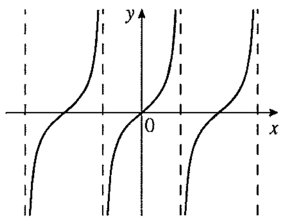
4.3 Curve Sketching

Concept Questions page 298

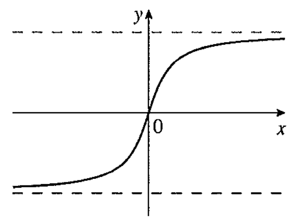
1. a. See the definition on page 293 of the text.

   b. See the definition on page 295 of the text.

2. a. There is no restriction to the number of vertical asymptotes the graph of a function can have.



   b. The graph of a function can have at most two horizontal asymptotes.



3. See the procedure given on page 292 of the text.

4. See the procedure given on page 295 of the text.

Exercises page 298

1. $y = 0$ is a horizontal asymptote.

2. $y = 0$ is a horizontal asymptote and $x = -1$ is a vertical asymptote.

3. $y = 0$ is a horizontal asymptote and $x = 0$ is a vertical asymptote.

4. $y = 0$ is a horizontal asymptote.

5. $y = 0$ is a horizontal asymptote and $x = -1$ and $x = 1$ are vertical asymptotes.

6. $y = 0$ is a horizontal asymptote.

7. $y = 3$ is a horizontal asymptote and $x = 0$ is a vertical asymptote.

8. $y = 0$ is a horizontal asymptote and $x = -2$ is a vertical asymptote.

9. $y = 1$ and $y = -1$ are horizontal asymptotes.

10. $y = 1$ is a horizontal asymptote and $x = \pm 1$ are vertical asymptotes.

11. $\lim_{x \to \infty} \frac{1}{x} = 0$, and so $y = 0$ is a horizontal asymptote. Next, since the numerator of the rational expression is not equal to zero and the denominator is zero at $x = 0$, we see that $x = 0$ is a vertical asymptote.

12. $\lim_{x \to \infty} \frac{1}{x+2} = 0$, and so $y = 0$ is a horizontal asymptote, Next, observe that the numerator of the rational function is not equal to zero but the denominator is equal to zero at $x = -2$, and so $x = -2$ is a vertical asymptote.

13. $f(x) = -\frac{2}{x^2}$, so $\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left(-\frac{2}{x^2}\right) = 0$. Thus, $y = 0$ is a horizontal asymptote. Next, the denominator of $f(x)$ is equal to zero at $x = 0$. Because the numerator of $f(x)$ is not equal to zero at $x = 0$, we see that $x = 0$ is a vertical asymptote.

14. $\lim_{x\to\infty} \frac{1}{1+2x^2} = 0$ and so $y = 0$ is a horizontal asymptote, Next, observe that the denominator $1 + 2x^2 \neq 0$, and so there is no vertical asymptote.

15. $\lim_{x\to\infty} \frac{x-2}{x+2} = \lim_{x\to\infty} \frac{1-\frac{2}{x}}{1+\frac{2}{x}} = 1$, and so $y = 1$ is a horizontal asymptote. Next, the denominator is equal to zero at $x = -2$ and the numerator is not equal to zero at this number, so $x = -2$ is a vertical asymptote.

16. $\lim_{t\to\infty} \frac{t+1}{2t-1} = \lim_{t\to\infty} \frac{1+\frac{1}{t}}{2-\frac{1}{t}} = \frac{1}{2}$, and so $y = \frac{1}{2}$ is a horizontal asymptote. Next, observe that the denominator of the rational expression is zero at $t = \frac{1}{2}$, but the numerator is not equal to zero at this number, and so $t = \frac{1}{2}$ is a vertical asymptote.

17. $h(x) = x^3 - 3x^2 + x + 1$. $h(x)$ is a polynomial function, and therefore it does not have any horizontal or vertical asymptotes.

18. The function g is a polynomial, and so the graph of g has no horizontal or vertical asymptotes.

19. $\lim_{t\to\infty} \frac{t^2}{t^2-16} = \lim_{t\to\infty} \frac{1}{1-\frac{16}{t^2}} = 1$, and so $y = 1$ is a horizontal asymptote. Next, observe that the denominator of the rational expression $t^2 - 16 = (t+4)(t-4) = 0$ if $t = -4$ or $t = 4$. But the numerator is not equal to zero at these numbers, so $t = -4$ and $t = 4$ are vertical asymptotes.

20. $\lim_{x\to\infty} \frac{x^3}{x^2-4} = \lim_{x\to\infty} \frac{x}{1-\frac{4}{x^2}} = \infty$, and similarly $\lim_{x\to\infty} \frac{x^3}{x^2-4} = -\infty$. Therefore, there is no horizontal asymptote. Next, note that the denominator of $g(x)$ equals zero at $x = \pm 2$. Because the numerator of $g(x)$ is not equal to zero at $x = \pm 2$, we see that $x = -2$ and $x = 2$ are vertical asymptotes.

21. $\lim_{x\to\infty} \frac{3x}{x^2-x-6} = \lim_{x\to\infty} \frac{\frac{3}{x}}{1-\frac{1}{x}-\frac{6}{x^2}} = 0$ and so $y = 0$ is a horizontal asymptote. Next, observe that the denominator $x^2 - x - 6 = (x-3)(x+2) = 0$ if $x = -2$ or $x = 3$. But the numerator $3x$ is not equal to zero at these numbers, so $x = -2$ and $x = 3$ are vertical asymptotes.

22. $\lim_{x\to\infty} \frac{2x}{x^2+x-2} = \lim_{x\to\infty} \frac{\frac{2}{x}}{1+\frac{1}{x}-\frac{2}{x^2}} = 0$, and so $y = 0$ is a horizontal asymptote. Next, observe that the denominator $x^2 + x - 2 = (x+2)(x-1) = 0$, if $x = -2$ or $x = 1$. The numerator is not equal to zero at these numbers, and so $x = -2$ and $x = 1$ are vertical asymptotes.

23. $\lim_{t\to\infty}\left[2 + \frac{5}{(t-2)^2}\right] = 2$, and so $y = 2$ is a horizontal asymptote. Next observe that

$\lim_{t\to 2^+} g(t) = \lim_{t\to 2^-}\left[2 + \frac{5}{(t-2)^2}\right] = \infty$, and so $t = 2$ is a vertical asymptote.

24. $\lim_{x\to\infty}\left(1+\frac{2}{x-3}\right)=1$ and $\lim_{x\to-\infty}\left(1+\frac{2}{x-3}\right)=1$, so $y=1$ is a horizontal asymptote. Next, we write $f(x)=1+\frac{2}{x-3}=\frac{x-3+2}{x-3}=\frac{x-1}{x-3}$, and observe that the denominator of $f(x)$ is equal to zero at $x=3$. However, since the numerator of $f(x)$ is not equal to zero at $x=3$, we see that $x=3$ is a vertical asymptote.

25. $\lim_{x\to\infty}\frac{x^2-2}{x^2-4}=\lim_{x\to\infty}\frac{1-\frac{2}{x^2}}{1-\frac{4}{x^2}}=1$, and so $y=1$ is a horizontal asymptote. Next, observe that the denominator $x^2-4=(x+2)(x-2)=0$ if $x=-2$ or 2. Because the numerator x^2-2 is not equal to zero at these numbers, the lines $x=-2$ and $x=2$ are vertical asymptotes.

26. $\lim_{x\to\infty}\frac{2-x^2}{x^2+x}=\lim_{x\to\infty}\frac{\frac{2}{x^2}-1}{1+\frac{1}{x}}=-1$, and so $y=1$ is a horizontal asymptote. Next, observe that the denominator $x^2+x=x(x+1)=0$ if $x=0$ or $x=-1$. Because the numerator $2-x^2$ is not equal to zero at these values of x, we see that $x=0$ and $x=-1$ are vertical asymptotes.

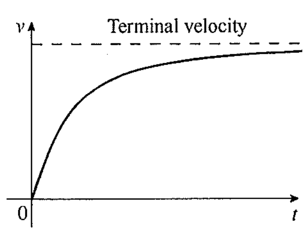
27. $g(x)=\frac{x^3-x}{x(x+1)}$. Rewrite $g(x)$ as $\frac{x^2-1}{x+1}$ for $x\neq 0$, and note that $\lim_{x\to-\infty}g(x)=\lim_{x\to-\infty}\frac{x-\frac{1}{x}}{1+\frac{1}{x}}=-\infty$ and $\lim_{x\to\infty}g(x)=\infty$. Therefore, there is no horizontal asymptote. Next, note that the denominator of $g(x)$ is equal to zero at $x=0$ and $x=-1$. However, since the numerator of $g(x)$ is also equal to zero when $x=0$, we see that $x=0$ is not a vertical asymptote. Also, the numerator of $g(x)$ is equal to zero when $x=-1$, so $x=-1$ is not a vertical asymptote.

