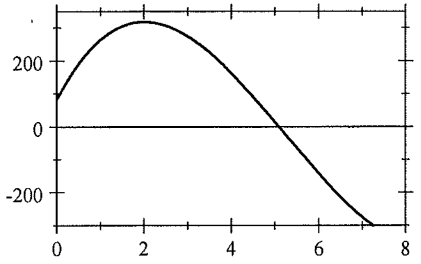
9. a. $N(t) = 1.2576t^4 - 26.357t^3 + 127.98t^2 + 82.3t + 43$, so $N'(t) = 5.0304t^3 - 79.071t^2 + 255.96t + 82.3$.

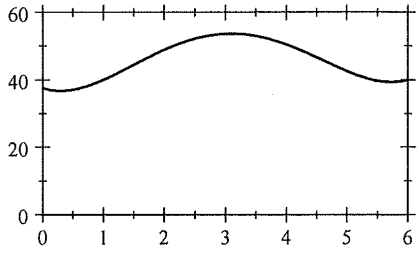


From the graph, we see that $N'(t)$ has a maximum when $t \approx 2$, on February 8.

b. From the graph in part (a), the maximum number of sickouts occurred when $N'(t) = 0$, that is, when $t = 5$. We calculate $N(5) \approx 1145$ canceled flights.

10. a. $A(t) = 0.28636t^4 - 3.4864t^3 + 11.689t^2 - 6.08t + 37.6$.

b.



c. The average account balance was lowest in the second year and highest early in the fourth year.

d. The lowest average balance was approximately \$37,000 (according to the model), and the highest was approximately \$55,500 (according to the figure for the third year).

4.5 Optimization II

Concept Questions

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1. We could solve the problem by sketching the graph of f and checking to see if there is an absolute extremum.

2. $S = 2\pi r^2 + 2\pi rh$. From $\pi r^2 h = 54$, we see that $r = \left(\frac{54}{\pi h}\right)^{1/2}$. Therefore,

$$S = 2\pi\left(\frac{54}{\pi h}\right) + 2\pi h\sqrt{\frac{54}{\pi}}\cdot\frac{1}{h^{1/2}} = \frac{108}{h} + 2\sqrt{54\pi}h^{1/2}, \text{ so } S' = -\frac{108}{h^2} + 2\sqrt{54\pi}\left(\frac{1}{2}h^{-1/2}\right) = -\frac{108}{h^2} + \frac{\sqrt{54\pi}}{h^{1/2}} = 0$$

implies $\frac{108}{h^2} = \frac{\sqrt{54\pi}}{h^{1/2}}$, $h^{3/2} = \frac{108}{\sqrt{54\pi}}$. Thus, $h = \left[\frac{108}{(54\pi)^{1/2}}\right]^{2/3} = \frac{108^{2/3}}{(54\pi)^{1/3}} = \left(\frac{108}{54\pi}\right)^{1/3} = \frac{6}{\sqrt[3]{\pi}}$, as obtained in Example 4. Writing S in terms of r seems to be a better choice.

Exercises

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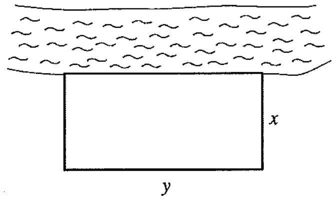
1. Let x and y denote the lengths of two adjacent sides of the rectangle. We want to maximize $A = xy$. But the perimeter is $2x + 2y$ and this is equal to 100, so $2x + 2y = 100$, and therefore $y = 50 - x$. Thus, $A = f(x) = x(50 - x) = -x^2 + 50x$, $0 \le x \le 50$. We allow the "degenerate"cases $x = 0$ and $x = 50$. $A' = -2x + 50 = 0$ implies that $x = 25$ is a critical number of f. $A(0) = 0$, $A(25) = 625$, and $A(50) = 0$, so we see that A is maximized for $x = 25$. The required dimensions are 25 ft by 25 ft.

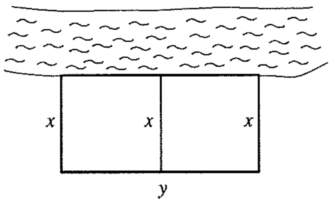
2. Let x and y denote the lengths of two adjacent sides of the rectangle. The quantity to be minimized is the perimeter of the rectangle, $P = 2x + 2y$. But $xy = 144$ and $y = \frac{144}{x}$, so $P = f(x) = 2x + \frac{288}{x}$ for $x > 0$, and $f'(x) = 2 - \frac{288}{x^2} = \frac{2x^2 - 288}{x^2}$, which has a zero when $2x^2 - 288 = 0$. The critical number in $(0, \infty)$ is $x = 12$. $f''(x) = \frac{576}{x^3} > 0$ on $(0, \infty)$, so f is concave upward. Therefore, $x = 12$ gives the absolute minimum value of f. The corresponding value of y is $y = \frac{144}{x} = \frac{144}{12} = 12$, so the dimensions are 12 ft by 12 ft.

3. We have $2x + y = 3000$ and we want to maximize the function $A = f(x) = xy = x(3000 - 2x) = 3000x - 2x^2$ on the interval $[0, 1500]$. The critical number of A is obtained by solving $f'(x) = 3000 - 4x = 0$, giving $x = 750$. From the table of values, we conclude that $x = 750$ yields the absolute maximum value of A. Thus, the required dimensions are 750×1500 yards. The maximum area is 1,125,000 yd^2.



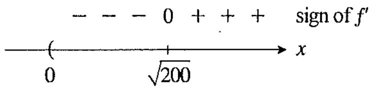
x	0	750	1500
$f(x)$	0	1,125,000	0

4. Let x denote the length of one of the sides. Then $y = 3000 - 3x = 3(1000 - x)$. The area is $A(x) = xy = 3x(1000 - x) = -3x^2 + 3000x$ for $0 \le x \le 1000$. Next, $A'(x) = -6x + 3000 = -6(x - 500)$. Setting $A'(x) = 0$ gives $x = 500$ as the critical number. From the table of values, we see that $f(500) = 750{,}000$ is the absolute maximum value. Next, $y = 3(1000 - 500) = 1500$. Therefore, the required dimensions are 500×1500 yd, and the area is 750,000 yd^2.



x	0	500	1000
$A(x)$	0	750,000	0

5. Let x denote the length of the side made of wood and y the length of the side made of steel. The cost of construction is $C = 6(2x) + 3y$, but $xy = 800$, so $y = \frac{800}{x}$. Therefore, $C = f(x) = 12x + 3\left(\frac{800}{x}\right) = 12x + \frac{2400}{x}$. To minimize C, we compute $f'(x) = 12 - \frac{2400}{x^2} = \frac{12x^2 - 2400}{x^2} = \frac{12(x^2 - 200)}{x^2}$. Setting $f'(x) = 0$ gives $x = \pm\sqrt{200}$ as critical numbers of f. The sign diagram of f' shows that $x = \pm\sqrt{200}$ gives a relative minimum of f. $f''(x) = \frac{4800}{x^3} > 0$ if $x > 0$, and so f is concave upward for $x > 0$. Therefore, $x = \sqrt{200} = 10\sqrt{2}$ yields the absolute minimum. Thus, the dimensions of the enclosure should be $10\sqrt{2}$ ft × $40\sqrt{2}$ ft, or 14.1 ft × 56.6 ft.



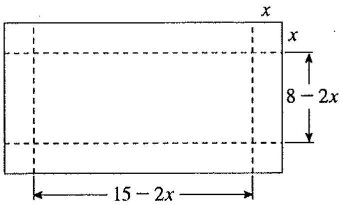
6. The volume of the box is given by

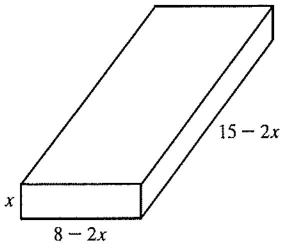
$V = f(x) = (8-2x)(15-2x)x = 4x^3 - 46x^2 + 120x.$

Because the sides of the box must be nonnegative, we must have $8 - 2x \geq 0$, so $x \leq 4$ and $15 - 2x \geq 0$, so $x \leq \frac{15}{2}$. The problem is equivalent to finding the absolute maximum of f on $[0, 4]$. Now

$$\begin{aligned} f'(x) &= 12x^2 - 92x + 120 \\ &= 4\left(3x^2 - 23x + 30\right) \\ &= 4(3x-5)(x-6), \end{aligned}$$

so $f'(x) = 0$ implies $x = \frac{5}{3}$ or $x = 6$. Because $x = 6$ is outside the interval $[0, 4]$, only $x = \frac{5}{3}$ qualifies as a critical number of f. From the table of values, we see that $x = \frac{5}{3}$ gives rise to an absolute maximum of f. Thus, the dimensions which yield the maximum volume are $\frac{14}{3}'' \times \frac{35}{3}'' \times \frac{5}{3}''$. The maximum volume is $\frac{2450}{27}$, or approximately 90.7 cubic inches.



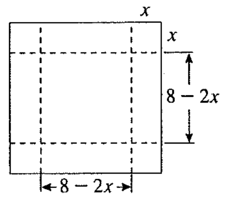


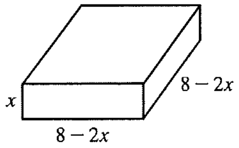
x	0	$\frac{5}{3}$	4
$f(x)$	0	$\frac{2450}{27}$	0

7. Let the dimensions of each square that is cut out be $x'' \times x''$. Then the dimensions of the box are $(8-2x)''$ by $(8-2x)''$ by x'', and its volume is be $V = f(x) = x(8-2x)^2$. We want to maximize f on $[0, 4]$.

$$\begin{aligned} f'(x) &= (8-2x)^2 + x(2)(8-2x)(-2) \quad \text{(by the Product Rule)} \\ &= (8-2x)[(8-2x) - 4x] \\ &= (8-2x)(8-6x) \\ &= 0 \text{ if } x = 4 \text{ or } \tfrac{4}{3}. \end{aligned}$$

The latter is a critical number in $(0, 4)$. From the table, we see that $x = \frac{4}{3}$ yields an absolute maximum for f, so the dimensions of the box should be $\frac{16}{3}'' \times \frac{16}{3}'' \times \frac{4}{3}''$.

