70. $N(t) = 1.42 \cdot x(t) = \dfrac{1.42 \cdot 7(t+10)^2}{(t+10)^2 + 2(t+15)^2} = \dfrac{9.94(t+10)^2}{(t+10)^2 + 2(t+15)^2}$. The number of jobs created 6 months from now will be $N(6) = \dfrac{9.94(16)^2}{(16)^2 + 2(21)^2} \approx 2.24$, or approximately 2.24 million jobs. The number of jobs created 12 months from now will be $N(12) = \dfrac{9.94(22)^2}{(22)^2 + 2(27)^2} \approx 2.48$, or approximately 2.48 million jobs.

71. **a.** $s = f + g + h = (f + g) + h = f + (g + h)$. This suggests we define the sum s by $s(x) = (f + g + h)(x) = f(x) + g(x) + h(x)$.

b. Let f, g, and h define the revenue (in dollars) in week t of three branches of a store. Then its total revenue (in dollars) in week t is $s(t) = (f + g + h)(t) = f(t) + g(t) + h(t)$.

72. **a.** $(h \circ g \circ f)(x) = h(g(f(x)))$

b. Let t denote time. Suppose f gives the number of people at time t in a town, g gives the number of cars as a function of the number of people in the town, and H gives the amount of carbon monoxide in the atmosphere. Then $(h \circ g \circ f)(t) = h(g(f(t)))$ gives the amount of carbon monoxide in the atmosphere at time t.

73. True. $(f + g)(x) = f(x) + g(x) = g(x) + f(x) = (g + f)(x)$.

74. False. Let $f(x) = x + 2$ and $g(x) = \sqrt{x}$. Then $(g \circ f)(x) = \sqrt{x + 2}$ is defined at $x = -1$, But $(f \circ g)(x) = \sqrt{x} + 2$ is not defined at $x = -1$.

75. False. Take $f(x) = \sqrt{x}$ and $g(x) = x + 1$. Then $(g \circ f)(x) = \sqrt{x} + 1$, but $(f \circ g)(x) = \sqrt{x + 1}$.

76. False. Take $f(x) = x + 1$. Then $(f \circ f)(x) = f(f(x)) = x + 2$, but $f^2(x) = [f(x)]^2 = (x + 1)^2 = x^2 + 2x + 1$.

77. True. $(h \circ (g \circ f))(x) = h((g \circ f)(x)) = h(g(f(x)))$ and $((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$.

78. False. Take $h(x) = \sqrt{x}$, $g(x) = x$, and $f(x) = x^2$. Then $(h \circ (g + f))(x) = h(x + x^2) = \sqrt{x + x^2} \neq ((h \circ g) + (h \circ f))(x) = h(g(x)) + h(f(x)) = \sqrt{x} + \sqrt{x^2}$.

2.3 Functions and Mathematical Models

Concept Questions

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1. See page 78 of the text. Answers will vary.

2. **a.** $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$, where $a_n \neq 0$ and n is a positive integer. An example is $P(x) = 4x^3 - 3x^2 + 2$.

b. $R(x) = \dfrac{P(x)}{Q(x)}$, where P and Q are polynomials with $Q(x) \neq 0$. An example is $R(x) = \dfrac{3x^4 - 2x^2 + 1}{x^2 + 3x + 5}$.

3. **a.** A demand function $p = D(x)$ gives the relationship between the unit price of a commodity p and the quantity x demanded. A supply function $p = S(x)$ gives the relationship between the unit price of a commodity p and the quantity x the supplier will make available in the marketplace.

b. Market equilibrium occurs when the quantity produced is equal to the quantity demanded. To find the market equilibrium, we solve the equations $p = D(x)$ and $p = S(x)$ simultaneously.

Exercises page 88

1. Yes. $2x+3y=6$ and so $y=-\frac{2}{3}x+2$.

2. Yes. $4y=2x+7$ and so $y=\frac{1}{2}x+\frac{7}{4}$.

3. Yes. $2y=x+4$ and so $y=\frac{1}{2}x+2$.

4. Yes. $3y=2x-8$ and so $y=\frac{2}{3}x-\frac{8}{3}$.

5. Yes. $4y=2x+9$ and so $y=\frac{1}{2}x+\frac{9}{4}$.

6. Yes. $6y=3x+7$ and so $y=\frac{1}{2}x+\frac{7}{6}$.

7. No, because of the term x^2.

8. No, because of the term $\sqrt{x}$.

9. f is a polynomial function in x of degree 6.

10. f is a rational function.

11. Expanding $G(x)=2(x^2-3)^3$, we have $G(x)=2x^6-18x^4+54x^2-54$, and we conclude that G is a polynomial function of degree 6 in x.

12. We can write $H(x)=\dfrac{2}{x^3}+\dfrac{5}{x^2}+6=\dfrac{2+5x+6x^3}{x^3}$ and conclude that H is a rational function.

13. f is neither a polynomial nor a rational function.

14. f is a rational function.

