = -\frac{1}{x}\Big|_1^2 = \frac{1}{4} - (-1) = \frac{1}{2}.$

32. $A = \int_0^3 \left(2 + \sqrt{x+1}\right)dx = \int_0^3 2\,dx + \int_0^3 \sqrt{x+1}\,dx$. Let $u = x + 1$ in the second integral, so $du = dx$. If $x = 0$, then $u = 1$ and if $x = 3$, then $u = 4$. Thus, $A = 2x\big|_0^3 + \int_1^4 u^{1/2}\,du = 6 + \left(\frac{2}{3}u^{3/2}\right)\Big|_1^4 = 6 + \frac{2}{3}(8-1) = \frac{32}{3}.$

33. $A = \int_{-1}^{2} e^{-x/2}\,dx = -2e^{-x/2}\big|_{-1}^{2} = -2\left(e^{-1} - e^{1/2}\right) = 2\left(\sqrt{e} - 1/e\right).$

34. The required area is $A = \displaystyle\int_1^2 f(x)\,dx = \frac{1}{4}\int_1^2 \frac{\ln x}{x}\,dx = \frac{1}{8}(\ln x)^2\Big|_1^2 = \frac{1}{8}\left[(\ln 2)^2 - (\ln 1)^2\right] = \frac{1}{8}(\ln 2)^2.$

35. The average value is $\frac{1}{2}\int_0^2 (2x+3)\,dx = \frac{1}{2}\left(x^2+3x\right)\Big|_0^2 = \frac{1}{2}(10) = 5.$

36. The average value is

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-1}\int_1^4 (8-x)dx = \frac{1}{3}\int_1^4 (8-x)\,dx = \frac{1}{3}\left(8x - \frac{1}{2}x^2\right)\Big|_1^4 = \frac{1}{3}\left[(32-8) - \left(8 - \frac{1}{2}\right)\right] = \frac{11}{2}.$$

37. The average value is $\frac{1}{2}\int_1^3 \left(2x^2-3\right)dx = \frac{1}{2}\left(\frac{2}{3}x^3 - 3x\right)\Big|_1^3 = \frac{1}{2}\left(9 + \frac{7}{3}\right) = \frac{17}{3}.$

38. The average value is $\frac{1}{5}\int_{-2}^{3}\left(4-x^2\right)dx = \frac{1}{5}\left(4x - \frac{1}{3}x^3\right)\Big|_{-2}^{3} = \frac{1}{5}\left[(12-9) - \left(-8 + \frac{8}{3}\right)\right] = \frac{5}{3}.$

39. The average value is

$$\frac{1}{3}\int_{-1}^{2}\left(x^2+2x-3\right)dx = \frac{1}{3}\left(\frac{1}{3}x^3 + x^2 - 3x\right)\Big|_{-1}^{2} = \frac{1}{3}\left[\left(\frac{8}{3}+4-6\right) - \left(-\frac{1}{3}+1+3\right)\right]$$
$$= \frac{1}{3}\left(\frac{8}{3} - 2 + \frac{1}{3} - 4\right) = -1.$$

40. The average value is $\frac{1}{2}\int_{-1}^{1} x^3\,dx = \frac{1}{2}\cdot\frac{1}{4}x^4\Big|_{-1}^{1} = \frac{1}{2}\left(\frac{1}{4} - \frac{1}{4}\right) = 0.$

41. The average value is $\frac{1}{4}\int_0^4 (2x+1)^{1/2}\,dx = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)(2x+1)^{3/2}\Big|_0^4 = \frac{1}{12}(27-1) = \frac{13}{6}.$

42. The average value is $\frac{1}{4-0}\int_0^4 e^{-x}\,dx = -\frac{1}{4}e^{-x}\Big|_0^4 = -\frac{1}{4}\left(e^{-4}-1\right) = \frac{1}{4}\left(1-e^{-4}\right) \approx 0.245.$

43. The average value is $\frac{1}{2}\int_0^2 xe^{x^2}\,dx = \dfrac{1}{4}e^{x^2}\Big|_0^2 = \frac{1}{4}\left(e^4-1\right).$

44. The average value is $\dfrac{1}{2}\displaystyle\int_0^2 \frac{dx}{x+1} = \frac{1}{2}\ln(x+1)\Big|_0^2 = \frac{1}{2}\ln 3.$

45. The distance traveled is $\int_0^4 3t\sqrt{16-t^2}\,dt = 3\left(-\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(16-t^2\right)^{3/2}\Big|_0^4 = 64$ ft.

46. The amount of oil that the well can be expected to yield is

$$\int_0^5 \left(\frac{600t^2}{t^3+32} + 5\right)dt = 600\int_0^5 \frac{t^2}{t^3+32}\,dt + (5t)\big|_0^5 = 600\left(\frac{1}{3}\right)\ln\left(t^3+32\right)\Big|_0^5 + 25$$
$$= 200\,(\ln 157 - \ln 32) + 25 \approx 343, \text{ or 343 thousand barrels.}$$

47. The amount is $\int_1^2 t\left(\frac{1}{2}t^2+1\right)^{1/2} dt$. Let $u = \frac{1}{2}t^2 + 1$, so $du = t\,dt$. Then

$$\int_1^2 t\left(\frac{1}{2}t^2+1\right)^{1/2} dt = \int_{3/2}^{3} u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big|_{3/2}^{3} = \frac{2}{3}\left[(3)^{3/2} - \left(\frac{3}{2}\right)^{3/2}\right] \approx \$2.24 \text{ million.}$$