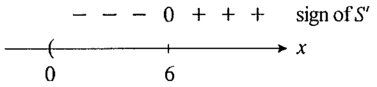




x	0	$\frac{4}{3}$	4
$f(x)$	0	$\frac{1024}{27}$	0

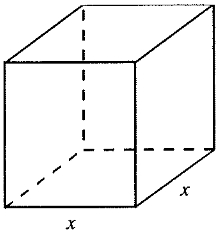
8. Let the dimensions of the box be $x'' \times x'' \times y''$. Because its volume is 108 cubic inches, we have $x^2y = 108$. We want to minimize $S = x^2 + 4xy$. But $y = 108/x^2$, so we minimize $S = x^2 + 4x\left(\frac{108}{x^2}\right) = x^2 + \frac{432}{x}$ for $x > 0$.

Now $S' = 2x - \frac{432}{x^2} = \frac{2\left(x^3 - 216\right)}{x^2}$. Setting $S' = 0$ gives $x = 6$ as a critical number of S. The sign diagram shows that $x = 6$ gives a relative minimum of S. Next, $S'' = 2 + \frac{864}{x^3} > 0$ for $x > 0$, and this says that S is concave upward on $(0, \infty)$. Therefore, $x = 6$ gives an absolute minimum, and so the dimensions of the box should be $6'' \times 6'' \times 3''$.



9. Let x denote the length of a side of the base and y the height of the cup, both measured in inches. Then the cost of constructing the cup is $C = 40x^2 + 15(4xy) = 40x^2 + 60xy$ (cents). The volume of the cup is 36 cubic inches, and so $x^2y = 36$ and $y = \frac{36}{x^2}$. Therefore, $C(x) = 40x^2 + 60x \cdot \frac{36}{x^2} = 40x^2 + \frac{2160}{x}$ and $C'(x) = 80x - \frac{2160}{x^2}$. Setting $C'(x) = 0$ gives $x^3 = 27$, so $x = 3$. Because $C''(3) = \left[80 + \frac{4320}{x^3}\right]_{x=3} > 0$, we see that $x = 3$ gives a relative (and absolute) minimum of C. Also, $y = \frac{36}{3^2} = 4$, so the required dimensions are 3″ × 3″ × 4″.

10. Let x denote the length of the sides of the box and y denote its height. Referring to the figure, we see that the volume of the box is given by $x^2y = 128$. The amount of material used is given by $S = f(x) = 2x^2 + 4xy = 2x^2 + 4x\left(\frac{128}{x^2}\right) = 2x^2 + \frac{512}{x}$ in^2. We want to minimize f subject to the condition that $x > 0$.



Now $f'(x) = 4x - \frac{512}{x^2} = \frac{4x^3 - 512}{x^2} = \frac{4(x^3 - 128)}{x^2}$.Setting $f'(x) = 0$ yields $x = 5.04$, a critical number of f. Next, $f''(x) = 4 + \frac{1024}{x^3} > 0$ for all $x > 0$. Thus, the graph of f is concave upward, and so $x = 5.04$ yields an absolute minimum of f. The required dimensions are 5.04″× 5.04″ × 5.04″.

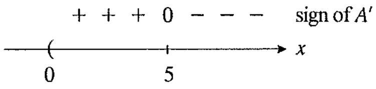
11. From the given figure, we see that $x^2y = 20$ and $y = 20/x^2$, and so $C = 30x^2 + 10(4xy) + 20x^2 = 50x^2 + 40x\left(\frac{20}{x^2}\right) = 50x^2 + \frac{800}{x}$. To find the critical numbers of C, we solve $C' = 100x - \frac{800}{x^2} = 0$, obtaining $100x^3 = 800$, $x^3 = 8$, and $x = 2$. Next, $C'' = \frac{1600}{x^3} > 0$ for all $x > 0$, so we see that $x = 2$ gives the absolute minimum value of C. Because $y = \frac{20}{4} = 5$, we see that the dimensions are 2 ft × 2 ft × 5 ft.

12. The length plus the girth of the box is $4x + h = 108$ and $h = 108 - 4x$. Then $V = x^2h = x^2(108 - 4x) = 108x^2 - 4x^3$ and $V' = 216x - 12x^2$. We want to maximize V on the interval $[0, 27]$. Setting $V'(x) = 0$ and solving for x, we obtain $x = 18$ and $x = 0$. We calculate $V(0) = 0$, $V(18) = 11{,}664$, and $V(27) = 0$. Thus, the dimensions of the box are 18″ × 18″ × 36″ and its maximum volume is approximately 11,664 in^3.

13. $xy = 50$ and so $y = 50/x$. The printed area is

$$A = (x-1)(y-2) = (x-1)\left(\frac{50}{x} - 2\right) = (x-1)\left(\frac{50-2x}{x}\right) = -2x + 52 - \frac{50}{x},\text{ so}$$

$A' = -2 + \frac{50}{x^2} = \frac{-2(x^2 - 25)}{x^2} = 0$ if $x = \pm 5$. From the sign diagram for A', we see that $x = 5$ yields a maximum. Because $A'' = -\frac{100}{x^3} < 0$ for $x > 0$, we see that the graph of A is concave downward on $(0, \infty)$ and so $x = 5$ yields an absolute maximum. The dimensions of the paper should therefore be 5″ × 10″.



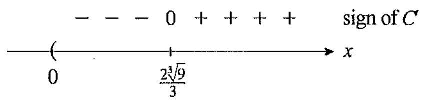
14. We take $2\pi r + \ell = 108$. We want to maximize $V = \pi r^2 \ell = \pi r^2 (-2\pi r + 108) = -2\pi^2 r^3 + 108\pi r^2$ subject to the condition that $0 \le r \le \frac{54}{\pi}$. Now $V'(r) = -6\pi^2 r^2 + 216\pi r = -6\pi r(\pi r - 36) = 0$ implies that $r = 0$ and $r = \frac{36}{\pi}$ are critical numbers of V. From the table, we conclude that the maximum volume occurs when $r = \frac{36}{\pi} \approx 11.5$ inches and $\ell = 108 - 2\pi\left(\frac{36}{\pi}\right) = 36$ inches and the volume of the parcel is $46{,}656/\pi$ in^3.

r	0	$\frac{36}{\pi}$	$\frac{54}{\pi}$
V	0	$\frac{46{,}656}{\pi}$	0

15. Denote the radius and height of the cup (in inches) by r and h respectively. Let k denote the price (in cents per square inch) of the material for the base of the cup. Then the cost of constructing the cup is $C = k\pi r^2 + \frac{3}{8}k(2\pi rh) = k\pi\left(r^2 + \frac{3}{4}rh\right)$. It suffices to minimize $F(r) = \dfrac{C(r)}{k\pi} = r^2 + \dfrac{3rh}{4}$. But $\pi r^2 h = 9\pi$, and so $h = \dfrac{9}{r^2}$. Thus, $F(r) = r^2 + \dfrac{3r}{4}\left(\dfrac{9}{r^2}\right) = r^2 + \dfrac{27}{4r}$. Now $F'(r) = 2r - \dfrac{27}{4r^2} = 0$ gives $8r^3 = 27$, so $r^3 = \frac{27}{8}$ and $r = \frac{3}{2}$. Because $F''(r) = 2 + \dfrac{27}{2r^2}$, we see that $F''\left(\frac{3}{2}\right) > 0$, and so F has a minimum at $r = \frac{3}{2}$. Also, $h = \frac{9}{(3/2)^2} = 4$, and so the required dimensions are a radius of 1.5 inches and a height of 4 inches.

16. Let r and h denote the radius and height of the container. Because its capacity is to be 36 in^3, we have $\pi r^2 h = 36$ or $h = 36/\pi r^2$. We want to minimize $S = 2\pi r^2 + 2\pi rh$ or $S = f(r) = 2\pi r^2 + 2\pi r\left(\dfrac{36}{\pi r^2}\right) = 2\pi r^2 + \dfrac{72}{r}$ over the interval $(0, \infty)$. Now $f'(r) = 4\pi r - \dfrac{72}{\pi r^2} = 0$ gives $4\pi r^3 = 72$, or $r = \left(\frac{18}{\pi}\right)^{1/3}$, as the only critical number of f. Next, observe that $f''(r) = 4\pi + \frac{144}{\pi r} > 0$ for r in $(0, \infty)$. Thus, f is concave upward on $(0, \infty)$ and $r = \left(\frac{18}{\pi}\right)^{1/3}$ gives rise to the absolute minimum of f. We find $h = \dfrac{36}{\pi\left(\frac{18}{\pi}\right)^{2/3}} = \dfrac{2 \cdot 18}{\pi^{1/3} 18^{2/3}} = 2\left(\dfrac{18}{\pi}\right)^{1/3}$ or twice the radius.

17. Let y denote the height and x the width of the cabinet. Then $y = \frac{3}{2}x$. Because the volume is to be 2.4 ft^3, we have $xyd = 2.4$, where d *is* the depth of the cabinet. Thus, $x\left(\frac{3}{2}x\right)d = 2.4$, so $d = \dfrac{2.4(2)}{3x^2} = \dfrac{1.6}{x^2}$. The cost for constructing the cabinet is $C = 40(2xd + 2yd) + 20(2xy) = 80\left[\dfrac{1.6}{x} + \left(\dfrac{3x}{2}\right)\left(\dfrac{1.6}{x^2}\right)\right] + 40x\left(\dfrac{3x}{2}\right) = \dfrac{320}{x} + 60x^2$, so $C'(x) = -\dfrac{320}{x^2} + 120x = \dfrac{120x^3 - 320}{x^2} = 0$ if $x = \sqrt[3]{\frac{8}{3}} = \frac{2}{\sqrt[3]{3}} = \frac{2}{3}\sqrt[3]{9}$. Therefore, $x = \frac{2}{3}\sqrt[3]{9}$ is a critical number of C. The sign diagram shows that $x = \frac{2}{3}\sqrt[3]{9}$ gives a relative minimum. Next, $C''(x) = \dfrac{640}{x^3} + 120 > 0$ for all $x > 0$, telling us that the graph of C is concave upward, so $x = \frac{2}{3}\sqrt[3]{9}$ yields an absolute minimum. The required dimensions are $\frac{2}{3}\sqrt[3]{9}' \times \sqrt[3]{9}' \times \frac{2}{5}\sqrt[3]{9}'$.



