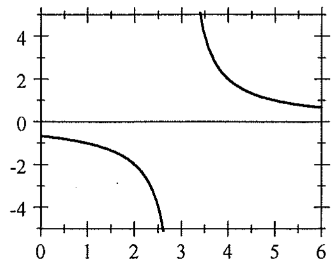
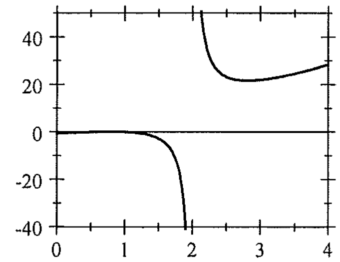
9.



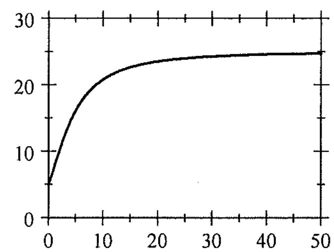
From the graph we see that $f(x)$ does not approach any finite number as x approaches 3.

10.



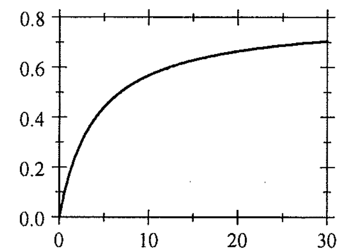
From the graph, we see that $f(x)$ does not approach any finite number as x approaches 2.

11. a.



b. $\lim_{t\to\infty} \frac{25t^2 + 125t + 200}{t^2 + 5t + 40} = 25$, so in the long run the population will approach 25,000.

12. a.



b. $\lim_{t\to\infty} \frac{0.8t}{t + 4.1} = \lim_{t\to\infty} \frac{0.8}{1 + \frac{4.1}{t}} = 0.8.$

2.5 One-Sided Limits and Continuity

Concept Questions page 129

1. $\lim_{x\to 3^-} f(x) = 2$ means $f(x)$ can be made as close to 2 as we please by taking x sufficiently close to but to the left of $x = 3$. $\lim_{x\to 3^+} f(x) = 4$ means $f(x)$ can be made as close to 4 as we please by taking x sufficiently close to but to the right of $x = 3$.

2. a. $\lim_{x\to 1} f(x)$ does not exist because the left- and right-hand limits at $x = 1$ are different.

b. Nothing, because the existence or value of f at $x = 1$ does not depend on the existence (or nonexistence) of the left- or right-hand, or two-sided, limits of f.

3. a. f is continuous at a if $\lim_{x\to a} f(x) = f(a)$.

b. f is continuous on an interval I if f is continuous at each point in I.

4. $f(a) = L = M$.

5. a. f is continuous because the plane does not suddenly jump from one point to another.

b. f is continuous.

c. f is discontinuous because the fare "jumps"after the cab has covered a certain distance or after a certain amount of time has elapsed.

d. f is discontinuous because the rates "jump"by a certain amount (up or down) when it is adjusted at certain times.

6. Refer to page 127 in the text. Answers will vary.

Exercises page 130

1. $\lim_{x\to 2^-} f(x) = 3$ and $\lim_{x\to 2^+} f(x) = 2$, so $\lim_{x\to 2} f(x)$ does not exist.

2. $\lim_{x\to 3^-} f(x) = 3$ and $\lim_{x\to 3^+} f(x) = 5$, so $\lim_{x\to 3} f(x)$ does not exist.

3. $\lim_{x\to -1^-} f(x) = \infty$ and $\lim_{x\to -1^+} f(x) = 2$, so $\lim_{x\to -1} f(x)$ does not exist.

4. $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 3$, so $\lim_{x\to 1} f(x) = 3$.

5. $\lim_{x\to 1^-} f(x) = 0$ and $\lim_{x\to 1^+} f(x) = 2$, so $\lim_{x\to 1} f(x)$ does not exist.

6. $\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = \infty$, so $\lim_{x\to 0} f(x)$ does not exist.

7. $\lim_{x\to 0^-} f(x) = -2$ and $\lim_{x\to 0^+} f(x) = 2$, so $\lim_{x\to 0} f(x)$ does not exist.

8. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 2$.

9. True. **10.** True. **11.** True. **12.** True. **13.** False. **14.** True.

15. True. **16.** True. **17.** False. **18.** True. **19.** True. **20.** False

21. $\lim_{x\to 1^+} (2x+4) = 6$.

22. $\lim_{x\to 1^-} (3x-4) = -1$.

23. $\lim_{x\to 2^-} \dfrac{x-3}{x+2} = \dfrac{2-3}{2+2} = -\dfrac{1}{4}$.

24. $\lim_{x\to 1^+} \dfrac{x+2}{x+1} = \dfrac{1+2}{1+1} = \dfrac{3}{2}$.