28. $\lim_{x\to\infty}\frac{x^4-x^2}{x(x-1)(x+2)}=\lim_{x\to\infty}\left[x\cdot\frac{1-\frac{1}{x^2}}{\left(1-\frac{1}{x}\right)\left(1+\frac{2}{x}\right)}\right]=\infty$, so there is no horizontal asymptote. Next, observe that the denominator is zero at $x=0$, $x=1$, and $x=-2$. Of these values, only $x=-2$ is a vertical asymptote because the numerator is not also equal to zero at this value.

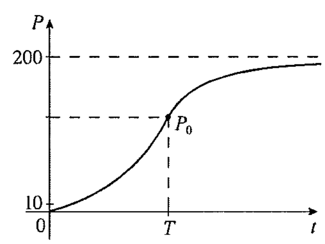
29. f is the derivative function of the function g. Observe that at a relative maximum or minimum of g, $f(x)=0$.

30. f is the derivative function of the function g. Observe that at a relative maximum or minimum of g, $f(x)=0$.

31.



32. a.



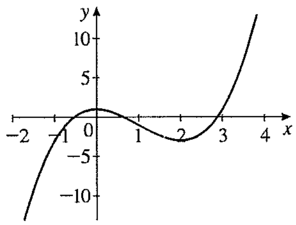
b. f is increasing on $(0,\infty)$.

c. Yes, $P=200$.

d. Concave upward on $(0,T)$ and concave downward on (T,∞).

e. Yes, at P_0. $P(t)$ is increasing fastest at $t=T$.

33.



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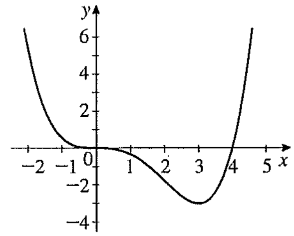

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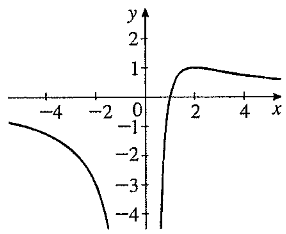

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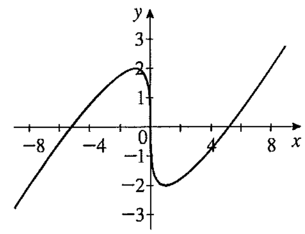
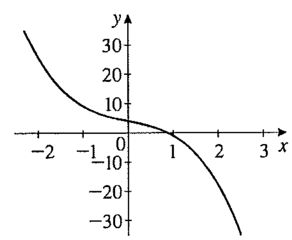


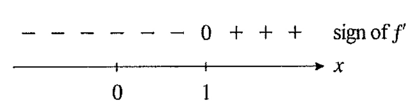
37. $g(x) = 4 - 3x - 2x^3$. We first gather the following information on f.

1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives $y = 4$ as the y-intercept. Setting $y = g(x) = 0$ gives a cubic equation which is not easily solved, and we will not attempt to find the x-intercepts.
3. $\lim_{x \to -\infty} g(x) = \infty$ and $\lim_{x \to \infty} g(x) = -\infty$.
4. The graph of g has no asymptote.
5. $g'(x) = -3 - 6x^2 = -3(2x^2 + 1) < 0$ for all values of x and so g is decreasing on $(-\infty, \infty)$.
6. The results of step 5 show that g has no critical number and hence no relative extremum.
7. $g''(x) = -12x$. Because $g''(x) > 0$ for $x < 0$ and $g''(x) < 0$ for $x > 0$, we see that g is concave upward on $(-\infty, 0)$ and concave downward on $(0, \infty)$.
8. From the results of step 7, we see that $(0, 4)$ is an inflection point of g.



38. $f(x) = x^2 - 2x + 3$. We first gather the following information on f.

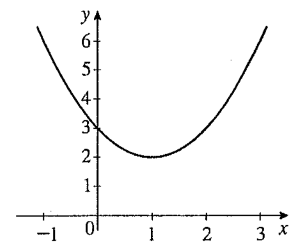
1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives the y-intercept as 3. There is no x-intercept because $x^2 - 2x + 3 = 0$ has no real solution.
3. $\lim_{x \to \infty} x^2 - 2x + 3 = \lim_{x \to -\infty} x^2 - 2x + 3 = \infty$.
4. There is no asymptote because $f(x)$ is a polynomial.
5. $f'(x) = 2x - 2 = 2(x - 1) = 0$ if $x = 1$. The sign diagram shows that f is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$.



6. The point $(1, 2)$ is a relative minimum.

7. $f''(x) = 2 > 0$ for all x and so the graph of f is concave upward on $(-\infty, \infty)$.

8. Because $f''(x) \neq 0$ for all values of x, there is no inflection point.



39. $h(x) = x^3 - 3x + 1$. We first gather the following information on h.

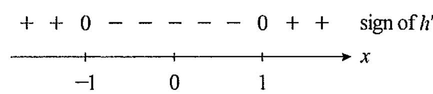
1. The domain of h is $(-\infty, \infty)$.

2. Setting $x = 0$ gives 1 as the y-intercept. We will not find the x-intercept.

3. $\lim_{x \to -\infty} \left(x^3 - 3x + 1\right) = -\infty$ and $\lim_{x \to \infty} \left(x^3 - 3x + 1\right) = \infty$.

4. There is no asymptote because $h(x)$ is a polynomial.

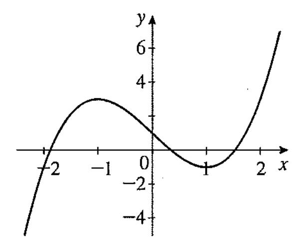
5. $h'(x) = 3x^2 - 3 = 3(x + 1)(x - 1)$, and we see that $x = -1$ and $x = 1$ are critical numbers. From the sign diagram, we see that h is increasing on $(-\infty, -1)$ and $(1, \infty)$ and decreasing on $(-1, 1)$.



6. The results of step 5 show that $(-1, 3)$ is a relative maximum and $(1, -1)$ is a relative minimum.

7. $h''(x) = 6x$, so $h''(x) < 0$ if $x < 0$ and $h''(x) > 0$ if $x > 0$. Thus, the graph of h is concave downward on $(-\infty, 0)$ and concave upward on $(0, \infty)$.

8. The results of step 7 show that $(0, 1)$ is an inflection point of h.



40. $f(x) = 2x^3 + 1$. We first gather the following information on f.

1. The domain of f is $(-\infty, \infty)$.

2. Setting $x = 0$ gives 1 as the y-intercept. Next, observe that $2x^3 = -1$ implies that $x^3 = -\frac{1}{2}$, so $x = -\frac{1}{\sqrt[3]{2}} \approx -0.8$ is the x-intercept.

3. $\lim_{x \to \infty} \left(2x^3 + 1\right) = \infty$ and $\lim_{x \to -\infty} \left(2x^3 + 1\right) = -\infty$.

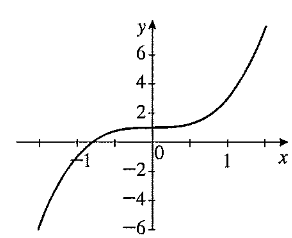
4. Because $f(x)$ is a polynomial, there is no asymptote.

5. $f'(x) = 6x^2 = 0$ if $x = 0$, a critical number of f. Because $f'(x) > 0$ for all $x \neq 0$, we see that f is increasing on $(-\infty, \infty)$.

6. Using the results of step 5, we see that f has no relative extremum.

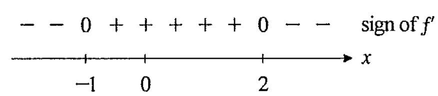
7. $f''(x) = 12x = 0$ if $x = 0$. Because $f''(x) < 0$ if $x < 0$ and $f''(x) > 0$ if $x > 0$, we see that f is concave downward if $x < 0$ and concave upward if $x > 0$.

8. The results of step 7 show that $(0, 1)$ is an inflection point.

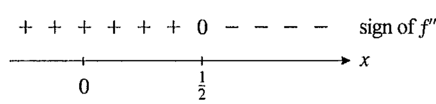


41. $f(x) = -2x^3 + 3x^2 + 12x + 2$. We first gather the following information on f.

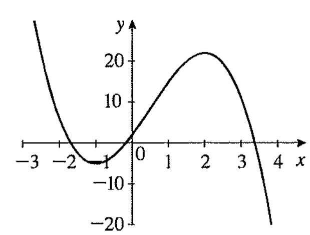
1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 2 as the y-intercept.
3. $\lim_{x\to-\infty} \left(-2x^3 + 3x^2 + 12x + 2\right) = \infty$ and $\lim_{x\to\infty} \left(-2x^3 + 3x^2 + 12x + 2\right) = -\infty$
4. There is no asymptote because $f(x)$ is a polynomial function.
5. $f'(x) = -6x^2 + 6x + 12 = -6\left(x^2 - x - 2\right) = -6(x-2)(x+1) = 0$ if $x = -1$ or $x = 2$, the critical numbers of f. From the sign diagram, we see that f is decreasing on $(-\infty, -1)$ and $(2, \infty)$ and increasing on $(-1, 2)$.



