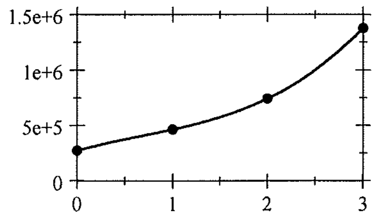
16. a. $y = 44{,}560t^3 - 89{,}394t^2 + 234{,}633t + 273{,}288$.

b.

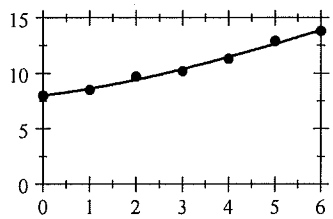


c.

t	$f(t)$
0	273,288
1	463,087
2	741,458
3	1,375,761

17. a. $f(t) = -0.0056t^3 + 0.112t^2 + 0.51t + 8$.

b.

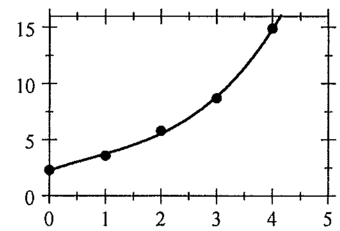


c.

t	0	3	6
$f(t)$	8	10.4	13.9

18. a. $f(t) = 0.2t^3 - 0.45t^2 + 1.75t + 2.26$.

b.

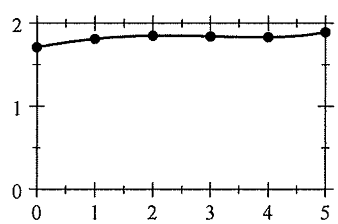


c.

t	0	1	2	3	4
$f(t)$	2.3	3.8	5.6	8.9	14.9

19. a. $f(t) = 0.00125t^4 + 0.0051t^3 - 0.0243t^2 + 0.129t + 1.71$.

b.



c.

t	0	1	2	3	4	5
$f(t)$	1.71	1.81	1.85	1.84	1.83	1.89

d. The average amount of nicotine in 2005 is $f(6) = 2.128$, or approximately 2.13 mg/cigarette.

20. $A(t) = 0.000008140t^4 - 0.00043833t^3 - 0.0001305t^2 + 0.02202t + 2.612$.

2.4 Limits

Concept Questions

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1. The values of $f(x)$ can be made as close to 3 as we please by taking x sufficiently close to $x = 2$.

2. a. Nothing. Whether $f(3)$ is defined or not does not depend on $\lim_{x \to 3} f(x)$.

b. Nothing. $\lim_{x \to 2} f(x)$ has nothing to do with the value of f at $x = 2$.

3. a. $\lim_{x\to 4}\sqrt{x}\left(2x^2+1\right) = \lim_{x\to 4}\left(\sqrt{x}\right)\lim_{x\to 4}\left(2x^2+1\right)$ (Rule 4)

$= \sqrt{4}\left[2(4)^2+1\right]$ (Rules 1 and 3)

$= 66$

b. $\lim_{x\to 1}\left(\frac{2x^2+x+5}{x^4+1}\right)^{3/2} = \left(\lim_{x\to 1}\frac{2x^2+x+5}{x^4+1}\right)^{3/2}$ (Rule 1)

$= \left(\frac{2+1+5}{1+1}\right)^{3/2}$ (Rules 2, 3, and 5)

$= 4^{3/2} = 8$

4. A limit that has the form $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$. For example, $\lim_{x\to 3}\frac{x^2-9}{x-3}$.

5. $\lim_{x\to\infty} f(x) = L$ means $f(x)$ can be made as close to L as we please by taking x sufficiently large.

$\lim_{x\to -\infty} f(x) = M$ means $f(x)$ can be made as close to M as we please by taking negative x as large as we please in absolute value.

Exercises

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1. $\lim_{x\to -2} f(x) = 3$.

2. $\lim_{x\to 1} f(x) = 2$.

3. $\lim_{x\to 3} f(x) = 3$.