25. $\lim_{x\to 0^+} \dfrac{1}{x}$ does not exist because $\dfrac{1}{x} \to \infty$ as $x \to 0$ from the right.

26. $\lim_{x\to 0^-} \dfrac{1}{x} = \infty$; that is, the limit does not exist.

27. $\lim_{x\to 0^+} \dfrac{x-1}{x^2+1} = \dfrac{-1}{1} = -1$.

28. $\lim_{x\to 2^+} \dfrac{x+1}{x^2-2x+3} = \dfrac{2+1}{4-4+3} = 1$.

29. $\lim_{x\to 0^+} \sqrt{x} = \sqrt{\lim_{x\to 0^+} x} = 0$.

30. $\lim_{x\to 2^+} 2\sqrt{x-2} = 2\cdot 0 = 0$.

31. $\lim_{x\to -2^+} \left(2x+\sqrt{2+x}\right) = \lim_{x\to -2^+} 2x + \lim_{x\to -2^+} \sqrt{2+x} = -4+0 = -4$.

32. $\lim_{x\to -5^+} x\left(1+\sqrt{5+x}\right) = -5\left[1+\sqrt{5+(-5)}\right] = -5$.

33. $\lim_{x\to 1^-} \dfrac{1+x}{1-x} = \infty$, that is, the limit does not exist.

34. $\lim_{x \to 1^+} \frac{1+x}{1-x} = -\infty.$

35. $\lim_{x \to 2^-} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2^-} \frac{(x+2)(x-2)}{x-2} = \lim_{x \to 2^-} (x+2) = 4.$

36. $\lim_{x \to -3^+} \frac{\sqrt{x+3}}{x^2+1} = \frac{0}{10} = 0.$

37. $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 = 0$ and $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 2x = 0.$

38. $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (2x+3) = 3$ and $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x+1) = 1.$

39. The function is discontinuous at $x = 0$. Conditions 2 and 3 are violated.

40. The function is not continuous because condition 3 for continuity is not satisfied.

41. The function is continuous everywhere.

42. The function is continuous everywhere.

43. The function is discontinuous at $x = 0$. Condition 3 is violated.

44. The function is not continuous at $x = -1$ because condition 3 for continuity is violated.

45. f is continuous for all values of x.

46. f is continuous for all values of x.

47. f is continuous for all values of x. Note that $x^2 + 1 \geq 1 > 0$.

48. f is continuous for all values of x. Note that $2x^2 + 1 \geq 1 > 0$.

49. f is discontinuous at $x = \frac{1}{2}$, where the denominator is 0. Thus, f is continuous on $\left(-\infty, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \infty\right)$.

50. f is discontinuous at $x = 1$, where the denominator is 0. Thus, f is continuous on $(-\infty, 1)$ and $(1, \infty)$.

51. Observe that $x^2 + x - 2 = (x+2)(x-1) = 0$ if $x = -2$ or $x = 1$, so f is discontinuous at these values of x. Thus, f is continuous on $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$.

52. Observe that $x^2 + 2x - 3 = (x+3)(x-1) = 0$ if $x = -3$ or $x = 1$, so, f is discontinuous at these values of x. Thus, f is continuous on $(-\infty, -3)$, $(-3, 1)$, and $(1, \infty)$.

53. f is continuous everywhere since all three conditions are satisfied.

54. f is continuous everywhere since all three conditions are satisfied.

55. f is continuous everywhere since all three conditions are satisfied.

56. f is not defined at $x = 1$ and is discontinuous there. It is continuous everywhere else.

57. Because the denominator $x^2 - 1 = (x-1)(x+1) = 0$ if $x = -1$ or 1, we see that f is discontinuous at -1 and 1.

58. The function f is not defined at $x = 1$ and $x = 2$. Therefore, f is discontinuous at 1 and 2.

59. Because $x^2 - 3x + 2 = (x - 2)(x - 1) = 0$ if $x = 1$ or 2, we see that the denominator is zero at these points and so f is discontinuous at these numbers.

60. The denominator of the function f is equal to zero when $x^2 - 2x = x(x - 2) = 0$; that is, when $x = 0$ or $x = 2$. Therefore, f is discontinuous at $x = 0$ and $x = 2$.

61. The function f is discontinuous at $x = 4, 5, 6, \ldots, 13$ because the limit of f does not exist at these points.

62. f is discontinuous at $t = 20$, 40, and 60. When $t = 0$, the inventory stands at 750 reams. The level drops to about 200 reams by the twentieth day at which time a new order of 500 reams arrives to replenish the supply. A similar interpretation holds for the other values of t.

63. Having made steady progress up to $x = x_1$, Michael's progress comes to a standstill at that point. Then at $x = x_2$ a sudden breakthrough occurs and he then continues to solve the problem.

64. The total deposits of Franklin make a jump at each of these points as the deposits of the ailing institutions become a part of the total deposits of the parent company.