6. The results of step 5 show that $(-1, -5)$ is a relative minimum and $(2, 22)$ is a relative maximum.
7. $f''(x) = -12x + 6 = 0$ if $x = \frac{1}{2}$. The sign diagram of f''shows that the graph of f is concave upward on $\left(-\infty, \frac{1}{2}\right)$ and concave downward on $\left(\frac{1}{2}, \infty\right)$.

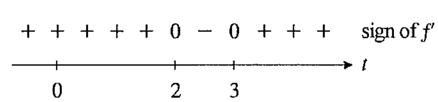


8. The results of step 7 show that $\left(\frac{1}{2}, \frac{17}{2}\right)$ is an inflection point.

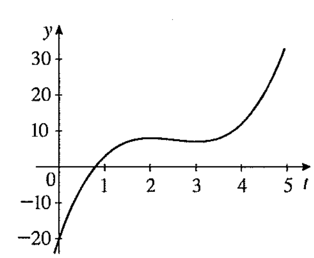


42. $f(t) = 2t^3 - 15t^2 + 36t - 20$. We first gather the following information on f.

1. The domain of f is $(-\infty, \infty)$.
2. Setting $t = 0$ gives -20 as the y-intercept. Setting $y = f(t) = 0$ leads to a cubic equation which is not easily solved and we will not attempt to find the t-intercepts.
3. $\lim_{t\to-\infty} f(t) = -\infty$ and $\lim_{t\to\infty} f(t) = \infty$.
4. f has no asymptote.
5. $f'(t) = 6t^2 - 30t + 36 = 6\left(t^2 - 5t + 6\right) = 6(t-3)(t-2)$. The sign diagram for f' shows that f is increasing on $(-\infty, 2)$ and $(3, \infty)$ and decreasing on $(2, 3)$.



6. The results of step 5 show that $(2, 8)$ is a relative maximum and $(3, 7)$ is a relative minimum.
7. $f''(t) = 12t - 30 = 6(2t - 5)$. Setting $f''(t) = 0$ gives $t = \frac{5}{2}$ as a candidate for an inflection point of f. Because $f''(t) < 0$ for $t < \frac{5}{2}$ and $f''(t) > 0$ for $t > \frac{5}{2}$, we see that f is concave downward on $\left(-\infty, \frac{5}{2}\right)$ and concave upward on $\left(\frac{5}{2}, \infty\right)$.
8. From the results of step 7, we see that $\left(\frac{5}{2}, \frac{15}{2}\right)$ is an inflection point of f.



43. $h(x) = \frac{3}{2}x^4 - 2x^3 - 6x^2 + 8$. We first gather the following information on h.

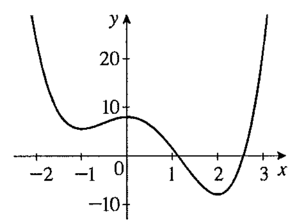
1. The domain of h is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 8 as the y-intercept.
3. $\lim_{x \to -\infty} h(x) = \lim_{x \to \infty} h(x) = \infty$
4. There is no asymptote.
5. $h'(x) = 6x^3 - 6x^2 - 12x = 6x(x^2 - x - 2) = 6x(x-2)(x+1) = 0$ if $x = -1$, 0, or 2, and these are the critical numbers of h. The sign diagram of h' shows that h is increasing on $(-1, 0)$ and $(2, \infty)$ and decreasing on $(-\infty, -1)$ and $(0, 2)$.

   − − 0 + 0 − − − 0 + + sign of h'
   x: −1, 0, 2

6. The results of step 5 show that $\left(-1, \frac{11}{2}\right)$ and $(2, -8)$ are relative minima of h and $(0, 8)$ is a relative maximum of h.
7. $h''(x) = 18x^2 - 12x - 12 = 6(3x^2 - 2x - 2)$. The zeros of h'' are $x = \frac{2 \pm \sqrt{4+24}}{6} \approx -0.5$ or 1.2. The sign diagram of h'' shows that the graph of h is concave upward on $(-\infty, -0.5)$ and $(1.2, \infty)$ and concave downward on $(-0.5, 1.2)$.

   + + 0 − − − − − − − 0 + + sign of h''
   x: ≈−0.5, 0, ≈1.2

8. The results of step 7 also show that $(-0.5, 6.8)$ and $(1.2, -1)$ are inflection points.



44. $f(t) = 3t^4 + 4t^3 = t^3(3t + 4)$. We first gather the following information on f.

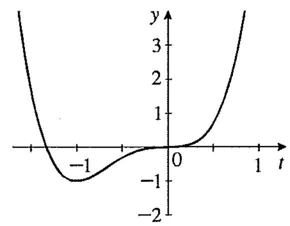
1. The domain of f is $(-\infty, \infty)$.
2. Setting $t = 0$ gives 0 as the y-intercept. Next, setting $y = f(t) = 0$ gives $3t^4 + 4t^3 = t^3(3t + 4) = 0$ and $t = -\frac{4}{3}$ and $t = 0$ as the t-intercepts.
3. $\lim_{t \to \infty} f(t) = \infty$ and $\lim_{t \to -\infty} f(t) = \infty$.
4. There is no asymptote.
5. $f'(t) = 12t^3 + 12t^2 = 12t^2(t + 1)$. From the sign diagram for f', we see that f is increasing on $(-1, \infty)$ and decreasing on $(-\infty, -1)$.

   − − − 0 + + + 0 + + + sign of f'
   t: −1, 0

6. From the results of step 5, we see that has a relative minimum at $(-1, -1)$.
7. $f''(t) = 36t^2 + 24t = 12t(3t + 2)$. Setting $f''(t) = 0$ gives $t = -\frac{2}{3}$ and $t = 0$ as candidates for inflection points of f. The sign diagram for f'' shows that f is concave upward on $\left(-\infty, -\frac{2}{3}\right)$ and $(0, \infty)$ and concave downward on $\left(-\frac{2}{3}, 0\right)$.

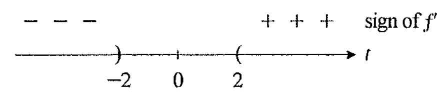
   + + + 0 − − − 0 + + + sign of f''
   t: $-\frac{2}{3}$, 0

8. The results of step 7 imply that $\left(-\frac{2}{3}, -\frac{16}{27}\right)$ and $(0, 0)$ are inflection points of f.

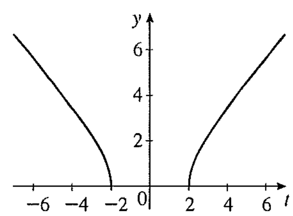


45. $f(t) = \sqrt{t^2 - 4}$. We first gather the following information on f.

1. The domain of f is found by solving $t^2 - 4 \geq 0$ to obtain $(-\infty, -2] \cup [2, \infty)$.
2. Because $t \neq 0$, there is no y-intercept. Next, setting $y = f(t) = 0$ gives the t-intercepts as -2 and 2.
3. $\lim_{t\to-\infty} f(t) = \lim_{t\to\infty} f(t) = \infty$.
4. There is no asymptote.
5. $f'(t) = \frac{1}{2}\left(t^2 - 4\right)^{-1/2}(2t) = t\left(t^2 - 4\right)^{-1/2} = \dfrac{t}{\sqrt{t^2 - 4}}$. Setting $f'(t) = 0$ gives $t = 0$. But $t = 0$ is not in the domain of f and so there is no critical number. From the sign diagram for f', we see that f is increasing on $(2, \infty)$ and decreasing on $(-\infty, -2)$.



6. From the results of step 5 we see that there is no relative extremum.
7. $f''(t) = \left(t^2 - 4\right)^{-1/2} + t\left(-\frac{1}{2}\right)\left(t^2 - 4\right)^{-3/2}(2t)$
$$= \left(t^2 - 4\right)^{-3/2}\left(t^2 - 4 - t^2\right) = -\frac{4}{\left(t^2 - 4\right)^{3/2}}.$$
8. Because $f''(t) < 0$ for all t in the domain of f, we see that f is concave downward everywhere. From the results of step 7, we see that there is no inflection point.

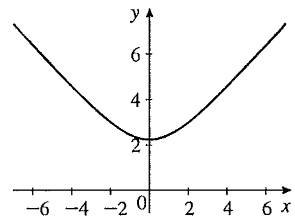


46. $f(x) = \sqrt{x^2 + 5}$. We first gather the following information on f.