4. $\lim_{x\to 1} f(x)$ does not exist. If we consider any value of x to the right of $x = 1$, we find that $f(x) = 3$. On the other hand, if we consider values of x to the left of $x = 1$, $f(x) \leq 1.5$, so that $f(x)$ does not approach a fixed number as x approaches 1.

5. $\lim_{x\to -2} f(x) = 3$.

6. $\lim_{x\to -2} f(x) = 3$.

7. The limit does not exist. If we consider any value of x to the right of $x = -2$, $f(x) \leq 2$. If we consider values of x to the left of $x = -2$, $f(x) \geq -2$. Because $f(x)$ does not approach any one number as x approaches $x = -2$, we conclude that the limit does not exist.

8. The limit does not exist.

9.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	4.61	4.9601	4.9960	5.004	5.0401	5.41

$\lim_{x\to 2}\left(x^2+1\right) = 5$.

10.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.62	0.9602	0.996002	1.004002	1.0402	1.42

$\lim_{x\to 1}(2x^2-1)=1.$

11.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	−1	−1	−1	1	1	1

The limit does not exist.

12.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	−1	−1	−1	1	1	1

The limit does not exist.

13.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	100	10,000	1,000,000	1,000,000	10,000	100

The limit does not exist.

14.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	−10	−100	−1000	1000	100	10

The limit does not exist.

15.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	2.9	2.99	2.999	3.001	3.01	3.1

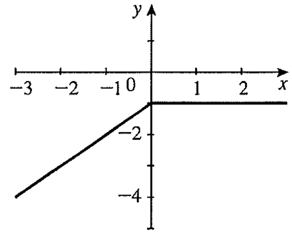
$\lim_{x\to 1}\frac{x^2+x-2}{x-1}=3.$

16.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	1	1	1	1	1	1

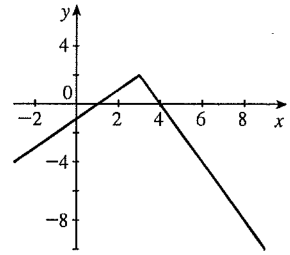
$\lim_{x\to 1}\frac{x-1}{x-1}=1.$

17.



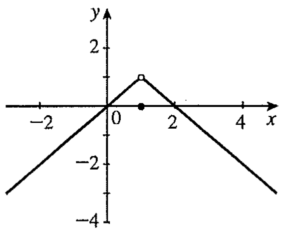
$\lim_{x\to 0}f(x)=-1.$

18.



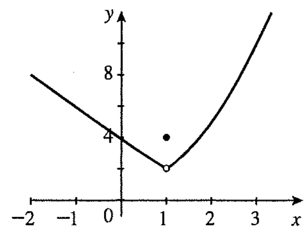
$\lim_{x\to 3}f(x)=2.$

19.



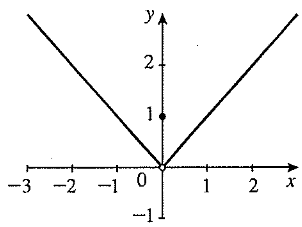
$\lim_{x\to 1} f(x) = 1.$

20.



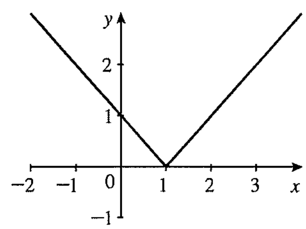
$\lim_{x\to 1} f(x) = 2.$

21.



$\lim_{x\to 0} f(x) = 0.$

22.



$\lim_{x\to 1} f(x) = 0.$

23. $\lim_{x\to 2} 3 = 3.$

24. $\lim_{x\to -2} -3 = -3.$

25. $\lim_{x\to 3} x = 3.$

26. $\lim_{x\to -2} -3x = -3(-2) = 6.$

27. $\lim_{x\to 1} \left(1 - 2x^2\right) = 1 - 2(1)^2 = -1.$

28. $\lim_{t\to 3} \left(4t^2 - 2t + 1\right) = 4(3)^2 - 2(3) + 1 = 31.$

29. $\lim_{x\to 1} \left(2x^3 - 3x^2 + x + 2\right) = 2(1)^3 - 3(1)^2 + 1 + 2$

$= 2.$

30. $\lim_{x\to 0} \left(4x^5 - 20x^2 + 2x + 1\right)$

$= 4(0)^5 - 20(0)^2 + 2(0) + 1 = 1.$

31. $\lim_{s\to 0} \left(2s^2 - 1\right)(2s + 4) = (-1)(4) = -4.$

32. $\lim_{x\to 2} \left(x^2 + 1\right)\left(x^2 - 4\right) = \left(2^2 + 1\right)\left(2^2 - 4\right) = 0.$