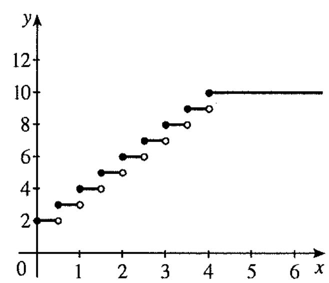
65. Conditions 2 and 3 are not satisfied at any of these points.

66. The function P is discontinuous at $t = 12$, 16, and 28. At $t = 12$, the prime interest rate jumped from $3\frac{1}{2}\%$ to 4%, at $t = 16$ it jumped to $4\frac{1}{2}\%$, and at $t = 28$ it jumped back down to 4%.

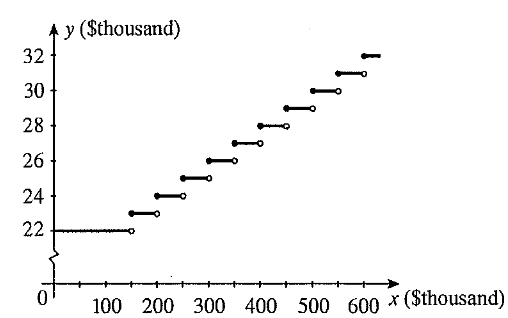
67.

$$f(x) = \begin{cases} 2 & \text{if } 0 < x \le \frac{1}{2} \\ 3 & \text{if } \frac{1}{2} < x \le 1 \\ \vdots & \vdots \\ 10 & \text{if } 4\frac{1}{2} < x \le 5 \end{cases}$$

f is discontinuous at $x = \frac{1}{2}, 1, 1\frac{1}{2}, \ldots, 4$.

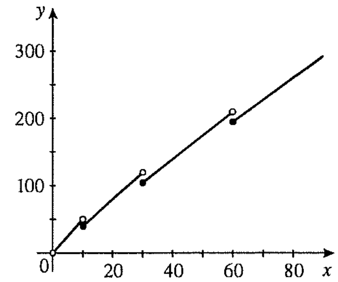


68.



f is discontinuous at $x = 150{,}000$, at $x = 200{,}000$, at $x = 250{,}000$, and so on.

69.



C is discontinuous at $x = 0$, 10, 30, and 60.

70. a. $\lim\limits_{v \to u^+} \dfrac{aLv^3}{v-u} = \infty$. This reflects the fact that when the speed of the fish is very close to that of the current, the energy expended by the fish will be enormous.

b. $\lim\limits_{v \to \infty} \dfrac{aLv^3}{v-u} = \infty$. This says that if the speed of the fish increases greatly, so does the amount of energy required to swim a distance of L ft.

71. a. $\lim\limits_{t \to 0^+} S(t) = \lim\limits_{t \to 0^+} \dfrac{a}{t} + b = \infty$. As the time taken to excite the tissue is made shorter and shorter, the electric current gets stronger and stronger.

b. $\lim\limits_{t \to \infty} \dfrac{a}{t} + b = b$. As the time taken to excite the tissue is made longer and longer, the electric current gets weaker and weaker and approaches b.

72. a. $\lim\limits_{D \to 0^+} L = \lim\limits_{D \to 0^+} \dfrac{Y - (1-D)R}{D} = \infty$, so if the investor puts down next to nothing to secure the loan, the leverage approaches infinity.

b. $\lim\limits_{D \to 1} L = \lim\limits_{D \to 1} \dfrac{Y - (1-D)R}{D} = Y$, so if the investor puts down all of the money to secure the loan, the leverage is equal to the yield.

73. We require that $f(1) = 1 + 2 = 3 = \lim\limits_{x \to 1^+} kx^2 = k$, so $k = 3$.

74. Because $\lim\limits_{x \to -2} \dfrac{x^2 - 4}{x + 2} = \lim\limits_{x \to -2} \dfrac{(x-2)(x+2)}{x+2} = \lim\limits_{x \to -2} (x-2) = -4$, we define $f(-2) = k = -4$, that is, take $k = -4$.

75. a. f is a polynomial of degree 2 and is therefore continuous everywhere, including the interval $[1, 3]$.

b. $f(1) = 3$ and $f(3) = -1$ and so f must have at least one zero in $(1, 3)$.

76. a. f is a polynomial of degree 3 and is thus continuous everywhere.

b. $f(0) = 14$ and $f(1) = -23$ and so f has at least one zero in $(0, 1)$.

77. a. f is a polynomial of degree 3 and is therefore continuous on $[-1, 1]$.

b. $f(-1) = (-1)^3 - 2(-1)^2 + 3(-1) + 2 = -1 - 2 - 3 + 2 = -4$ and $f(1) = 1 - 2 + 3 + 2 = 4$. Because $f(-1)$ and $f(1)$ have opposite signs, we see that f has at least one zero in $(-1, 1)$.