1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives $\sqrt{5} \approx 2.2$ as the y-intercept.
3. $\lim_{x\to-\infty} \sqrt{x^2 + 5} = \lim_{x\to\infty} \sqrt{x^2 + 5} = \infty$.
4. The results of step 3 show that there is no horizontal asymptote. There is also no vertical asymptote.
5. $f'(x) = \frac{1}{2}\left(x^2 + 5\right)^{-1/2}(2x) = \dfrac{x}{\sqrt{x^2 + 5}}$ and this shows that $x = 0$ is a critical number of f. Because $f'(x) < 0$ if $x < 0$ and $f'(x) > 0$ is $x > 0$, we see that f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.
6. The results of step 5 show that $\left(0, \sqrt{5}\right)$ is a relative minimum of f.
7. $f''(x) = \dfrac{d}{dx} x\left(x^2 + 5\right)^{-1/2} = \left(x^2 + 5\right)^{-1/2} + x\left(-\frac{1}{2}\right)\left(x^2 + 5\right)^{-3/2}(2x) = \left(x^2 + 5\right)^{-3/2}\left[\left(x^2 + 5\right) - x^2\right]$
$$= \frac{5}{\left(x^2 + 5\right)^{3/2}}.$$

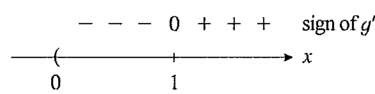
This expression is positive for all values of x, so the graph of f is concave upward on $(-\infty, \infty)$.

8. The results of step 7 also show that there is no inflection point.

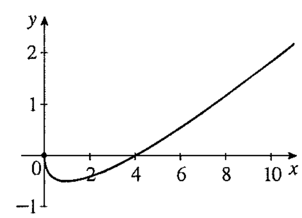


47. $g(x) = \frac{1}{2}x - \sqrt{x}$. We first gather the following information on g.

1. The domain of g is $[0, \infty)$.
2. The y-intercept is 0. To find the x-intercept(s), set $y = 0$, giving $\frac{1}{2}x - \sqrt{x} = 0$, $x = 2\sqrt{x}$, $x^2 = 4x$, $x(x-4) = 0$, and so $x = 0$ or $x = 4$.
3. $\lim_{x\to\infty} \left(\frac{1}{2}x - \sqrt{x}\right) = \lim_{x\to\infty} \frac{1}{2}x\left(1 - \frac{2}{\sqrt{x}}\right) = \infty$.
4. There is no asymptote.
5. $g'(x) = \frac{1}{2} - \frac{1}{2}x^{-1/2} = \frac{1}{2}x^{-1/2}\left(x^{1/2} - 1\right) = \frac{\sqrt{x} - 1}{2\sqrt{x}}$, which is zero when $x = 1$. From the sign diagram for g', we see that g is decreasing on $(0, 1)$ and increasing on $(1, \infty)$.

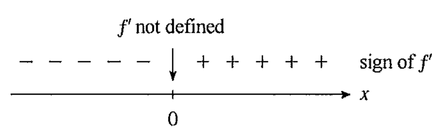


6. From the results of part 5, we see that $g(1) = -\frac{1}{2}$ is a relative minimum.
7. $g''(x) = \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)x^{-3/2} = \frac{1}{4x^{3/2}} > 0$ for $x > 0$, and so g is concave upward on $(0, \infty)$.
8. There is no inflection point.

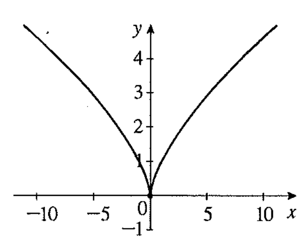


48. $f(x) = \sqrt[3]{x^2}$. We first gather the following information on f.

1. The domain of f is $(-\infty, \infty)$ because $x^2 \geq 0$ for all x.
2. Setting $x = 0$ gives the y-intercept as 0. Similarly, setting $y = 0$ gives 0 as the x-intercept.
3. $\lim_{x\to-\infty} \sqrt[3]{x^2} = \lim_{x\to\infty} \sqrt[3]{x^2} = \infty$.
4. There is no asymptote.
5. $f'(x) = \frac{d}{dx}x^{2/3} = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$. The sign diagram of f' shows that f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

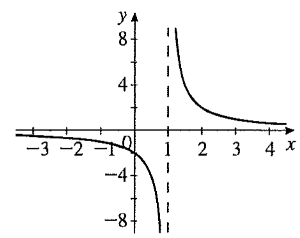


6. Because f has no critical number, f has no relative extremum.
7. $f''(x) = \frac{d}{dx}\left(\frac{2}{3}x^{-1/3}\right) = -\frac{2}{9}x^{-4/3} = -\frac{2}{9x^{4/3}} > 0$ for all $x \neq 0$, and so f is concave downward on $(-\infty, 0)$ and $(0, \infty)$.
8. Because $f''(x) \neq 0$, there is no inflection point.



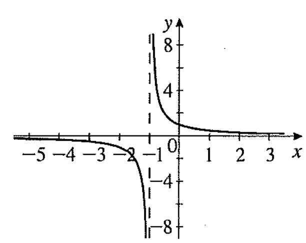
49. $g(x) = \dfrac{2}{x-1}$. We first gather the following information on g.

1. The domain of g is $(-\infty, 1) \cup (1, \infty)$.
2. Setting $x = 0$ gives -2 as the y-intercept. There is no x-intercept because $\dfrac{2}{x-1} \neq 0$ for all x.
3. $\lim\limits_{x\to-\infty} \dfrac{2}{x-1} = 0$ and $\lim\limits_{x\to\infty} \dfrac{2}{x-1} = 0$.
4. The results of step 3 show that $y = 0$ is a horizontal asymptote. Furthermore, the denominator of $g(x)$ is equal to zero at $x = 1$ but the numerator is not equal to zero there. Therefore, $x = 1$ is a vertical asymptote.
5. $g'(x) = -2(x-1)^{-2} = -\dfrac{2}{(x-1)^2} < 0$ for all $x \neq 1$ and so g is decreasing on $(-\infty, 1)$ and $(1, \infty)$.
6. Because g has no critical number, there is no relative extremum.
7. $g''(x) = \dfrac{4}{(x-1)^3}$ and so $g''(x) < 0$ if $x < 1$ and $g''(x) > 0$ if $x > 1$.
   Therefore, the graph of g is concave downward on $(-\infty, 1)$ and concave upward on $(1, \infty)$.
8. Because $g''(x) \neq 0$, there is no inflection point.



50. $f(x) = \dfrac{1}{x+1}$. We first gather the following information on f.

1. Because the denominator is zero when $x = -1$, we see that the domain of f is $(-\infty, -1) \cup (-1, \infty)$.
2. Setting $x = 0$ gives the y-intercept as 1. Because $y \neq 0$, there is no x-intercept.
3. $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to\infty} f(x) = 0$.
4. From the results of step 3, we see that $y = 0$ is a horizontal asymptote of f. Next, setting the denominator of f equal to zero gives $x = -1$. Furthermore, $\lim\limits_{x\to-1^-} f(x) = -\infty$ and $\lim\limits_{x\to-1^+} f(x) = \infty$, and so $x = -1$ is a vertical asymptote of f.
5. $f'(x) = -\dfrac{1}{(x+1)^2}$. Note that $f'(x)$ is not defined at $x = -1$. Because $f'(x) < 0$ whenever x is defined, we see that f is decreasing everywhere.
6. The results of step 5 show that there is no critical numbers ($x = -1$ is not in the domain of f.) Thus, there is no relative extremum.
7. $f''(x) = \dfrac{2}{(x+1)^3}$. We see that $f''(x) < 0$ for $x < -1$ and $f''(x) > 0$ for $x > -1$. Therefore, f is concave downward on $(-\infty, -1)$ and concave upward on $(-1, \infty)$.
8. Because f' has no critical number, f has no inflection point.

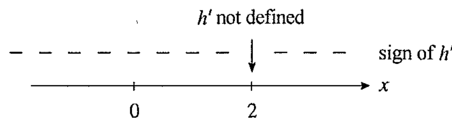


51. $h(x) = \dfrac{x+2}{x-2}$. We first gather the following information on h.

1. The domain of h is $(-\infty, 2) \cup (2, \infty)$.
2. Setting $x = 0$ gives $y = -1$ as the y-intercept. Next, setting $y = 0$ gives $x = -2$ as the x-intercept.
3. $\lim\limits_{x\to\infty} h(x) = \lim\limits_{x\to-\infty} \dfrac{1 + \frac{2}{x}}{1 - \frac{2}{x}} = \lim\limits_{x\to-\infty} h(x) = 1$.

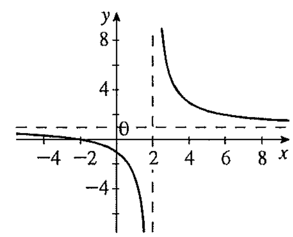
4. Setting $x-2=0$ gives $x=2$. Furthermore, $\lim_{x\to 2^+}\frac{x+2}{x-2}=\infty$, so $x=2$ is a vertical asymptote of h. Also, from the results of step 3, we see that $y=1$ is a horizontal asymptote of h.

5. $h'(x)=\frac{(x-2)(1)-(x+2)(1)}{(x-2)^2}=-\frac{4}{(x-2)^2}$. We see that h has no critical number. (Note that $x=2$ is not in the domain of h.) The sign diagram of h' shows that h is decreasing on $(-\infty,2)$ and $(2,\infty)$.



6. From the results of step 5, we see that there is no relative extremum.

7. $h''(x)=\frac{8}{(x-2)^3}$. Note that $x=2$ is not a candidate for an inflection point because $h(2)$ is not defined. Because $h''(x)<0$ for $x<2$ and $h''(x)>0$ for $x>2$, we see that h is concave downward on $(-\infty,2)$ and concave upward on $(2,\infty)$.