33. $\lim_{x\to 2} \frac{2x + 1}{x + 2} = \frac{2(2) + 1}{2 + 2} = \frac{5}{4}.$

34. $\lim_{x\to 1} \frac{x^3 + 1}{2x^3 + 2} = \frac{1^3 + 1}{2\left(1^3\right) + 2} = \frac{2}{4} = \frac{1}{2}.$

35. $\lim_{x\to 2} \sqrt{x + 2} = \sqrt{2 + 2} = 2.$

36. $\lim_{x\to -2} \sqrt[3]{5x + 2} = \sqrt[3]{5(-2) + 2} = \sqrt[3]{-8} = -2.$

37. $\lim_{x\to -3} \sqrt{2x^4 + x^2} = \sqrt{2(-3)^4 + (-3)^2} = \sqrt{162 + 9}$

$= \sqrt{171} = 3\sqrt{19}.$

38. $\lim_{x\to 2} \sqrt{\frac{2x^3 + 4}{x^2 + 1}} = \sqrt{\frac{2(8) + 4}{4 + 1}} = 2.$

39. $\lim_{x\to -1} \frac{\sqrt{x^2 + 8}}{2x + 4} = \frac{\sqrt{(-1)^2 + 8}}{2(-1) + 4} = \frac{\sqrt{9}}{2} = \frac{3}{2}.$

40. $\lim_{x\to 3} \frac{x\sqrt{x^2 + 7}}{2x - \sqrt{2x + 3}} = \frac{3\sqrt{3^2 + 7}}{2(3) - \sqrt{2(3) + 3}} = \frac{12}{3} = 4.$

41. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = 3 - 4 = -1.$

42. $\lim_{x \to a} 2f(x) = 2(3) = 6.$

43. $\lim_{x \to a} [2f(x) - 3g(x)] = \lim_{x \to a} 2f(x) - \lim_{x \to a} 3g(x) = 2(3) - 3(4) = -6.$

44. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = 3 \cdot 4 = 12.$

45. $\lim_{x \to a} \sqrt{g(x)} = \lim_{x \to a} \sqrt{4} = 2.$

46. $\lim_{x \to a} \sqrt[3]{5f(x) + 3g(x)} = \sqrt[3]{5(3) + 3(4)} = \sqrt[3]{27} = 3.$

47. $\lim_{x \to a} \frac{2f(x) - g(x)}{f(x)g(x)} = \frac{2(3) - (4)}{(3)(4)} = \frac{2}{12} = \frac{1}{6}.$

48. $\lim_{x \to a} \frac{g(x) - f(x)}{f(x) + \sqrt{g(x)}} = \frac{4 - 3}{3 + 2} = \frac{1}{5}.$

49. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x - 1} = \lim_{x \to 1} (x + 1) = 1 + 1 = 2.$

50. $\lim_{x \to -2} \frac{x^2 - 4}{x + 2} = \lim_{x \to -2} \frac{(x-2)(x+2)}{x + 2} = \lim_{x \to -2} (x - 2) = -2 - 2 = -4.$

51. $\lim_{x \to 0} \frac{x^2 - x}{x} = \lim_{x \to 0} \frac{x(x-1)}{x} = \lim_{x \to 0} (x - 1) = 0 - 1 = -1.$

52. $\lim_{x \to 0} \frac{2x^2 - 3x}{x} = \lim_{x \to 0} \frac{x(2x-3)}{x} = \lim_{x \to 0} (2x - 3) = -3.$

53. $\lim_{x \to -5} \frac{x^2 - 25}{x + 5} = \lim_{x \to -5} \frac{(x+5)(x-5)}{x + 5} = \lim_{x \to -5} (x - 5) = -10.$

54. $\lim_{b \to -3} \frac{b + 1}{b + 3}$ does not exist.