18. Because the perimeter of the window is 28 ft, we have $2x + 2y + \pi x = 28$ or $y = \frac{1}{2}(28 - \pi x - 2x)$. We want to maximize $A = 2xy + \frac{1}{2}\pi x^2 = \frac{1}{2}\pi x^2 + x(28 - \pi x - 2x) = \frac{1}{2}\pi x^2 + 28x - \pi x^2 - 2x^2 = 28x - \frac{\pi}{2}x^2 - 2x^2$. Now $A' = 28 - \pi x - 4x = 0$ gives $x = \frac{28}{4+\pi}$ as a critical number of A. Because $A'' = -\pi - 4 < 0$, the point yields a maximum of A. Finally, $y = \dfrac{1}{2}\left(28 - \dfrac{28\pi}{4+\pi} - \dfrac{56}{4+\pi}\right) = \dfrac{1}{2}\left(\dfrac{112 + 28\pi - 28\pi - 56}{4+\pi}\right) = \dfrac{28}{4+\pi}$.

19. Let x denote the number of passengers beyond the 200th. We want to maximize the function $R(x) = (200 + x)(300 - x) = -x^2 + 100x + 60{,}000$. Now $R'(x) = -2x + 100 = 0$ gives $x = 50$, and this is a critical number of R. Because $R''(x) = -2 < 0$, we see that $x = 50$ gives an absolute maximum of R. Therefore, the number of passengers should be 250. The fare will then be \$250/passenger and the revenue will be \$62,500.

20. Let x denote the number of trees beyond 22 per acre. Then the yield is $Y = (36 - 2x)(22 + x) = -2x^2 - 8x + 792$. Next, $Y' = -4x - 8 = 0$ gives $x = -2$ as the critical number of Y. Now $Y'' = -4 < 0$ and so $x = -2$ gives the absolute maximum of Y. So 20 trees/acre should be planted.

21. Let x denote the number of people beyond 20 who sign up for the cruise. Then the revenue is $R(x) = (20 + x)(600 - 4x) = -4x^2 + 520x + 12{,}000$. We want to maximize R on the closed bounded interval $[0, 70]$. $R'(x) = -8x + 520 = 0$ implies $x = 65$, a critical number of R. Evaluating R at this critical number and the endpoints, we see that R is maximized if $x = 65$. Therefore, 85 passengers will result in a maximum revenue of \$28,900. The fare in this case is \$340/passenger.

x	0	65	70
$R(x)$	12,000	28,900	28,800

22. Let x denote the number of bottles beyond 10,000. Then the profit is $P(x) = (10{,}000 + x)(5 - 0.0002x) = -0.0002x^2 + 3x + 50{,}000$. We want to maximize P on $[0, \infty)$. $P'(x) = -0.0004x + 3 = 0$ implies $x = 7500$. Because $P''(x) = -0.0004 < 0$, the graph of P is concave downward, and we see that $x = 7500$ gives the absolute maximum of P. So Phillip should produce 17,500 bottles of wine for a profit of $P(7500) = -0.0002(7500)^2 + 3(7500) + 50{,}000$ or \$61,250.

23. The fuel cost is $x/600$ dollars per mile and the labor cost is $18/x$ dollars per mile. Therefore, the total cost is $C(x) = \dfrac{18}{x} + \dfrac{3x}{600}$. We calculate $C'(x) = -\dfrac{18}{x^2} + \dfrac{3}{600} = 0$, giving $-\dfrac{18}{x^2} = -\dfrac{3}{600}$, $3x^2 = 18(600)$, $x^2 = 3600$, and so $x = 60$. Next, $C''(x) = \dfrac{48}{x^3} > 0$ for all $x > 0$ so C is concave upward. Therefore, $x = 60$ gives the absolute minimum. The most economical speed is 60 mph.

24. Suppose the distance between the two ports is D miles. Then it takes the ship D/v hours to travel from one port to the other. Therefore, the total cost incurred in making the trip is $C = (a + bv^3)\left(\dfrac{1}{v}\right) = \dfrac{a}{v} + bv^2$ dollars. We want to minimize C for $v > 0$. Setting $C' = 0$, we have $C' = -\dfrac{a}{v^2} + 2bv = \dfrac{-a + 2bv^3}{v^2} = 0$, $2bv^3 = a$, and so $v = \left(\dfrac{a}{2b}\right)^{1/3}$. Because $C'' = \dfrac{2a}{v^3} + 2b > 0$ for $v > 0$, the graph of C is concave upward and so the critical number $\sqrt[3]{\dfrac{a}{2b}}$ gives rise to the absolute minimum value of C. So the ship should sail at $\sqrt[3]{\dfrac{a}{2b}}$ mph.

25. We want to maximize $S = kh^2w$. But $h^2 + w^2 = 24^2$, or $h^2 = 576 - w^2$, so $S = f(w) = kw\left(576 - w^2\right) = k\left(576w - w^3\right)$. Now, setting $f'(w) = k\left(576 - 3w^2\right) = 0$ gives $w = \pm\sqrt{192} \approx \pm 13.86$. Only the positive root is a critical number of interest. Next, we find $f''(w) = -6kw$, and in particular, $f''\left(\sqrt{192}\right) = -6\sqrt{192}k < 0$, so that $w \approx 13.86$ gives a relative maximum of f. Because $f''(w) < 0$ for $w > 0$, we see that the graph of f is concave downward on $(0, \infty)$, and so $w = \sqrt{192}$ gives an absolute maximum of f. We find $h^2 = 576 - 192 = 384$ and so $h \approx 19.60$, so the width and height of the log should be approximately 13.86 inches and 19.60 inches, respectively.

26. We want to minimize $S = 3\pi r^2 + 2\pi rh$. But $\pi r^2 h + \frac{2}{3}\pi r^3 = 504\pi$, or $h = \dfrac{1}{r^2}\left(504 - \frac{2}{3}r^3\right)$. Therefore, $S = f(r) = 3\pi r^2 + 2\pi r \cdot \dfrac{1}{r^2}\left(504 - \frac{2}{3}r^3\right) = 3\pi r^2 + \dfrac{1008\pi}{r} - \dfrac{4\pi r^2}{3} = \dfrac{5\pi r^2}{3} + \dfrac{1008\pi}{r}$. Now $f'(r) = \dfrac{10\pi r}{3} - \dfrac{1008\pi}{r^2} = \dfrac{10\pi r^3 - 3024\pi}{3r^2}$, so $f'(r) = 0$ if $r^3 = \frac{3024\pi}{10\pi}$. Thus, $r = \left(\frac{1512}{5}\right)^{1/3} \approx 6.7$ is a critical number of f. Because $f''(r) = \dfrac{10\pi}{3} + \dfrac{2016\pi}{r^3} > 0$ for all r in $(0, \infty)$, we see that $r \approx 6.7$ does yield an absolute minimum of h. Therefore, the radius should be approximately 6.7 ft and the height should be approximately 6.7 ft.

27. We want to minimize $C(x) = 1.50\,(10{,}000 - x) + 2.50\sqrt{3000^2 + x^2}$ subject to $0 \le x \le 10{,}000$. Now $C'(x) = -1.50 + 2.5\left(\frac{1}{2}\right)\left(9{,}000{,}000 + x^2\right)^{-1/2}(2x) = -1.50 + \dfrac{2.50x}{\sqrt{9{,}000{,}000 + x^2}} = 0$ if $2.5x = 1.50\sqrt{9{,}000{,}000 + x^2}$, or $6.25x^2 = 2.25\left(9{,}000{,}000 + x^2\right)$, or $4x^2 = 20{,}250{,}000$, giving $x = 2250$. From the table, we see that $x = 2250$, or 2250 ft, gives the absolute minimum.

x	0	2250	10,000
$C(x)$	22,500	21,000	26,101

28. We need to maximize $\hat{V} = \dfrac{16r^2}{\left(r + \frac{1}{2}\right)^2} - r^2$. Now

$$\hat{V}' = \frac{\left(r + \frac{1}{2}\right)^2 (32r) - 16r^2 \cdot 2\left(r + \frac{1}{2}\right)}{\left(r + \frac{1}{2}\right)^4} - 2r = \frac{32r\left(r + \frac{1}{2}\right) - 32r^2\left(r + \frac{1}{2} - r\right)}{\left(r + \frac{1}{2}\right)^3} - 2r = \frac{16r - 2r\left(r + \frac{1}{2}\right)^3}{\left(r + \frac{1}{2}\right)^3}$$

$$= \frac{2r\left[8 - \left(r + \frac{1}{2}\right)^3\right]}{\left(r + \frac{1}{2}\right)^3}.$$

$\hat{V}' = 0$ implies $8 - \left(r + \frac{1}{2}\right)^3 = 0$, $\left(r + \frac{1}{2}\right)^3 = 8$, $r + \frac{1}{2} = 2$, and $r = \frac{3}{2}$. Next,

$$\hat{V}\left(\tfrac{3}{2}\right) = \frac{16\left(\frac{3}{2}\right)^2}{2^2} - \left(\tfrac{3}{2}\right)^2 = \left(\tfrac{3}{2}\right)^2(4 - 1) = 3\left(\tfrac{9}{4}\right) = \tfrac{27}{4}, \text{ so } h = \frac{16}{\left(r + \frac{1}{2}\right)^2} - 1 = \tfrac{16}{4} - 1 = 3.$$

Thus, the dimensions are $r = \frac{3}{2}$ and $h = 3$. From the table, we see that $\hat{V}$ is maximized if $r = \frac{3}{2}$, so the radius is 1.5 ft and the height is 3 ft.

r	0	$\frac{3}{2}$	$\frac{7}{2}$
V	0	$\frac{27}{4}$	0

29. The time of flight is $T = f(x) = \dfrac{12-x}{6} + \dfrac{\sqrt{x^2+9}}{4}$, so $f'(x) = -\frac{1}{6} + \frac{1}{4}\left(\frac{1}{2}\right)\left(x^2+9\right)^{-1/2}(2x) = -\dfrac{1}{6} + \dfrac{x}{4\sqrt{x^2+9}} = \dfrac{3x-2\sqrt{x^2+9}}{12\sqrt{x^2+9}}$. Setting $f'(x) = 0$ gives $3x = 2\sqrt{x^2+9}$, $9x^2 = 4\left(x^2+9\right)$, and $5x^2 = 36$. Therefore, $x = \pm\frac{6}{\sqrt{5}} = \pm\frac{6\sqrt{5}}{5}$. Only the critical number $x = \frac{6\sqrt{5}}{5}$ is of interest. The nature of the problem suggests $x \approx 2.68$ gives an absolute minimum for T.