8. From the results of step 7, we see that there is no inflection point.

52. $g(x)=\frac{x}{x-1}$. We first gather the following information on g.

1. The domain of g is $(-\infty,1)\cup(1,\infty)$.

2. Setting $x=0$ gives 0 as the y-intercept. Similarly, setting $y=0$ gives 0 as the x-intercept.

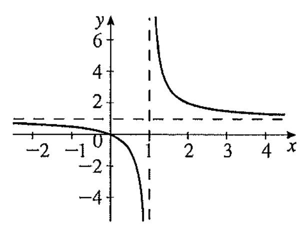
3. $\lim_{x\to-\infty}\frac{x}{x-1}=1$ and $\lim_{x\to\infty}\frac{x}{x-1}=1$.

4. The results of step 3 show that $y=1$ is a horizontal asymptote. Next, because the denominator is zero at $x=1$ but the numerator is not equal to zero at this value of x, we see that $x=1$ is a vertical asymptote of g.

5. $g'(x)=\frac{(x-1)(1)-x(1)}{(x-1)^2}=-\frac{1}{(x-1)^2}<0$ if $x\neq 1$ and so f is decreasing on $(-\infty,1)$ and $(1,\infty)$.

6. Because $g'(x)\neq 0$ for all x, there is no critical number and so g has no relative extremum.

7. $g''(x)=\frac{2}{(x-1)^3}$ and so $g''(x)<0$ if $x<1$ and $g''(x)>0$ if $x>1$. Therefore, the graph of g is concave downward on $(-\infty,1)$ and concave upward on $(1,\infty)$.



8. Because $g''(x)\neq 0$ for all x, we see that there is no inflection point.

53. $f(t)=\frac{t^2}{1+t^2}$. We first gather the following information on f.

1. The domain of f is $(-\infty,\infty)$.

2. Setting $t=0$ gives the y-intercept as 0. Similarly, setting $y=0$ gives the t-intercept as 0.

3. $\lim_{t\to-\infty}\frac{t^2}{1+t^2}=\lim_{t\to\infty}\frac{t^2}{1+t^2}=1$.

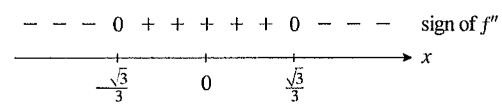
4. The results of step 3 show that $y=1$ is a horizontal asymptote. There is no vertical asymptote since the denominator is never zero.

5. $f'(t) = \dfrac{(1+t^2)(2t) - t^2(2t)}{(1+t^2)^2} = \dfrac{2t}{(1+t^2)^2} = 0$, if $t = 0$, the only critical number of f. Because $f'(t) < 0$ if $t < 0$ and $f'(t) > 0$ if $t > 0$, we see that f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

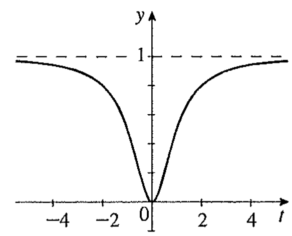
6. The results of step 5 show that $(0, 0)$ is a relative minimum.

7. $f''(t) = \dfrac{(1+t^2)^2(2) - 2t(2)(1+t^2)(2t)}{(1+t^2)^4} = \dfrac{2(1+t^2)\left[(1+t^2) - 4t^2\right]}{(1+t^2)^4} = \dfrac{2(1-3t^2)}{(1+t^2)^3} = 0$ if $t = \pm\frac{\sqrt{3}}{3}$.

The sign diagram of f'' shows that f is concave downward on $\left(-\infty, -\frac{\sqrt{3}}{3}\right)$ and $\left(\frac{\sqrt{3}}{3}, \infty\right)$ and concave upward on $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$.



8. The results of step 7 show that $\left(-\frac{\sqrt{3}}{3}, \frac{1}{4}\right)$ and $\left(\frac{\sqrt{3}}{3}, \frac{1}{4}\right)$ are inflection points.



54. $g(x) = \dfrac{x}{x^2 - 4} = \dfrac{x}{(x+2)(x-2)}$. We first gather the following information on g.

1. The denominator of $g(x)$ is zero when $x = \pm 2$, and so the domain of g is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

2. Setting $x = 0$ gives 0 as the y-intercept and setting $y = 0$ gives 0 as the x-intercept.

3. $\displaystyle\lim_{x\to-\infty} \frac{x}{x^2 - 4} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1 - \frac{4}{x^2}} = 0$. Similarly, $\displaystyle\lim_{x\to\infty} g(x) = 0$.

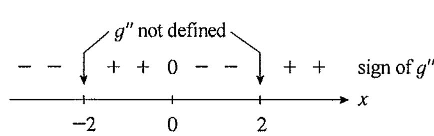
4. From the results of step 3, we see that $y = 0$ is a horizontal asymptote of g. Next, observe that the denominator of $g(x)$ is zero when $x = \pm 2$. Now $\displaystyle\lim_{x\to-2^-} \frac{x}{(x+2)(x-2)} = -\infty$, $\displaystyle\lim_{x\to-2^+} \frac{x}{(x+2)(x-2)} = \infty$, $\displaystyle\lim_{x\to2^-} \frac{x}{(x+2)(x-2)} = -\infty$, and $\displaystyle\lim_{x\to2^+} \frac{x}{(x+2)(x-2)} = \infty$. Therefore, $x = -2$ and $x = 2$ are vertical asymptotes.

5. $g'(x) = \dfrac{(x^2-4)(1) - x(2x)}{(x^2-4)^2} = -\dfrac{x^2+4}{(x^2-4)^2}$. Because $g'(x) < 0$ whenever it is defined, we see that g is decreasing everywhere.

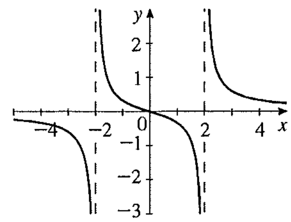
6. From the results of step 5, we see that there is no relative extremum.

7. $g''(x) = \dfrac{(x^2-4)^2(-2x) + (x^2+4)\,2(x^2-4)(2x)}{(x^2-4)^4} = \dfrac{2x(x^2-4)(-x^2+4+2x^2+8)}{(x^2-4)^4} = \dfrac{2x(x^2+12)}{(x^2-4)^3}$.

Setting $g''(x) = 0$ gives $x = 0$ as the only candidate for a point of inflection. The sign diagram for g'' shows that g is concave upward on $(-2, 0)$ and $(2, \infty)$ and concave downward on $(-\infty, -2)$ and $(0, 2)$.



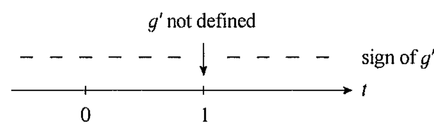
8. From the results of step 7, we see that $(0, 0)$ is an inflection point of g.



55. $g(t) = -\frac{t^2 - 2}{t - 1}$. We first gather the following information on g.

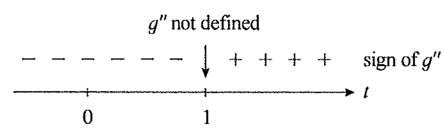
1. The domain of g is $(-\infty, 1) \cup (1, \infty)$
2. Setting $t = 0$ gives -2 as the y-intercept.
3. $\lim_{t \to -\infty} \left(-\frac{t^2 - 2}{t - 1}\right) = \infty$ and $\lim_{t \to \infty} \left(-\frac{t^2 - 2}{t - 1}\right) = -\infty$.
4. There is no horizontal asymptotes. The denominator is equal to zero at $t = 1$ at which number the numerator is not equal to zero. Therefore $t = 1$ is a vertical asymptote.
5. $g'(t) = -\frac{(t - 1)(2t) - (t^2 - 2)(1)}{(t - 1)^2}$

   $= -\frac{t^2 - 2t + 2}{(t - 1)^2} \neq 0$ for all values of t.

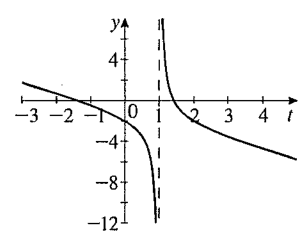
   

   The sign diagram of g' shows that g is decreasing on $(-\infty, 1)$ and $(1, \infty)$.
6. Because there is no critical number, g has no relative extremum.
7. $g''(t) = -\frac{(t - 1)^2 (2t - 2) - (t^2 - 2t + 2)(2)(t - 1)}{(t - 1)^4}$

   $= \frac{-2(t - 1)(t^2 - 2t + 1 - t^2 + 2t - 2)}{(t - 1)^4} = \frac{2}{(t - 1)^3}$.

   The sign diagram of g''shows that the graph of g is concave upward on $(1, \infty)$ and concave downward on $(-\infty, 1)$.
8. There is no inflection point because $g''(x) \neq 0$ for all x.