55. $\lim_{x \to 1} \frac{x}{x - 1}$ does not exist.

56. $\lim_{x \to 2} \frac{x + 2}{x - 2}$ does not exist.

57. $\lim_{x \to -2} \frac{x^2 - x - 6}{x^2 + x - 2} = \lim_{x \to -2} \frac{(x-3)(x+2)}{(x+2)(x-1)} = \lim_{x \to -2} \frac{x - 3}{x - 1} = \frac{-2 - 3}{-2 - 1} = \frac{5}{3}.$

58. $\lim_{z \to 2} \frac{z^3 - 8}{z - 2} = \lim_{z \to 2} \frac{(z-2)(z^2 + 2z + 4)}{z - 2} = \lim_{z \to 2} (z^2 + 2z + 4) = 2^2 + 2(2) + 4 = 12.$

59. $\lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} = \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} \cdot \frac{\sqrt{x} + 1}{\sqrt{x} + 1} = \lim_{x \to 1} \frac{x - 1}{(x-1)(\sqrt{x} + 1)} = \lim_{x \to 1} \frac{1}{\sqrt{x} + 1} = \frac{1}{2}.$

60. $\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} = \lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2} = \lim_{x \to 4} \sqrt{x} + 2 = 2 + 2 = 4.$

61. $\lim_{x \to 1} \frac{x - 1}{x^3 + x^2 - 2x} = \lim_{x \to 1} \frac{x - 1}{x(x-1)(x+2)} = \lim_{x \to 1} \frac{1}{x(x+2)} = \frac{1}{3}.$

62. $\lim_{x \to -2} \frac{4 - x^2}{2x^2 + x^3} = \lim_{x \to -2} \frac{(2-x)(2+x)}{x^2(2+x)} = \lim_{x \to -2} \frac{2 - x}{x^2} = \frac{2 - (-2)}{(-2)^2} = 1.$

63. $\lim_{x \to \infty} f(x) = \infty$ (does not exist) and $\lim_{x \to -\infty} f(x) = \infty$ (does not exist).

64. $\lim_{x\to\infty} f(x) = \infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

65. $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$.

66. $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = 1$.

67. $\lim_{x\to\infty} f(x) = -\infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

68. $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = \infty$ (does not exist).

69. $f(x) = \dfrac{1}{x^2+1}$.

x	1	10	100	1000
$f(x)$	0.5	0.009901	0.0001	0.000001

x	-1	-10	-100	-1000
$f(x)$	0.5	0.009901	0.0001	0.000001

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$.

70. $f(x) = \dfrac{2x}{x+1}$.

x	1	10	100	1000
$f(x)$	1	1.818	1.980	1.998

x	-5	-10	-100	-1000
$f(x)$	2.5	2.222	2.020	2.002

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 2$.

71. $f(x) = 3x^3 - x^2 + 10$.

x	1	5	10	100	1000
$f(x)$	12	360	2910	2.99×10^6	2.999×10^9

x	-1	-5	-10	-100	-1000
$f(x)$	6	-390	-3090	-3.01×10^6	-3.0×10^9

$\lim_{x\to\infty} f(x) = \infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

72. $f(x) = \dfrac{|x|}{x}$

x	1	10	100
$f(x)$	1	1	1

x	-1	-10	-100
$f(x)$	-1	-1	-1

$\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = -1$.

73. $\displaystyle\lim_{x\to\infty} \frac{3x+2}{x-5} = \lim_{x\to\infty} \frac{3+\frac{2}{x}}{1-\frac{5}{x}} = \frac{3}{1} = 3.$

74. $\lim_{x\to-\infty}\dfrac{4x^2-1}{x+2}=\lim_{x\to-\infty}\dfrac{4x-\frac{1}{x}}{1+\frac{2}{x}}=-\infty$; that is, the limit does not exist.