30. The time taken to get to Q is $T(x) = \dfrac{\sqrt{x^2+1}}{3} + \dfrac{10-x}{4}$ for $0 \le x \le 10$. Next, $T'(x) = \dfrac{1}{3}\dfrac{d}{dx}\left(x^2+1\right)^{1/2} + \dfrac{1}{4}\dfrac{d}{dx}(10-x) = \frac{1}{3}\left(\frac{1}{2}\right)\left(x^2+1\right)^{-1/2}(2x) - \dfrac{1}{4} = \dfrac{x}{3\sqrt{x^2+1}} - \dfrac{1}{4}$. Setting $T'(x) = 0$ gives $\dfrac{x}{3\sqrt{x^2+1}} = \dfrac{1}{4}$, $4x = 3\sqrt{x^2+1}$, $16x^2 = 9\left(x^2+1\right)$, $7x^2 = 9$, and $x = \frac{3\sqrt{7}}{7}$, since x must be positive. $T(0) = \frac{17}{6} \approx 2.83$, $T\left(\frac{3\sqrt{7}}{7}\right) \approx 2.72$, and $T(10) = \frac{\sqrt{101}}{3} \approx 3.35$. We see that $x = \frac{3\sqrt{7}}{7} \approx 1.134$ yields the absolute minimum value for T, so she should land at the point R located about 1.134 miles from P.

31. The area enclosed by the rectangular region of the racetrack is $A = (\ell)(2r) = 2r\ell$. The length of the racetrack is $2\pi r + 2\ell$, and is equal to 1760. That is, $2(\pi r + \ell) = 1760$, and $\pi r + \ell = 880$. Therefore, we want to maximize $A = f(r) = 2r(880 - \pi r) = 1760r - 2\pi r^2$. The restriction on r is $0 \le r \le \frac{880}{\pi}$. To maximize A, we compute $f'(r) = 1760 - 4\pi r$. Setting $f'(r) = 0$ gives $r = \frac{1760}{4\pi} = \frac{440}{\pi} \approx 140$. Because $f(0) = f\left(\frac{880}{\pi}\right) = 0$, we see that the maximum rectangular area is enclosed if we take $r = \frac{440}{\pi}$ and $\ell = 880 - \pi\left(\frac{440}{\pi}\right) = 440$. So $r = 140$ and $\ell = 440$. The total area enclosed is $2r\ell + \pi r^2 = 2\left(\frac{440}{\pi}\right)(440) + \pi\left(\frac{440}{\pi}\right)^2 = \frac{2(440)^2}{\pi} + \frac{440^2}{\pi} = \frac{580{,}800}{\pi} \approx 184{,}874\text{ ft}^2$.

32. Let x denote the number of motorcycle tires in each order. We want to minimize $C(x) = 400\left(\dfrac{40{,}000}{x}\right) + x = \dfrac{16{,}000{,}000}{x} + x$. We compute $C'(x) = -\dfrac{16{,}000{,}000}{x^2} + 1 = \dfrac{x^2 - 16{,}000{,}000}{x^2}$. Setting $C'(x) = 0$ gives $x = 4000$, a critical number of C. Because $C''(x) = \dfrac{32{,}000{,}000}{x^3} > 0$ for all $x > 0$, we see that the graph of C is concave upward and so $x = 4000$ gives an absolute minimum of C. So there should be 10 orders per year, each for 4000 tires.

33. Let x denote the number of bottles in each order. We want to minimize $C(x) = 200\left(\dfrac{2{,}000{,}000}{x}\right) + \dfrac{x}{2}(0.40) = \dfrac{400{,}000{,}000}{x} + 0.2x$. We compute $C'(x) = -\dfrac{400{,}000{,}000}{x^2} + 0.2$. Setting $C'(x) = 0$ gives $x^2 = \dfrac{400{,}000{,}000}{0.2} = 2{,}000{,}000{,}000$, or $x = 44{,}721$, a critical number of C. $C''(x) = \dfrac{800{,}000{,}000}{x^3} > 0$ for all $x > 0$, and we see that the graph of C is concave upward and so $x = 44{,}721$ gives an absolute minimum of C. Therefore, there should be $2{,}000{,}000/x \approx 45$ orders per year (since we can not have fractions of an order.) Each order should be for $2{,}000{,}000/45 \approx 44{,}445$ bottles.

34. We want to minimize the function $C(x) = \dfrac{500{,}000{,}000}{x} + 0.2x + 500{,}000$ on the interval $(0, 1000000)$. Differentiating $C(x)$, we have $C'(x) = -\dfrac{500{,}000{,}000}{x^2} + 0.2$. Setting $C'(x) = 0$ and solving the resulting equation, we find $0.2x^2 = 500{,}000{,}000$ and $x = \sqrt{2{,}500{,}000{,}000}$ or $x = 50{,}000$. Next, we find $C''(x) = \dfrac{1{,}000{,}000{,}000}{x^3} > 0$ for all x, and so the graph of C is concave upward on $(0, \infty)$. Thus, $x = 50{,}000$ gives rise to the absolute minimum of C. The company should produce 50,000 containers of cookies per production run.

35. a. Because the sales are assumed to be steady and D units are expected to be sold per year, the number of orders per year is D/x. Because is costs $\$K$ per order, the ordering cost is KD/x. The purchasing cost is pD (cost per item times number purchased). Finally, the holding cost is $\frac{1}{2}xh$ (the average number on hand times holding cost per item). Therefore, $C(x) = \dfrac{KD}{x} + pD + \dfrac{hx}{2}$.

b. $C'(x) = -\dfrac{KD}{x^2} + \dfrac{h}{2} = 0$ implies $\dfrac{KD}{x^2} = \dfrac{h}{2}$, so $x^2 = \dfrac{2KD}{h}$ and $x = \pm\sqrt{\dfrac{2KD}{h}}$. We reject the negative root. So $x = \sqrt{\dfrac{2KD}{h}}$ is the only critical number. Next, $C''(x) = \dfrac{2KD}{x^3} > 0$ for $x > 0$, so $C''\left(\sqrt{\dfrac{2KD}{h}}\right) > 0$ and the Second Derivative Test shows that $x = \sqrt{\dfrac{2KD}{h}}$ does give a relative minimum. Because C is concave upward, this is also the absolute minimum.

36. a. We use the result of Exercise 35 with $D = 960$, $K = 10$, $p = 80$, and $h = 12$. Thus, the EOQ is $x = \sqrt{\dfrac{2KD}{h}} = \sqrt{\dfrac{2(10)(960)}{12}} = 40$.

b. The number of orders to be placed each year is $\frac{960}{40} = 24$.

c. The interval between orders is $\frac{12}{24} = \frac{1}{2}$, or one-half month.

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1. a. $f(x_1) < f(x_2)$ **b.** $f(x_1) > f(x_2)$

2. a. increasing **b.** $f'(x) < 0$ **c.** constant

3. a. $f(x) \le f(c)$ **b.** $f(x) \ge f(c)$

4. a. domain, $= 0$, exist **b.** critical number **c.** relative extremum

5. a. $f'(x)$ **b.** > 0 **c.** concavity **d.** relative maximum; relative extremum

6. $\pm\infty, \pm\infty$ **7.** 0, 0 **8.** b, b

9. a. $f(x) \le f(c)$, absolute maximum value **b.** $f(x) \ge f(c)$, open interval

10. continuous, absolute, absolute

CHAPTER 4 Review Exercises page 333

1. a. $f(x) = \frac{1}{3}x^3 - x^2 + x - 6$, so $f'(x) = x^2 - 2x + 1 = (x-1)^2$. $f'(x) = 0$ gives $x = 1$, the critical number of f. Now $f'(x) > 0$ for all $x \neq 1$. Thus, f is increasing on $(-\infty, \infty)$.

b. Because $f'(x)$ does not change sign as we move across the critical number $x = 1$, the First Derivative Test implies that $x = 1$ does not give a relative extremum of f.

c. $f''(x) = 2(x-1)$. Setting $f''(x) = 0$ gives $x = 1$ as a candidate for an inflection point of f. Because $f''(x) < 0$ for $x < 1$, and $f''(x) > 0$ for $x > 1$, we see that f is concave downward on $(-\infty, 1)$ and concave upward on $(1, \infty)$.

d. The results of part (c) imply that $\left(1, -\frac{17}{3}\right)$ is an inflection point.

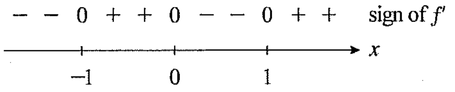
2. a. $f(x) = (x-2)^3$, so $f'(x) = 3(x-2)^2 > 0$ for all $x \neq 2$. Therefore, f is increasing on $(-\infty, \infty)$.

b. There is no relative extremum.

c. $f''(x) = 6(x-2)$. Because $f''(x) < 0$ if $x < 2$ and $f''(x) > 0$ if $x > 2$, we see that f is concave downward on $(-\infty, 2)$ and concave upward on $(2, \infty)$.