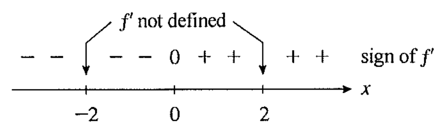


56. $f(x) = \frac{x^2 - 9}{x^2 - 4}$. We first gather the following information on f.

1. The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.
2. The y-intercept is $\frac{9}{4}$ and the x-intercepts are -3 and 3.
3. $\lim_{x \to \infty} \frac{x^2 - 9}{x^2 - 4} = \lim_{x \to \infty} \frac{1 - \frac{9}{x^2}}{1 - \frac{4}{x^2}} = 1$, and similarly $\lim_{x \to -\infty} \frac{x^2 - 9}{x^2 - 4} = 1$.

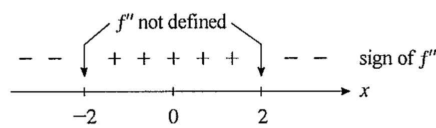
4. From the results of step 3, we see that $y = 1$ is a horizontal asymptote. Next, $x^2 - 4 = 0$ implies $x = \pm 2$. Because the numerator $x^2 - 9$ is not zero at $x = \pm 2$, we see that $x = -2$ and $x = 2$ are vertical asymptotes.

5. $f'(x) = \dfrac{(x^2-4)(2x) - (x^2-9)(2x)}{(x^2-4)^2} = \dfrac{10x}{(x^2-4)^2}$ is equal to 0 at $x = 0$ and is discontinuous at $x = \pm 2$. From the sign diagram for f', we see that f is increasing on $(0, 2)$ and $(2, \infty)$ and decreasing on $(-\infty, -2)$ and $(-2, 0)$.

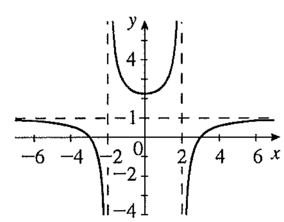


6. The point $\left(0, \frac{9}{4}\right)$ is a relative minimum.

7. $f''(x) = \dfrac{(x^2-4)^2(10) - (10x)(2)(x^2-4)(2x)}{(x^2-4)^4} = \dfrac{10(x^2-4)(x^2-4-4x^2)}{(x^2-4)^4} = \dfrac{-10(3x^2+4)}{(x^2-4)^3}$, which is not defined at $x = \pm 2$. From the sign diagram for f'', we see that f is concave upward on $(-2, 2)$ and concave downward on $(-\infty, -2)$ and $(2, \infty)$.



8. There is no inflection point. Note that both $x = -2$ and $x = 2$ lie outside the domain of f.



57. $g(t) = \dfrac{t^2}{t^2 - 1}$. We first gather the following information on g.

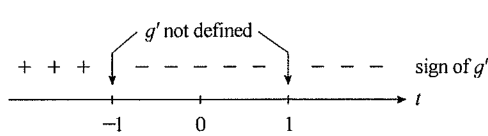
1. Because $t^2 - 1 = 0$ if $t = \pm 1$, we see that the domain of g is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

2. Setting $t = 0$ gives 0 as the y-intercept. Setting $y = 0$ gives 0 as the t-intercept.

3. $\lim_{t \to -\infty} g(t) = \lim_{t \to \infty} g(t) = 1$.

4. The results of step 3 show that $y = 1$ is a horizontal asymptote. Because the denominator (but not the numerator) is zero at $t = \pm 1$, we see that $t = \pm 1$ are vertical asymptotes.

5. $g'(t) = \dfrac{(t^2-1)(2t) - (t^2)(2t)}{(t^2-1)^2} = -\dfrac{2t}{(t^2-1)^2} = 0$ if $t = 0$. From the sign diagram of g', we see that g is increasing on $(-\infty, -1)$ and $(-1, 0)$ and decreasing on $(0, 1)$ and $(1, \infty)$.

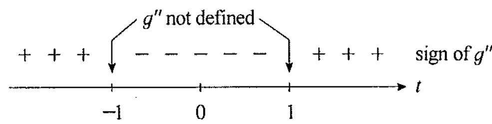


6. From the results of step 5, we see that g has a relative maximum at $t = 0$.

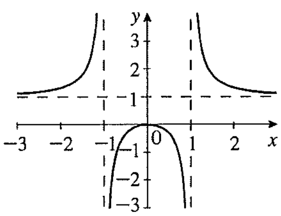
7. $g''(t) = \dfrac{(t^2-1)^2(-2) - (-2t)(2)(t^2-1)(2t)}{(t^2-1)^4} = \dfrac{2(t^2-1)\left[-(t^2-1)+4t^2\right]}{(t^2-1)^4} = \dfrac{2(-t^2+1+4t^2)}{(t^2-1)^3}$

$= \dfrac{2(3t^2+1)}{(t^2-1)^3}.$

From the sign diagram, we see that the graph of g is concave upward on $(-\infty, -1)$ and $(1, \infty)$ and concave downward on $(-1, 1)$.



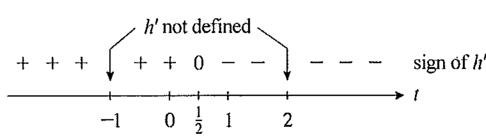
8. Because g is undefined at ± 1, the graph of g has no inflection point.



58. $h(x) = \dfrac{1}{x^2 - x - 2}$. We first gather the following information on h.

1. Because $x^2 - x - 2 = (x-2)(x+1) = 0$ if $x = -1$ or $x = 2$, we see that the domain of h is $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$.
2. Setting $x = 0$ gives $-\frac{1}{2}$ as the y-intercept.
3. $\lim\limits_{x\to-\infty} h(x) = \lim\limits_{x\to\infty} h(x) = 0.$
4. The results of step 3 show that $y = 0$ is a horizontal asymptote. Furthermore, the denominator is equal to zero at $x = -1$ and $x = 2$, where the numerator is not equal to zero. Therefore, $x = -1$ and $x = 2$ are vertical asymptotes.
5. $h'(x) = \dfrac{d}{dx}(x^2 - x - 2)^{-1} = -(x^2 - x - 2)^{-2}(2x - 1) = \dfrac{1 - 2x}{(x^2 - x - 2)^2}.$

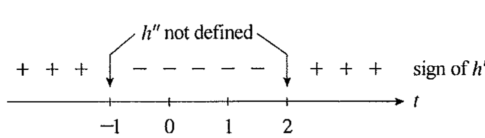
   Setting $h'(x) = 0$ gives $x = \frac{1}{2}$ as a critical number. The sign diagram of h' shows us that h is increasing on $(-\infty, -1)$ and $\left(-1, \frac{1}{2}\right)$ and decreasing on $\left(\frac{1}{2}, 2\right)$ and $(2, \infty)$.



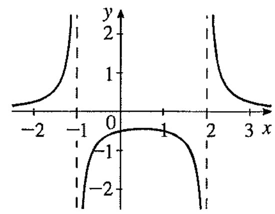
6. The results of step 5 show that $\left(\frac{1}{2}, -\frac{4}{9}\right)$ is a relative maximum.
7. $h''(x) = \dfrac{(x^2-x-2)^2(-2) - (1-2x)2(x^2-x-2)(2x-1)}{(x^2-x-2)^4}$

   $= \dfrac{2(x^2-x-2)\left[-(x^2-x-2)+(2x-1)^2\right]}{(x^2-x-2)^4} = \dfrac{6(x^2-x+1)}{(x^2-x-2)^3}.$

   $h''(x)$ has no zero and is discontinuous at $x = -1$ and $x = 2$. The sign diagram of h'' shows that the graph of h is concave upward on $(-\infty, -1)$ and $(2, \infty)$ and concave downward on $(-1, 2)$.



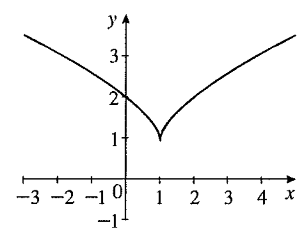
8. Because $h''(x) \neq 0$, there is no inflection point.



59. $h(x) = (x-1)^{2/3} + 1$. We begin by obtaining the following information on h.

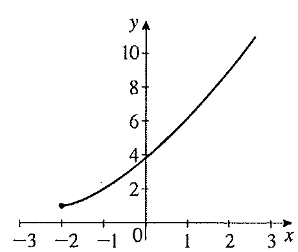
1. The domain of h is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 2 as the y-intercept; since $h(x) \neq 0$ there is no x-intercept.
3. $\lim_{x\to\infty} \left[(x-1)^{2/3} + 1\right] = \infty$ and $\lim_{x\to-\infty} \left[(x-1)^{2/3} + 1\right] = \infty$.
4. There is no asymptote.
5. $h'(x) = \frac{2}{3}(x-1)^{-1/3}$ and is positive if $x > 1$ and negative if $x < 1$. Thus, h is increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$.
6. From step 5, we see that h has a relative minimum at $(1, 1)$.
7. $h''(x) = \frac{2}{3}\left(-\frac{1}{3}\right)(x-1)^{-4/3} = -\frac{2}{9}(x-1)^{-4/3} = -\dfrac{2}{(x-1)^{4/3}}$.