75. $\lim_{x\to-\infty}\dfrac{3x^3+x^2+1}{x^3+1}=\lim_{x\to-\infty}\dfrac{3+\frac{1}{x}+\frac{1}{x^3}}{1+\frac{1}{x^3}}=3.$

76. $\lim_{x\to\infty}\dfrac{2x^2+3x+1}{x^4-x^2}=\lim_{x\to\infty}\dfrac{\frac{2}{x^2}+\frac{3}{x^3}+\frac{1}{x^4}}{1-\frac{1}{x^2}}=0.$

77. $\lim_{x\to-\infty}\dfrac{x^4+1}{x^3-1}=\lim_{x\to-\infty}\dfrac{x+\frac{1}{x^3}}{1-\frac{1}{x^3}}=-\infty$; that is, the limit does not exist.

78. $\lim_{x\to\infty}\dfrac{4x^4-3x^2+1}{2x^4+x^3+x^2+x+1}=\lim_{x\to\infty}\dfrac{4-\frac{3}{x^2}+\frac{1}{x^4}}{2+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}}=2.$

79. $\lim_{x\to\infty}\dfrac{x^5-x^3+x-1}{x^6+2x^2+1}=\lim_{x\to\infty}\dfrac{\frac{1}{x}-\frac{1}{x^3}+\frac{1}{x^5}-\frac{1}{x^6}}{1+\frac{2}{x^4}+\frac{1}{x^6}}=0.$

80. $\lim_{x\to\infty}\dfrac{2x^2-1}{x^3+x^2+1}=\lim_{x\to\infty}\dfrac{\frac{2}{x}-\frac{1}{x^3}}{1+\frac{1}{x}+\frac{1}{x^3}}=0.$

81. **a.** The cost of removing 50% of the pollutant is $C(50)=\dfrac{0.5(50)}{100-50}=0.5$, or \$500,000. Similarly, we find that the cost of removing 60%, 70%, 80%, 90%, and 95% of the pollutant is \$750,000, \$1,166,667, \$2,000,000, \$4,500,000, and \$9,500,000, respectively.

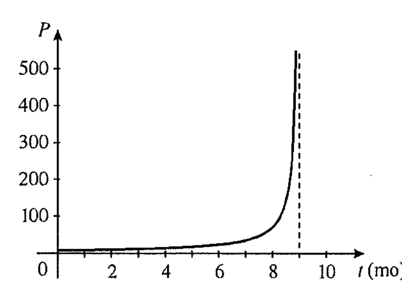
b. $\lim_{x\to100}\dfrac{0.5x}{100-x}=\infty$, which means that the cost of removing the pollutant increases without bound if we wish to remove almost all of the pollutant.

82. **a.** The number present initially is given by $P(0)=\dfrac{72}{9-0}=8.$

b. As t approaches 9 (remember that $0<t<9$), the denominator approaches 0 while the numerator remains constant at 72. Therefore, $P(t)$ gets larger and larger. Thus,

$\lim_{t\to9}P(t)=\lim_{t\to9}\dfrac{72}{9-t}=\infty.$

c.



83. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty} 2.2 + \frac{2500}{x} = 2.2$, or \$2.20 per DVD. In the long run, the average cost of producing x DVDs approaches \$2.20/disc.

84. $\lim_{t\to\infty} C(t) = \lim_{t\to\infty} \frac{0.2t}{t^2+1} = \lim_{t\to\infty} \frac{\frac{0.2}{t}}{1+\frac{1}{t^2}} = 0$, which says that the concentration of drug in the bloodstream eventually decreases to zero.

85. a. $T(1) = \frac{120}{1+4} = 24$, or \$24 million. $T(2) = \frac{120(2)^2}{8} = 60$, or \$60 million. $T(3) = \frac{120(3)^2}{13} = 83.1$, or \$83.1 million.