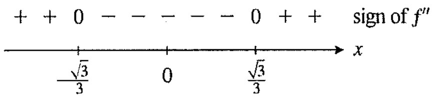
d. The results of part (c) show that $(2, 0)$ is an inflection point.

3. a. $f(x) = x^4 - 2x^2$, so $f'(x) = 4x^3 - 4x = 4x\left(x^2 - 1\right) = 4x(x+1)(x-1)$. The sign diagram of f' shows that f is decreasing on $(-\infty, -1)$ and $(0, 1)$ and increasing on $(-1, 0)$ and $(1, \infty)$.



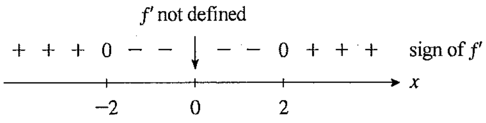
b. The results of part (a) and the First Derivative Test show that $(-1, -1)$ and $(1, -1)$ are relative minima and $(0, 0)$ is a relative maximum.

c. $f''(x) = 12x^2 - 4 = 4\left(3x^2 - 1\right) = 0$ if $x = \pm\frac{\sqrt{3}}{3}$. The sign diagram shows that f is concave upward on $\left(-\infty, -\frac{\sqrt{3}}{3}\right))$ and $\left(\frac{\sqrt{3}}{3}, \infty\right)$ and concave downward on $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$.



d. The results of part (c) show that $\left(-\frac{\sqrt{3}}{3}, -\frac{5}{9}\right)$ and $\left(\frac{\sqrt{3}}{3}, -\frac{5}{9}\right)$ are inflection points.

4. a. $f(x) = x + \dfrac{4}{x}$, so $f'(x) = 1 - \dfrac{4}{x^2} = \dfrac{x^2 - 4}{x^2} = \dfrac{(x-2)(x+2)}{x^2}$. Setting $f'(x) = 0$ gives $x = -2$ and $x = 2$ as critical numbers of f. $f'(x)$ is undefined at $x = 0$ as well. The sign diagram for f' shows that f is increasing on $(-\infty, -2)$ and $(2, \infty)$ and decreasing on $(-2, 0)$ and $(0, 2)$.



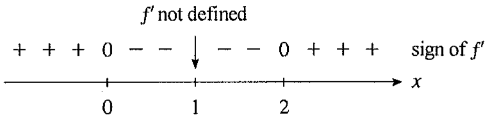
b. $f(-2) = -4$ is a relative maximum and $f(2) = 4$ is a relative minimum.

c. $f''(x) = \dfrac{8}{x^3}$. Because $f''(x) < 0$ for $x < 0$ and $f''(x) > 0$ for $x > 0$, we see that f is concave downward on $(-\infty, 0)$ and concave upward on $(0, \infty)$.

d. There is no inflection point. Note that $x = 0$ is not in the domain of f and is therefore not a candidate for an inflection point.

5. **a.** $f(x) = \dfrac{x^2}{x-1}$, so $f'(x) = \dfrac{(x-1)(2x) - x^2(1)}{(x-1)^2} = \dfrac{x^2-2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$.

The sign diagram of f' shows that f is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 1)$ and $(1, 2)$.



b. The results of part (a) show that (0, 0) is a relative maximum and (2, 4) is a relative minimum.

c. $f''(x) = \dfrac{(x-1)^2(2x-2) - x(x-2)2(x-1)}{(x-1)^4} = \dfrac{2(x-1)\left[(x-1)^2 - x(x-2)\right]}{(x-1)^4} = \dfrac{2}{(x-1)^3}$. Because $f''(x) < 0$ if $x < 1$ and $f''(x) > 0$ if $x > 1$, we see that f is concave downward on $(-\infty, 1)$ and concave upward on $(1, \infty)$.

d. Because $x = 1$ is not in the domain of f, there is no inflection point.

6. **a.** $f(x) = \sqrt{x-1}$, so $f'(x) = \frac{1}{2}(x-1)^{-1/2} = \dfrac{1}{2\sqrt{x-1}}$. Because $f'(x) > 0$ for $x > 1$, we see that f is increasing on $(1, \infty)$.

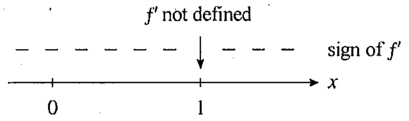
b. Because there is no critical numbers in $(1, \infty)$, f has no relative extremum.

c. $f''(x) = -\frac{1}{4}(x-1)^{-3/2} = -\dfrac{1}{4(x-1)^{3/2}} < 0$ if $x > 1$, and so f is concave downward on $(1, \infty)$.

d. There are no inflection points because $f''(x) \neq 0$ for all x in $(1, \infty)$.

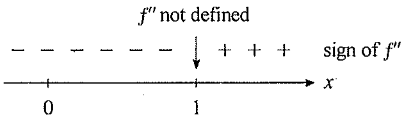
7. **a.** $f(x) = (1-x)^{1/3}$, so

$f'(x) = -\frac{1}{3}(1-x)^{-2/3} = -\dfrac{1}{3(1-x)^{2/3}}$. The sign diagram for f' shows that f is decreasing on $(-\infty, \infty)$.



b. There is no relative extremum.

c. $f''(x) = -\frac{2}{9}(1-x)^{-5/3} = -\dfrac{2}{9(1-x)^{5/3}}$. The sign diagram for f'' shows that f is concave downward on $(-\infty, 1)$ and concave upward on $(1, \infty)$.



d. $x = 1$ is a candidate for an inflection point of f. Referring to the sign diagram for f'', we see that (1, 0) is an inflection point.

8. **a.** $f(x) = x\sqrt{x-1} = x(x-1)^{1/2}$, so

$f'(x) = x\left(\frac{1}{2}\right)(x-1)^{-1/2} + (x-1)^{1/2} = \frac{1}{2}(x-1)^{-1/2}[x + 2(x-1)] = \dfrac{3x-2}{2(x-1)^{1/2}}$. Setting $f'(x) = 0$ gives $x = \frac{2}{3}$. But this point lies outside the domain of f, which is $[1, \infty)$. Thus, f has no critical number. Now, $f'(x) > 0$ for all $x \in (1, \infty)$ so f is increasing on $(1, \infty)$.

b. Because there is no critical numbers, f has no relative extremum.

c. $f''(x) = \frac{1}{2}\left[\frac{(x-1)^{1/2}(3) - (3x-2)\frac{1}{2}(x-1)^{-1/2}}{x-1}\right] = \frac{1}{2}\left[\frac{\frac{1}{2}(x-1)^{-1/2}[6(x-1)-(3x-2)]}{x-1}\right]$
$= \frac{3x-4}{4(x-1)^{3/2}}$.

$f''(x) = 0$ implies that $x = \frac{4}{3}$. $f''(x) < 0$ if $x < \frac{4}{3}$ and $f''(x) > 0$ if $x > \frac{4}{3}$, so f is concave downward on $\left(1, \frac{4}{3}\right)$ and concave upward on $\left(\frac{4}{3}, \infty\right)$.

d. From the results of part (c), we conclude that $\left(\frac{4}{3}, \frac{4\sqrt{3}}{9}\right)$ is an inflection point of f.

9. a. $f(x) = \frac{2x}{x+1}$, so $f'(x) = \frac{(x+1)(2) - 2x(1)}{(x+1)^2} = \frac{2}{(x+1)^2} > 0$ if $x \neq -1$. Therefore f is increasing on $(-\infty, -1)$ and $(-1, \infty)$.

b. Because there is no critical number, f has no relative extremum.

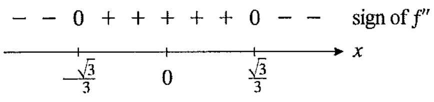
c. $f''(x) = -4(x+1)^{-3} = -\frac{4}{(x+1)^3}$. Because $f''(x) > 0$ if $x < -1$ and $f''(x) < 0$ if $x > -1$, we see that f is concave upward on $(-\infty, -1)$ and concave downward on $(-1, \infty)$.

d. There is no inflection point because $f''(x) \neq 0$ for all x in the domain of f.

10. a. $f(x) = -\frac{1}{1+x^2}$, so $f'(x) = \frac{2x}{\left(1+x^2\right)^2}$. Setting $f'(x) = 0$ gives $x = 0$ as the only critical number of f. For $x < 0$, $f'(x) < 0$ and for $x > 0$, $f'(x) > 0$. Therefore, f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

b. f has a relative minimum at $f(0) = -1$.

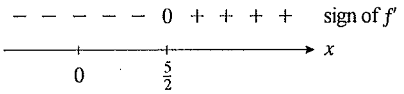
c. $f''(x) = \frac{\left(1+x^2\right)^2(2) - 2x(2)\left(1+x^2\right)(2x)}{\left(1+x^2\right)^4} = \frac{2\left(1+x^2\right)\left(1+x^2-4x^2\right)}{\left(1+x^2\right)^4} = -\frac{2\left(3x^2-1\right)}{\left(1+x^2\right)^3}$, and we see that $x = \pm\frac{\sqrt{3}}{3}$ are candidates for inflection points of f. The sign diagram for f'' shows that f is concave downward on $\left(-\infty, -\frac{\sqrt{3}}{3}\right)$ and $\left(\frac{\sqrt{3}}{3}, \infty\right)$ and concave upward on $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$.



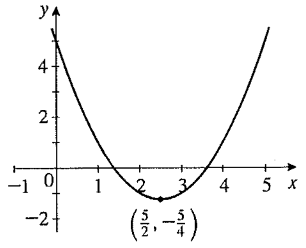
d. $\left(-\frac{\sqrt{3}}{3}, -\frac{3}{4}\right)$ and $\left(\frac{\sqrt{3}}{3}, -\frac{3}{4}\right)$ are inflection points of f.