   Because $h''(x) < 0$ on $(-\infty, 1)$ and $(1, \infty)$, we see that h is concave downward on $(-\infty, 1)$ and $(1, \infty)$. Note that $h''(x)$ is not defined at $x = 1$.
8. From the results of step 7, we see h has no inflection point.



60. $g(x) = (x+2)^{3/2} + 1$. We first gather the following information on g.

1. The domain of g is $[-2, \infty)$.
2. Setting $x = 0$ gives $2^{3/2} + 1 \approx 3.8$ as the y-intercept.
3. $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} (x+2)^{3/2} + 1 = \infty$.
4. There is no asymptote.
5. $g'(x) = \frac{3}{2}(x+2)^{1/2} \geq 0$ if $x \geq -2$, and so g is increasing on $(-2, \infty)$.
6. There is no relative extremum because $g'(x) \neq 0$ on $(-2, \infty)$.
7. $g''(x) = \frac{3}{4}(x+2)^{-1/2} = \dfrac{3}{4\sqrt{x+2}} > 0$ if $x > 0$, and so the graph of g is concave upward on $(2, \infty)$.
8. There is no inflection point since $g''(x) \neq 0$.



61. a. The denominator of $C(x)$ is equal to zero if $x = 100$. Also, $\lim_{x\to100^-} \dfrac{0.5x}{100-x} = \infty$ and $\lim_{x\to100^+} \dfrac{0.5x}{100-x} = -\infty$. Therefore, $x = 100$ is a vertical asymptote of C.

b. No, because the denominator is equal to zero in that case.

62. a. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty} \left(2.2 + \dfrac{2500}{x}\right) = 2.2$, and so $y = 2.2$ is a horizontal asymptote.

b. The limiting value is 2.2, or \$2.20 per disc.

63. a. Because $\lim_{t\to\infty} C(t) = \lim_{t\to\infty} \frac{0.2t}{t^2+1} = \lim_{t\to\infty} \left(\frac{0.2}{t + \frac{1}{t^2}} \right) = 0$, $y = 0$ is a horizontal asymptote.

b. Our results reveal that as time passes, the concentration of the drug decreases and approaches zero.

64. a. $\lim_{x\to\infty} \frac{ax}{x+b} = \lim_{x\to\infty} \frac{a}{1+\frac{b}{x}} = a$, so the horizontal asymptote is $V = a$.

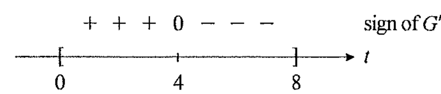
b. The initial speed of the reaction approaches a moles/liter/sec as the amount of substrate becomes arbitrarily large.

65. $G(t) = -0.2t^3 + 2.4t^2 + 60$. We first gather the following information on G.

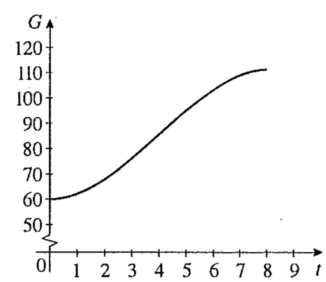
1. The domain of G is restricted to $[0, 8]$.
2. Setting $t = 0$ gives 60 as the y-intercept.

Step 3 is unnecessary in this case because of the restricted domain.

4. There is no asymptote because G is a polynomial function.
5. $G'(t) = -0.6t^2 + 4.8t = -0.6t(t-8) = 0$ if $t = 0$ or $t = 8$, critical numbers of G. But $G'(t) > 0$ on $(0, 8)$, so G is increasing on its domain.
6. The results of step 5 tell us that there is no relative extremum.
7. $G''(t) = -1.2t + 4.8 = -1.2(t-4)$. The sign diagram of G'' shows that G is concave upward on $(0, 4)$ and concave downward on $(4, 8)$.



8. The results of step 7 show that $(4, 85.6)$ is an inflection point.

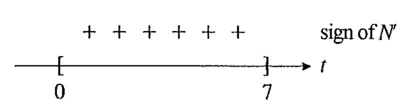


66. $N(t) = -0.1t^3 + 1.5t^2 + 80$, $0 \le t \le 7$. We first gather the following information on N.

1. The domain of N is $[0, 7]$.
2. Setting $t = 0$ gives 80 as the y-intercept.

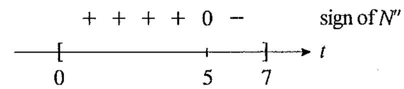
Step 3 is unnecessary in this case because of the restricted domain.

4. There is no asymptote because N is a polynomial function.
5. $N'(t) = -0.3t^2 + 3t = -0.3t(t-10) = 0$ if $t = 0$ or $t = 10$, but $t = 10$ lies outside the domain of N. The sign diagram of N' shows that N is increasing on $(0, 7)$.

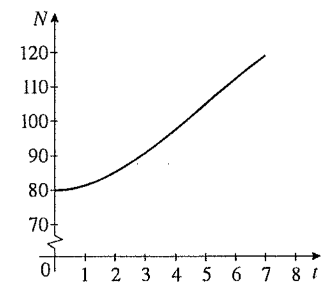


6. The results of step 5 tell us that $f(0) = 80$ is a relative minimum.

7. $N''(t) = -0.6t + 3 = -0.6(t - 5)$. The sign diagram of N'' shows that N is concave upward on $(0, 5)$ and concave downward on $(5, 7)$.



8. The results of step 7 show that $(5, 105)$ is an inflection point. Because N is concave upward on $(0, 5)$ and concave downward on $(5, 7)$ it follows that N' (the rate of increase of N) was increasing from 0 to 5 and decreasing thereafter. This indicates that the program is working.

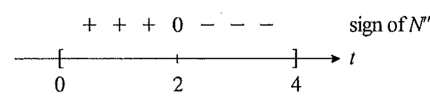


67. $N(t) = -\frac{1}{2}t^3 + 3t^2 + 10t$, $0 \le t \le 4$. We first gather the following information on N.

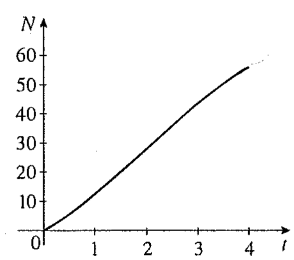
1. The domain of N is restricted to $[0, 4]$.
2. The y-intercept is 0.

Step 3 does not apply because the domain of $N(t)$ is $[0, 4]$.

4. There is no asymptote.
5. $N'(t) = -\frac{3}{2}t^2 + 6t + 10 = -\frac{1}{2}(3t^2 - 12t - 20)$ is never zero. Therefore, N is increasing on $(0, 4)$.
6. There is no relative extremum in $(0, 4)$.
7. $N''(t) = -3t + 6 = -3(t - 2) = 0$ at $t = 2$. From the sign diagram of N'', we see that N is concave upward on $(0, 2)$ and concave downward on $(2, 4)$.

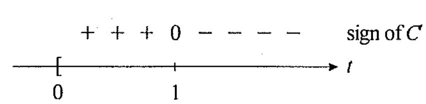


8. The point $(2, 28)$ is an inflection point.



68. $C(t) = \dfrac{0.2t}{t^2 + 1}$. We first gather the following information on C.

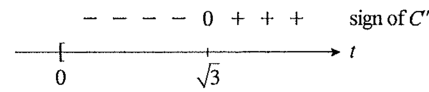
1. The domain of C is $[0, \infty)$.
2. If $t = 0$, then $y = 0$. Also, if $y = 0$, then $t = 0$.
3. $\lim\limits_{t \to \infty} \dfrac{0.2t}{t^2 + 1} = 0$.
4. The results of step 3 imply that $y = 0$ is a horizontal asymptote.
5. $C'(t) = \dfrac{(t^2 + 1)(0.2) - 0.2t(2t)}{(t^2 + 1)^2} = \dfrac{0.2(t^2 + 1 - 2t^2)}{(t^2 + 1)^2} = \dfrac{0.2(1 - t^2)}{(t^2 + 1)^2} = 0$ at $t = \pm 1$, so $t = 1$ is a critical number of C. The sign diagram of C' shows that C is decreasing on $(1, \infty)$ and increasing on $(0, 1)$.



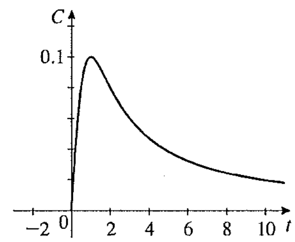
6. The results of step 5 tell us that $(1, 0.1)$ is a relative maximum.

7. $$C''(t) = 0.2\left[\frac{(t^2+1)^2(-2t) - (1-t^2)\,2\,(t^2+1)(2t)}{(t^2+1)^4}\right] = \frac{0.2\,(t^2+1)(2t)(-t^2-1-2+2t^2)}{(t^2+1)^4} = \frac{0.4t\,(t^2-3)}{(t^2+1)^3}.$$

The sign diagram of C'' shows that the graph of C is concave downward on $(0, \sqrt{3})$ and concave upward on $(\sqrt{3}, \infty)$.