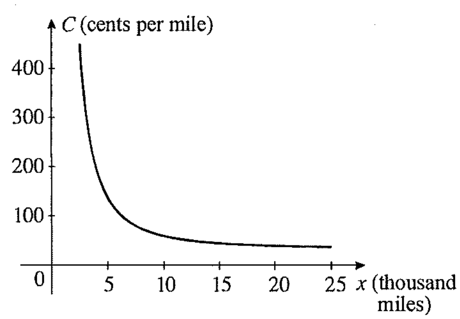
b. In the long run, the movie will gross $\lim_{x\to\infty} \frac{120x^2}{x^2+4} = \lim_{x\to\infty} \frac{120}{1+\frac{4}{x^2}} = 120$, or \$120 million.

86. a. The current population is $P(0) = \frac{200}{40} = 5$, or 5000.

b. The population in the long run will be $\lim_{t\to\infty} \frac{25t^2+125t+200}{t^2+5t+40} = \lim_{t\to\infty} \frac{25+\frac{125}{t}+\frac{200}{t^2}}{1+\frac{5}{t}+\frac{40}{t^2}} = 25$, or 25,000.

87. a. The average cost of driving 5000 miles per year is $C(5) = \frac{2410}{5^{1.95}} + 32.8 \approx 137.28$, or 137.3 cents per mile. Similarly, we see that the average costs of driving 10, 15, 20, and 25 thousand miles per year are 59.8, 45.1, 39.8, and 37.3 cents per mile, respectively.

b.



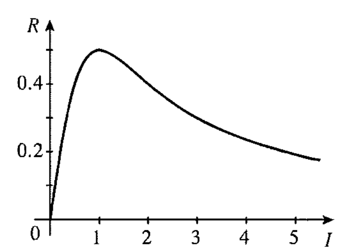
c. It approaches 32.8 cents per mile.

88. a. $R(I) = \frac{I}{1+I^2}$

I	0	1	2	3	4	5
$R(I)$	0	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{4}{17}$	$\frac{5}{26}$

b. $\lim_{I\to\infty} R(I) = \lim_{I\to\infty} \frac{I}{1+I^2} = 0.$

c.



89. False. Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ Then $\lim_{x\to 0} f(x) = 1$, but $f(1)$ is not defined.

90. True.

91. True. Division by zero is not permitted.

92. False. Let $f(x) = (x-3)^2$ and $g(x) = x - 3$. Then $\lim_{x\to 3} f(x) = 0$ and $\lim_{x\to 3} g(x) = 0$, but

$$\lim_{x\to 3} \frac{f(x)}{g(x)} = \lim_{x\to 3} \frac{(x-3)^2}{x-3} = \lim_{x\to 3} (x-3) = 0.$$

93. True. Each limit in the sum exists. Therefore, $\lim_{x\to 2} \left(\frac{x}{x+1} + \frac{3}{x-1} \right) = \lim_{x\to 2} \frac{x}{x+1} + \lim_{x\to 2} \frac{3}{x-1} = \frac{2}{3} + \frac{3}{1} = \frac{11}{3}$.

94. False. Neither of the limits $\lim_{x\to 1} \frac{2x}{x-1}$ and $\lim_{x\to 1} \frac{2}{x-1}$ exists.

95. $\lim_{x\to\infty} \frac{ax}{x+b} = \lim_{x\to\infty} \frac{a}{1+\frac{b}{x}} = a$. As the amount of substrate becomes very large, the initial speed approaches the constant a moles per liter per second.

96. Consider the functions $f(x) = 1/x$ and $g(x) = -1/x$. Observe that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} [f(x) + g(x)] = \lim_{x\to 0} 0 = 0$. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

97. Consider the functions $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x \geq 0 \end{cases}$ Then $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} [f(x) g(x)] = \lim_{x\to 0} (-1) = -1$. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

98. Take $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$, and $a = 0$. Then $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{1}{x} \cdot \frac{x^2}{1} = \lim_{x\to 0} x = a$ exists. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

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1. 5

2. 11

3. 3

4. 0

5. $\frac{2}{3}$

6. $\frac{10}{11}$

7. $e^2 \approx 7.38906$

8. $\ln 2 \approx 0.693147$