11. $f(x) = x^2 - 5x + 5$.

1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 5 as the y-intercept.
3. $\lim_{x\to-\infty}\left(x^2 - 5x + 5\right) = \lim_{x\to\infty}\left(x^2 - 5x + 5\right) = \infty$.
4. There is no asymptote because f is a quadratic function.
5. $f'(x) = 2x - 5 = 0$ if $x = \frac{5}{2}$. The sign diagram shows that f is increasing on $\left(\frac{5}{2}, \infty\right)$ and decreasing on $\left(-\infty, \frac{5}{2}\right)$.

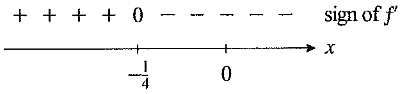


6. The First Derivative Test implies that $\left(\frac{5}{2}, -\frac{5}{4}\right)$ is a relative minimum.
7. $f''(x) = 2 > 0$ and so f is concave upward on $(-\infty, \infty)$.
8. There is no inflection point.

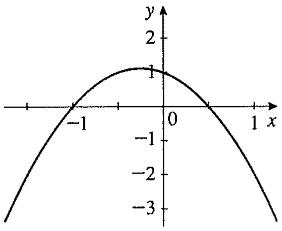


12. $f(x) = -2x^2 - x + 1$.

1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 1 as the y-intercept.
3. $\lim_{x\to-\infty}\left(-2x^2 - x + 1\right) = \lim_{x\to\infty}\left(-2x^2 - x + 1\right) = -\infty$.
4. There is no asymptote because f is a polynomial function.
5. $f'(x) = -4x - 1 = 0$ if $x = -\frac{1}{4}$. The sign diagram of f' shows that f is increasing on $\left(-\infty, -\frac{1}{4}\right)$ and decreasing on $\left(-\frac{1}{4}, \infty\right)$.

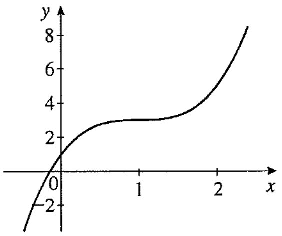


6. The results of step 5 show that $\left(-\frac{1}{4}, \frac{9}{8}\right)$ is a relative maximum.
7. $f''(x) = -4 < 0$ for all x in $(-\infty, \infty)$, and so f is concave downward on $(-\infty, \infty)$.
8. There is no inflection point.



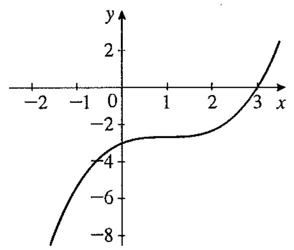
13. $g(x) = 2x^3 - 6x^2 + 6x + 1$.

1. The domain of g is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 1 as the y-intercept.
3. $\lim_{x\to-\infty} g(x) = -\infty$ and $\lim_{x\to\infty} g(x) = \infty$.
4. There is no vertical or horizontal asymptote.
5. $g'(x) = 6x^2 - 12x + 6 = 6\left(x^2 - 2x + 1\right) = 6(x-1)^2$. Because $g'(x) > 0$ for all $x \neq 1$, we see that g is increasing on $(-\infty, 1)$ and $(1, \infty)$.
6. $g'(x)$ does not change sign as we move across the critical number $x = 1$, so there is no extremum.
7. $g''(x) = 12x - 12 = 12(x-1)$. Because $g''(x) < 0$ if $x < 1$ and $g''(x) > 0$ if $x > 1$, we see that g is concave upward on $(1, \infty)$ and concave downward on $(-\infty, 1)$.
8. The point $x = 1$ gives rise to the inflection point $(1, 3)$.



14. $g(x) = \frac{1}{3}x^3 - x^2 + x - 3$

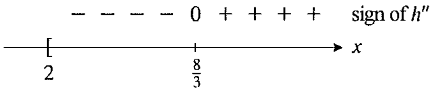
1. The domain of g is $(-\infty, \infty)$.
2. Setting $x = 0$ gives -3 as the y-intercept.
3. $\lim_{x \to -\infty} \left(\frac{1}{3}x^3 - x^2 + x - 3\right) = -\infty$ and $\lim_{x \to \infty} \left(\frac{1}{3}x^3 - x^2 + x - 3\right) = \infty$.
4. There is no asymptote because $g(x)$ is a polynomial.
5. $g'(x) = x^2 - 2x + 1 = (x-1)^2 = 0$ if $x = 1$, a critical number of g. Observe that $g'(x) > 0$ if $x \neq 1$, and so g is increasing on $(-\infty, 1)$ and $(1, \infty)$.
6. The results of step 5 show that there is no relative extremum.
7. $g'(x) = 2x - 2 = 2(x-1) = 0$ if $x = 1$. Observe that $g'(x) < 0$ if $x < 1$ and $g''(x) > 0$ if $x > 1$ and so g is concave downward on $(-\infty, 1)$ and concave upward on $(1, \infty)$.
8. The results of step 7 show that $\left(1, -\frac{8}{3}\right)$ is an inflection point.



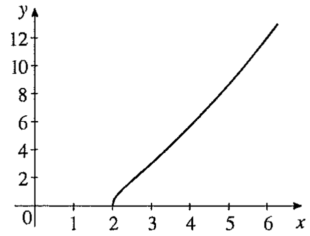
15. $h(x) = x\sqrt{x-2}$.

1. The domain of h is $[2, \infty)$.
2. There is no y-intercept. Setting $y = 0$ gives 2 as the x-intercept.
3. $\lim_{x \to \infty} x\sqrt{x-2} = \infty$.
4. There is no asymptote.
5. $h'(x) = (x-2)^{1/2} + x\left(\frac{1}{2}\right)(x-2)^{-1/2} = \frac{1}{2}(x-2)^{-1/2}[2(x-2)+x] = \dfrac{3x-4}{2\sqrt{x-2}} > 0$ on $[2, \infty)$, and so h is increasing on $[2, \infty)$.
6. Because h has no critical number in $(2, \infty)$, there is no relative extremum.
7. $h''(x) = \dfrac{1}{2}\left[\dfrac{(x-2)^{1/2}(3) - (3x-4)\frac{1}{2}(x-2)^{-1/2}}{x-2}\right] = \dfrac{(x-2)^{-1/2}[6(x-2)-(3x-4)]}{4(x-2)} = \dfrac{3x-8}{4(x-2)^{3/2}}$.

   The sign diagram for h'' shows that h is concave downward on $\left(2, \frac{8}{3}\right)$ and concave upward on $\left(\frac{8}{3}, \infty\right)$.

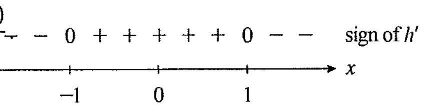


8. The results of step 7 tell us that $\left(\frac{8}{3}, \frac{8\sqrt{6}}{9}\right)$ is an inflection point.



16. $h(x) = \dfrac{2x}{1+x^2}$.

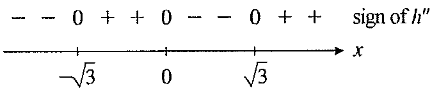
1. The domain of h is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 0 as the y-intercept.
3. $\displaystyle\lim_{x\to-\infty} \frac{2x}{1+x^2} = \lim_{x\to\infty} \frac{2x}{1+x^2} = 0.$
4. The results of step 3 tell us that $y = 0$ is a horizontal asymptote.
5. $h'(x) = \dfrac{(1+x^2)(2) - 2x(2x)}{(1+x^2)^2} = \dfrac{2(1-x^2)}{(1+x^2)^2} = \dfrac{2(1-x)(1+x)}{(1+x^2)^2}$



The sign diagram of h' shows us that h is decreasing on $(-\infty, -1)$ and $(1, \infty)$ and increasing on $(-1, 1)$.

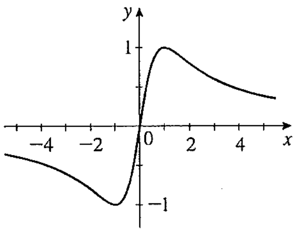
6. The results of step 6 show that $(-1, -1)$ is a relative minimum and $(1, 1)$ is a relative maximum.
7. $h''(x) = 2\left[\dfrac{(1+x^2)^2(-2x) - (1-x^2)2(1+x^2)(2x)}{(1+x^2)^4}\right] = \dfrac{4x(1+x^2)\left[-(1+x^2) - 2(1-x^2)\right]}{(1+x^2)^4}$

$$= \frac{4x(x^2-3)}{(1+x^2)^3}.$$



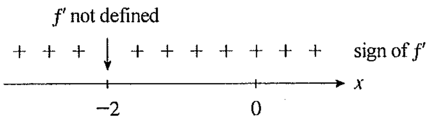
The sign diagram of h'' shows that h is concave downward on $\left(-\infty, -\sqrt{3}\right)$ and $\left(0, \sqrt{3}\right)$ and concave upward on $\left(-\sqrt{3}, 0\right)$ and $\left(\sqrt{3}, \infty\right)$.

8. The results of step 6 also tell us that $\left(-\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$ and $\left(\sqrt{3}, \frac{\sqrt{3}}{2}\right)$ are inflection points.



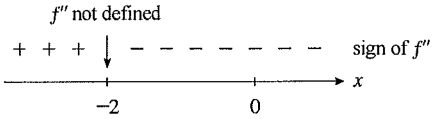
17. $f(x) = \dfrac{x-2}{x+2}$.

1. The domain of f is $(-\infty, -2) \cup (-2, \infty)$.
2. Setting $x = 0$ gives -1 as the y-intercept. Setting $y = 0$ gives 2 as the x-intercept.
3. $\displaystyle\lim_{x\to-\infty} \frac{x-2}{x+2} = \lim_{x\to\infty} \frac{x-2}{x+2} = 1.$
4. The results of step 3 tell us that $y = 1$ is a horizontal asymptote. Next, observe that the denominator of $f(x)$ is equal to zero at $x = -2$, but its numerator is not equal to zero there. Therefore, $x = -2$ is a vertical asymptote.
5. $f'(x) = \dfrac{(x+2)(1) - (x-2)(1)}{(x+2)^2} = \dfrac{4}{(x+2)^2}$. The sign diagram of f' tells us that f is increasing on $(-\infty, -2)$ and $(-2, \infty)$.