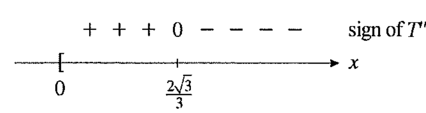
8. The results of step 7 show that $(\sqrt{3}, 0.05\sqrt{3})$ is an inflection point.



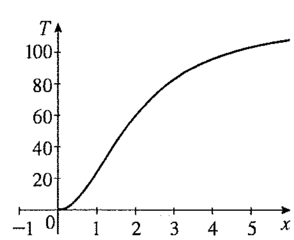
69. $T(x) = \dfrac{120x^2}{x^2+4}$. We first gather the following information on T.

1. The domain of T is $[0, \infty)$.
2. Setting $x = 0$ gives 0 as the y-intercept.
3. $\displaystyle\lim_{x\to\infty} \frac{120x^2}{x^2+4} = 120$.
4. The results of step 3 show that $y = 120$ is a horizontal asymptote.
5. $T'(x) = 120\left[\dfrac{(x^2+4)\,2x - x^2(2x)}{(x^2+4)^2}\right] = \dfrac{960x}{(x^2+4)^2}$. Because $T'(x) > 0$ if $x > 0$, we see that T is increasing on $(0, \infty)$.
6. There is no relative extremum.
7. $$T''(x) = 960\left[\frac{(x^2+4)^2 - x(2)(x^2+4)(2x)}{(x^2+4)^4}\right] = \frac{960(x^2+4)\left[(x^2+4)-4x^2\right]}{(x^2+4)^4} = \frac{960(4-3x^2)}{(x^2+4)^3}.$$

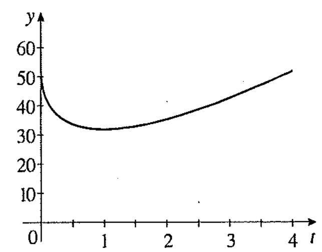
The sign diagram for T'' shows that T is concave downward on $\left(\frac{2\sqrt{3}}{3}, \infty\right)$ and concave upward on $\left(0, \frac{2\sqrt{3}}{3}\right)$.



8. We see from the results of step 7 that $\left(\frac{2\sqrt{3}}{3}, 30\right)$ is an inflection point.



70. Using the curve-sketching guidelines, we obtain the following graph of $f(t) = 20t - 40\sqrt{t} + 52$. The speed of traffic flow decreases until it reaches a minimum of 32 mph at 7 a.m., and then increases again to 52 mph.

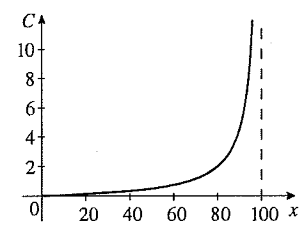


71. $C(x) = \dfrac{0.5x}{100 - x}$. We first gather the following information on C.

1. The domain of C is $[0, 100)$.
2. Setting $x = 0$ gives the y-intercept as 0.

Because of the restricted domain, we omit steps 3 and 4.

5. $C'(x) = 0.5\left[\dfrac{(100 - x)(1) - x(-1)}{(100 - x)^2}\right] = \dfrac{50}{(100 - x)^2} > 0$ for all $x \neq 100$. Therefore C is increasing on $(0, 100)$.
6. There is no relative extremum.
7. $C''(x) = -\dfrac{100}{(100 - x)^3}$, so $C''(x) > 0$ if $x < 100$ and the graph of C is concave upward on $(0, 100)$.
8. There is no inflection point.



72. False. Consider $f(x) = \begin{cases} 0 & x \le 0 \\ 1/x & x > 0 \end{cases}$ f has a vertical asymptote at $x = 0$, but $\lim_{x \to 0^-} f(x) = 0$.

73. False. Consider $f(x) = \begin{cases} 0 & x \le 0 \\ 1/x & x > 0 \end{cases}$ The graph of f intersects its vertical asymptote at the point $(0, 0)$.

74. False. Consider $f(x) = \begin{cases} \dfrac{2x}{x^2 + 4} & x \le 2 \\ -\frac{3}{2}x + \frac{7}{2} & x > 2 \end{cases}$ Since $\lim_{x \to \infty} f(x) = 0$, we see that $y = 0$ is a horizontal asymptote of f. But $f(0) = f\left(\frac{7}{3}\right) = 0$, so the graph of f intersects its asymptote at $(0, 0)$ and $\left(\frac{7}{3}, 0\right)$.

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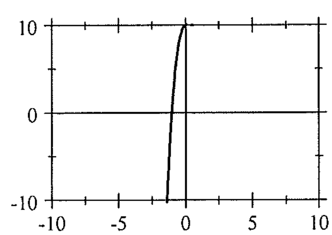
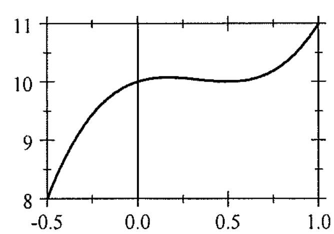
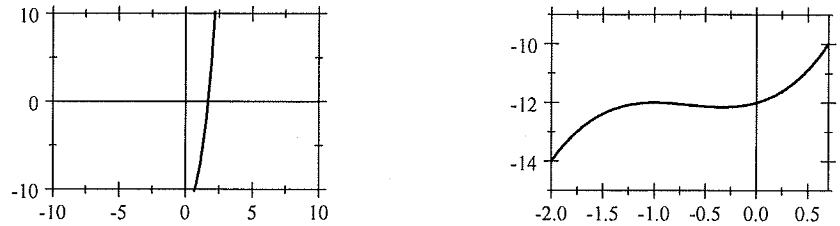
1.





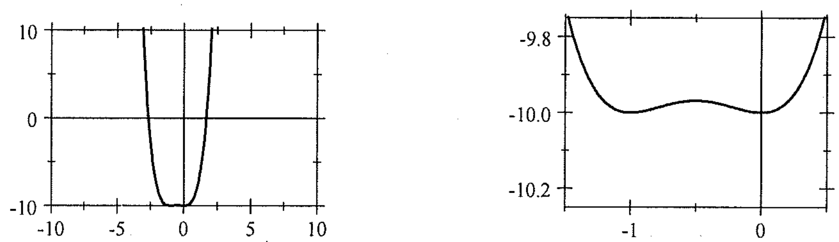
$f(x) = 4x^3 - 4x^2 + x + 10$, so $f'(x) = 12x^2 - 8x + 1 = (6x - 1)(2x - 1) = 0$ if $x = \frac{1}{6}$ or $x = \frac{1}{2}$. The second graph shows that f has a maximum at $x = \frac{1}{6}$ and a minimum at $x = \frac{1}{2}$.

2.



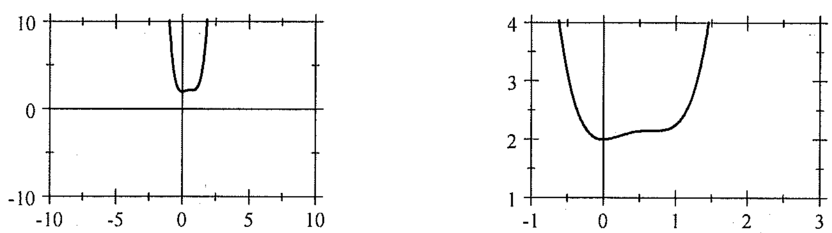
$f(x) = x^3 + 2x^2 + x - 12$, so $f'(x) = 3x^2 + 4x + 1 = (x+1)(3x+1) = 0$ if $x = -1$ or $-\frac{1}{3}$. The second graph shows that f has a maximum at $x = -1$ and a minimum at $x = -\frac{1}{3}$.

3.



$f(x) = \frac{1}{2}x^4 + x^3 + \frac{1}{2}x^2 - 10$, so $f'(x) = 2x^3 + 3x^2 + x = x(x+1)(2x+1) = 0$ if $x = -1, -\frac{1}{2}$, or 0. The second graph shows that x has minima at $x = -1$ and $x = 0$ and a maximum at $x = -\frac{1}{2}$.

4.



$f(x) = 2.25x^4 - 4x^3 + 2x^2 + 2$, so $f'(x) = 9x^3 - 12x^2 + 4x = x(3x-2)^2$ if $x = 0$ or $\frac{2}{3}$. The second graph shows that f has a minimum at $x = 0$ and no extremum at $\frac{2}{3}$.

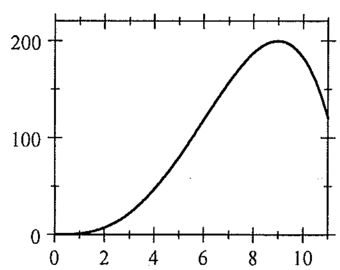
5. -0.9733, 2.3165, and 4.6569.

6. -1.5962, 0.6647, 1.8137, and 3.1178.

7. 1.5142

8. 0.7071

9.



10.