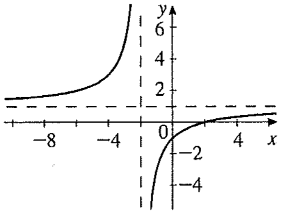


6. The results of step 5 tell us that there is no relative extremum.

7. $f''(x) = -\dfrac{8}{(x+2)^3}$. The sign diagram of f'' shows that f is concave upward on $(-\infty, -2)$ and concave downward on $(-2, \infty)$.



8. There is no inflection point.



18. $f(x) = x - \dfrac{1}{x}$.

1. The domain of f is $(-\infty, 0) \cup (0, \infty)$.

2. There is no y-intercept. Setting $y = 0$ gives $\dfrac{x^2 - 1}{x} = \dfrac{(x+1)(x-1)}{x} = 0$, and so the x-intercepts are -1 and 1.

3. $\displaystyle\lim_{x \to -\infty} \left(x - \frac{1}{x}\right) = -\infty$ and $\displaystyle\lim_{x \to \infty} \left(x - \frac{1}{x}\right) = \infty$.

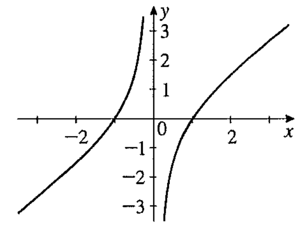
4. There is no horizontal asymptote. From $f(x) = \dfrac{x^2 - 1}{x}$, we see that the denominator of $f(x)$ is equal to zero at $x = 0$. Because the numerator is not equal to zero there, we conclude that $x = 0$ is a vertical asymptote.

5. $f'(x) = 1 + \dfrac{1}{x^2} = \dfrac{x^2 + 1}{x^2} > 0$ for all $x \neq 0$. Therefore, f is increasing on $(-\infty, 0)$ and $(0, \infty)$.

6. The results of step 5 show that f has no relative extremum.

7. $f''(x) = -\dfrac{2}{x^3}$. Observe that $f''(x) > 0$ if $x < 0$ and $f''(x) < 0$ if $x > 0$. Therefore, f is concave upward on $(-\infty, 0)$ and concave downward on $(0, \infty)$.

8. There is no inflection point.



19. $\displaystyle\lim_{x \to -\infty} \frac{1}{2x+3} = \lim_{x \to \infty} \frac{1}{2x+3} = 0$ and so $y = 0$ is a horizontal asymptote. Because the denominator is equal to zero at $x = -\frac{3}{2}$ but the numerator is not equal to zero there, we see that $x = -\frac{3}{2}$ is a vertical asymptote.

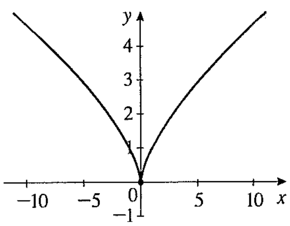
20. $\displaystyle\lim_{x \to -\infty} \frac{2x}{x+1} = \lim_{x \to \infty} \frac{2x}{x+1} = 2$ and so $y = 2$ is a horizontal asymptote. Because the denominator is equal to zero at $x = -1$, but the numerator is not equal to zero there, we see that $x = -1$ is a vertical asymptote.

21. $\displaystyle\lim_{x \to -\infty} \frac{5x}{x^2 - 2x - 8} = \lim_{x \to \infty} \frac{5x}{x^2 - 2x - 8} = 0$, so $y = 0$ is a horizontal asymptote. Next, note that the denominator is zero if $x^2 - 2x - 8 = (x-4)(x+2) = 0$, that is, if $x = -2$ or $x = 4$. Because the numerator is not equal to zero at these points, we see that $x = -2$ and $x = 4$ are vertical asymptotes.

22. $\lim\limits_{x\to-\infty}\dfrac{x^2+x}{x^2-x} = \lim\limits_{x\to\infty}\dfrac{x^2+x}{x^2-x} = 1$, we see that $y = 1$ is a horizontal asymptote. Next, observe that the denominator is equal to zero at $x = 0$ or $x = 1$. Because the numerator is not equal to zero at $x = 1$, we see that $x = 1$ is a vertical asymptote.

23. $f(x) = 2x^2 + 3x - 2$, so $f'(x) = 4x + 3$. Setting $f'(x) = 0$ gives $x = -\frac{3}{4}$ as a critical number of f. Next, $f''(x) = 4 > 0$ for all x, so f is concave upward on $(-\infty, \infty)$. Therefore, $f\left(-\frac{3}{4}\right) = -\frac{25}{8}$ is an absolute minimum of f. There is no absolute maximum.

24. $g(x) = x^{2/3}$, so $g'(x) = \frac{2}{3}x^{-1/3} = \dfrac{2}{3x^{1/3}}$, so $x = 0$ is a critical number. Because $g'(x) < 0$ if $x < 0$ and $g'(x) > 0$ if $x > 0$, we see that $(0, 0)$ is a relative minimum. The graph of g shows that $(0, 0)$ is an absolute minimum.



25. $g(t) = \sqrt{25 - t^2} = \left(25 - t^2\right)^{1/2}$. Differentiating $g(t)$, we have $g'(t) = \frac{1}{2}\left(25 - t^2\right)^{-1/2}(-2t) = -\dfrac{t}{\sqrt{25 - t^2}}$. Setting $g'(t) = 0$ gives $t = 0$ as a critical number of g. The domain of g is given by solving the inequality $25 - t^2 \geq 0$ or $(5 - t)(5 + t) \geq 0$ which implies that $t \in [-5, 5]$. From the table, we conclude that $g(0) = 5$ is the absolute maximum of g and $g(-5) = 0$ and $g(5) = 0$ is the absolute minimum value of g.

t	-5	0	5
$g(t)$	0	5	0

26. $f(x) = \frac{1}{3}x^3 - x^2 + x + 1$, so $f'(x) = x^2 - 2x + 1 = (x - 1)^2$. Therefore, $x = 1$ is a critical number of f. From the table, we see that $f(0) = 1$ is the absolute minimum value and $f(2) = \frac{5}{3}$ is the absolute maximum value of f.

x	0	1	2
$f(x)$	1	$\frac{4}{3}$	$\frac{5}{3}$

27. $h(t) = t^3 - 6t^2$, so $h'(t) = 3t^2 - 12t = 3t(t - 4) = 0$ if $t = 0$ or $t = 4$, critical numbers of h. But only $t = 4$ lies in $(2, 5)$. From the table, we see that h has an absolute minimum at $(4, -32)$ and an absolute maximum at $(2, -16)$.

t	2	4	5
$h(t)$	-16	-32	-25

28. $g(x) = \dfrac{x}{x^2 + 1}$, so $g'(x) = \dfrac{\left(x^2 + 1\right)(1) - x(2x)}{\left(x^2 + 1\right)^2} = \dfrac{1 - x^2}{\left(x^2 + 1\right)^2} = 0$ if $x = \pm 1$. But only the critical number $x = 1$ lies in $(0, 5)$. From the table, we see that $(0, 0)$ is an absolute minimum and $\left(1, \frac{1}{2}\right)$ is an absolute maximum of g.

x	0	1	5
$g(x)$	0	$\frac{1}{2}$	$\frac{5}{26}$

29. $f(x) = x - \dfrac{1}{x}$ on $[1, 3]$, so $f'(x) = 1 + \dfrac{1}{x^2}$. Because $f'(x)$ is never zero, f has no critical number. Calculating $f(x)$ at the endpoints, we see that $f(1) = 0$ is the absolute minimum value and $f(3) = \frac{8}{3}$ is the absolute maximum value.

30. $h(t) = 8t - \frac{1}{t^2}$ on $[1, 3]$, so $h'(t) = 8 + \frac{2}{t^3} = \frac{8t^3 + 2}{t^3} = 0$ if $t = -\frac{1}{4^{1/3}}$, but this number does not lie in $(1, 3)$. $\frac{8t^3 + 2}{t^3}$ is undefined at $t = 0$, but this value also lies outside $(1, 3)$. Evaluating $h(t)$ at the endpoints, we see that h has an absolute minimum at $(1, 7)$ and an absolute maximum at $\left(3, \frac{215}{9}\right)$.

31. $f(s) = s\sqrt{1 - s^2}$ on $[-1, 1]$. The function f is continuous on $[-1, 1]$ and differentiable on $(-1, 1)$. Next, $f'(s) = \left(1 - s^2\right)^{1/2} + s\left(\frac{1}{2}\right)\left(1 - s^2\right)^{-1/2}(-2s) = \frac{1 - 2s^2}{\sqrt{1 - s^2}}$. Setting $f'(s) = 0$, we find that $s = \pm\frac{\sqrt{2}}{2}$ are critical numbers of f. From the table, we see that $f\left(-\frac{\sqrt{2}}{2}\right) = -\frac{1}{2}$ is the absolute minimum value and $f\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$ is the absolute maximum value of f.

x	-1	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$f(x)$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0

32. $f(x) = \frac{x^2}{x - 1}$. Observe that $\lim_{x \to 1^-} \frac{x^2}{x - 1} = -\infty$ and $\lim_{x \to 1^+} \frac{x^2}{x - 1} = \infty$. Therefore, there is no absolute extremum.

33. a. The sign of R_1' is negative and the sign of R_2' is positive on $(0, T)$. The sign of R_1'' is negative and the sign of R_2'' is positive on $(0, T)$.

b. The revenue of the neighborhood bookstore is decreasing at an increasing rate, while the revenue of the new bookstore is increasing at an increasing rate.

34. The rumor spreads initially with increasing speed. The rate at which the rumor is spread reaches a maximum at the time corresponding to the t-coordinate of the point P on the curve. Thereafter, the speed at which the rumor is spread decreases.

35. We want to maximize $P(x) = -x^2 + 8x + 20$. Now, $P'(x) = -2x + 8 = 0$ if $x = 4$, a critical number of P. Because $P''(x) = -2 < 0$, the graph of P is concave downward. Therefore, the critical number $x = 4$ yields an absolute maximum. So, to maximize profit, the company should spend \$4000 per month on advertising.

36. a. $S'(t) = \frac{d}{dt}\left(0.195t^2 + 0.32t + 23.7\right) = 0.39t + 0.32 > 0$ on $[0, 7]$, so sales were increasing through the years in question.

b. $S''(t) = 0.39 > 0$ on $[0, 7]$, so sales continued to accelerate through the years.

37. a. $f'(t) = \frac{d}{dt}\left(0.0117t^3 + 0.0037t^2 + 0.7563t + 4.1\right) = 0.0351t^2 + 0.0074t + 0.7563 \geq 0.7563$ for all t in the interval $[0, 9]$. This shows that f is increasing on $(0, 9)$, which tells us that the projected amount of AMT will keep on increasing over the years in question.

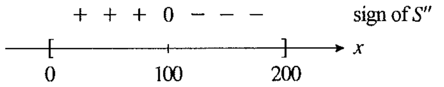
b. $f''(t) = \frac{d}{dt}\left(0.0351t^2 + 0.0074t + 0.7563\right) = 0.0702t + 0.0074 \geq 0.0074$. This shows that f' is increasing on $(0, 9)$. Our result tells us that not only is the amount of AMT paid increasing over the period in question, but it is actually accelerating.

38. **a.** The number of measles deaths in 1999 is given by $N(0) = 506$, or 506,000. The number of measles deaths in 2005 is given by $N(6) = -2.42\left(6^3\right) + 24.5\left(6^2\right) - 123.3(6) + 506 \approx 125.48$, or approximately 125,480.

b. $N'(t) = \dfrac{d}{dt}\left(-2.42t^3 + 24.5t^2 - 123.3t + 506\right) = -7.26t^2 + 49t - 123.3$. Because $(49)^2 - 4(-7.26)(-123.3) = -1179.6 < 0$, we see that $N'(t)$ has no zero. Because $N'(0) = -123.3 < 0$, we conclude that $N'(t) < 0$ on $(0, 6)$. This shows that N is decreasing on $(0, 6)$, so the number of measles deaths was dropping from 1999 through 2005.

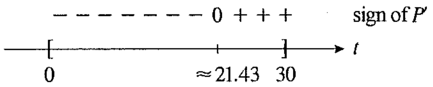
c. $N''(t) = -14.52t + 49 = 0$ implies that $t \approx 3.37$, so the number of measles deaths was decreasing most rapidly in April 2002. The rate is given by $N'(3.37) = -7.26(3.37)^2 + 49(3.37) - 123.3 \approx -40.62$, or approximately -41 deaths/yr.

39. $S(x) = -0.002x^3 + 0.6x^2 + x + 500$, so $S'(x) = -0.006x^2 + 1.2x + 1$ and $S''(x) = -0.012x + 1.2$. $x = 100$ is a candidate for an inflection point of S. The sign diagram for S'' shows that $(100, 4600)$ is an inflection point of S.



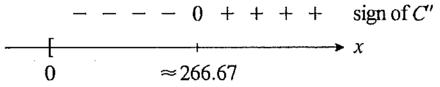
40. **a.** $P(t) = 0.00093t^3 - 0.018t^2 - 0.51t + 25$, so $P'(t) = 0.00279t^2 - 0.036t - 0.51$. Setting $P'(t) = 0$ and solving the resulting equation, we have $t = \dfrac{0.036 \pm \sqrt{(-0.036)^2 - 4(0.00279)(-0.51)}}{2(0.00279)} \approx -8.53$ or 21.43.

From the sign diagram, we see that P is decreasing on $(0, 21.43)$ and increasing on $(21.43, 30)$.



b. The percentage of men 65 and older in the workforce decreased from 1970 through about the middle of 1991, and then increased through the year 2000.

41. $C(x) = 0.0001x^3 - 0.08x^2 + 40x + 5000$, so $C'(x) = 0.0003x^2 - 0.16x + 40$ and $C''(x) = 0.0006x - 0.16$. Thus, $x = 266.67$ is a candidate for an inflection point of C. The sign diagram for C'' shows that C has an inflection point at $(266.67, 11874.08)$.



42. **a.** $S(t) = 6.8(t + 1.03)^{0.49}$, so $S'(t) = 6.8(0.49)(t + 1.03)^{-0.51} = \dfrac{3.332}{(t + 1.03)^{0.51}} > 0$ on $(0, 4)$, so S is increasing on $(0, 4)$. This tells us that the sales are increasing from 2003 through 2007.

b. $S''(t) = 3.332(-0.51)(t + 1.03)^{-1.51} = -\dfrac{1.69932}{(t + 1.03)^{1.51}} < 0$ on $(0, 4)$. This tells us that the graph of S is concave downward on $(0, 4)$, and that the sales are increasing but at a decreasing rate.

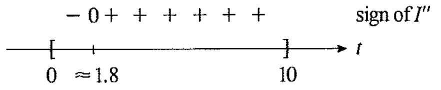
43. **a.** $f(0) \approx 12.98$, so the proportion in 2005 was approximately 13.0%. The projected proportion in 2015 is given by $f(10) \approx 22.21$, or approximately 22.2%.

b. $f'(t) = \dfrac{\left(59 - t^{1/2}\right)(150)\left(\frac{1}{2}t^{-1/2}\right) - \left(150t^{1/2} + 766\right)\left(-\frac{1}{2}t^{-1/2}\right)}{\left(59 - t^{1/2}\right)^2} \doteq \dfrac{4.808}{\sqrt{t}\left(59 - t^{1/2}\right)^2} > 0$ for $0 < t \leq 10$, so f is increasing on $(0, 10)$. This says that the percentage of small and lower-midsize vehicles will be growing over the period from 2005 to 2015.

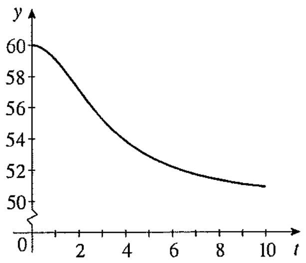
44. **a.** $I(t) = \frac{50t^2+600}{t^2+10}$, so $I'(t) = \frac{(t^2+10)(100t)-(50t^2+600)(2t)}{(t^2+10)^2} = -\frac{200t}{(t^2+10)^2} < 0$ on (0, 10), and so I is decreasing on (0, 10).

b. $I''(t) = -200\left[\frac{(t^2+10)^2(1)-t(2)(t^2+10)(2t)}{(t^2+10)^4}\right] = \frac{-200(t^2+10)\left[(t^2+10)-4t^2\right]}{(t^2+10)^4} = -\frac{200(10-3t^2)}{(t^2+10)^3}$.

The sign diagram of I'' for $t > 0$ shows that I is concave downward on $\left(0, \sqrt{\frac{10}{3}}\right)$ and concave upward on $\left(\sqrt{\frac{10}{3}}, \infty\right)$.



c.



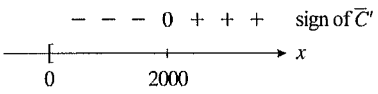
d. The rate of decline in the environmental quality of the wildlife was increasing for the first 1.8 years. After that time the rate of decline decreased.

45. The revenue is $R(x) = px = x\left(-0.0005x^2+60\right) = -0.0005x^3 + 60x$. Therefore, the total profit is $P(x) = R(x) - C(x) = -0.0005x^3 + 0.001x^2 + 42x - 4000$.
$P'(x) = -0.0015x^2 + 0.002x + 42$, and setting $P'(x) = 0$ gives $3x^2 - 4x - 84{,}000 = 0$. Solving for x, we find $x = \frac{4 \pm \sqrt{16-4(3)(84{,}000)}}{2(3)} = \frac{4 \pm 1004}{6} = 168$ or -167. We reject the negative root. Next, $P''(x) = -0.003x + 0.002$ and $P''(168) = -0.003(168) + 0.002 = -0.502 < 0$. By the Second Derivative Test, $x = 168$ gives a relative maximum. Therefore, the required level of production is 168 DVDs.

46. $P(x) = -0.04x^2 + 240x - 10{,}000$, so $P'(x) = -0.08x + 240 = 0$ if $x = 3000$. The graph of P is a parabola that opens downward and so $x = 3000$ gives rise to the absolute maximum of P. Thus, to maximize profits, the company should produce 3000 cameras per month.

47. **a.** $C(x) = 0.001x^2 + 100x + 4000$, so $\overline{C}(x) = \frac{C(x)}{x} = \frac{0.001x^2+100x+4000}{x} = 0.001x + 100 + \frac{4000}{x}$.

b. $\overline{C}'(x) = 0.001 - \frac{4000}{x^2} = \frac{0.001x^2-4000}{x^2} = \frac{0.001\left(x^2-4{,}000{,}000\right)}{x^2}$. Setting $\overline{C}'(x) = 0$ gives $x = \pm 2000$. We reject the negative root. The sign diagram of $\overline{C}'$ shows that $x = 2000$ gives rise to a relative minimum of $\overline{C}$. Because $\overline{C}''(x) = \frac{8000}{x^3} > 0$ if $x > 0$, we see that $\overline{C}$ is concave upward on $(0, \infty)$, and so $x = 2000$ yields an absolute minimum. The required production level is 2000 units.



48. $N(t) = -2t^3 + 12t^2 + 2t$. We wish to find the inflection point of the function N. $N'(t) = -6t^2 + 24t + 2$ and $N''(t) = -12t + 24 = -12(t-2) = 0$ if $t = 2$. Furthermore, $N''(t) > 0$ when $t < 2$ and $N''(t) < 0$ when $t > 2$. Therefore, $t = 2$ gives an inflection point of N. The average worker is performing at peak efficiency at 10 a.m.