15. $f(0)=2$ gives $f(0)=m(0)+b=b=2$. Next, $f(3)=-1$ gives $f(3)=m(3)+b=-1$. Substituting $b=2$ in this last equation, we have $3m+2=-1$, or $3m=-3$, and therefore, $m=-1$ and $b=2$.

16. $f(2)=4$ gives $f(2)=2m+b=4$. We also know that $m=-1$. Therefore, we have $2(-1)+b=4$ and so $b=6$.

17. **a.** $C(x)=8x+40{,}000$.
    b. $R(x)=12x$.
    c. $P(x)=R(x)-C(x)=12x-(8x+40{,}000)=4x-40{,}000$.
    d. $P(8000)=4(8000)-40{,}000=-8000$, or a loss of \$8000. $P(12{,}000)=4(12{,}000)-40{,}000=8000$, or a profit of \$8000.

18. **a.** $C(x)=14x+100{,}000$.
    b. $R(x)=20x$.
    c. $P(x)=R(x)-C(x)=20x-(14x+100{,}000)=6x-100{,}000$.
    d. $P(12{,}000)=6(12{,}000)-100{,}000=-28{,}000$, or a loss of \$28,000.
    $P(20{,}000)=6(20{,}000)-100{,}000=20{,}000$, or a profit of \$20,000.

19. The individual's disposable income is $D=(1-0.28)\cdot 60{,}000=43{,}200$, or \$43,200.

20. The child should receive $D(0.4)=\dfrac{(0.4)(500)}{1.7}\approx 117.65$, or approximately 118 mg.

21. The child should receive $D(4)=\left(\dfrac{4+1}{24}\right)(500)\approx 104.17$, or approximately 104 mg.

22. **a.** The graph of f passes through the points $P_1(0,17.5)$ and $P_2(10,10.3)$. Its slope is $\dfrac{10.3-17.5}{10-0}=-0.72$. An equation of the line is $y-17.5=-0.72(t-0)$ or $y=-0.72t+17.5$, so the linear function is $f(t)=-0.72t+17.5$.

b. The rate was decreasing at 0.72% per year.

c. The percentage of high school students who drink and drive at the beginning of 2014 is projected to be $f(13) = -0.72(13) + 17.5 = 8.14$, or 8.14%.

23. a. The slope of the graph of f is a line with slope -13.2 passing through the point $(0, 400)$, so an equation of the line is $y - 400 = -13.2(t - 0)$ or $y = -13.2t + 400$, and the required function is $f(t) = -13.2t + 400$.

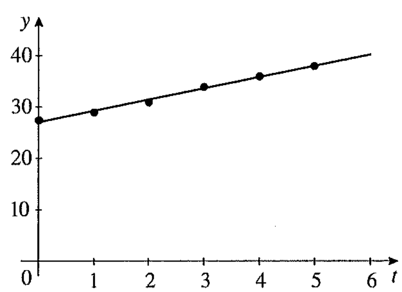
b. The emissions cap is projected to be $f(2) = -13.2(2) + 400 = 373.6$, or 373.6 million metric tons of carbon dioxide equivalent.

24. a. The graph of f is a line through the points $P_1(0, 0.7)$ and $P_2(20, 1.2)$, so it has slope $\frac{1.2 - 0.7}{20 - 0} = 0.025$. Its equation is $y - 0.7 = 0.025(t - 0)$ or $y = 0.025t + 0.7$. The required function is thus $f(t) = 0.025t + 0.7$.

b. The projected annual rate of growth is the slope of the graph of f, that is, 0.025 billion per year, or 25 million per year.

c. The projected number of boardings per year in 2022 is $f(10) = 0.025(10) + 0.7 = 0.95$, or 950 million boardings per year.

25. a.



b. The projected revenue in 2010 is projected to be $f(6) = 2.19(6) + 27.12 = 40.26$, or \$40.26 billion.

c. The rate of increase is the slope of the graph of f, that is, 2.19 (billion dollars per year).

26. Two hours after starting work, the average worker will be assembling at the rate of $f(2) = -\frac{3}{2}(2)^2 + 6(2) + 10 = 16$, or 16 phones per hour.

27. $P(28) = -\frac{1}{8}(28)^2 + 7(28) + 30 = 128$, or \$128,000.

28. a. The amount paid out in 2010 was $S(0) = 0.72$, or \$0.72 trillion (or \$720 billion).

b. The amount paid out in 2030 is projected to be $S(3) = 0.1375(3)^2 + 0.5185(3) + 0.72 = 3.513$, or \$3.513 trillion.

29. a. The average time spent per day in 2009 was $f(0) = 21.76$ (minutes).

b. The average time spent per day in 2013 is projected to be $f(4) = 2.25(4)^2 + 13.41(4) + 21.76 = 111.4$ (minutes).

30. a. The GDP in 2011 was $G(0) = 15$, or \$15 trillion.

b. The projected GDP in 2015 is $G(4) = 0.064(4)^2 + 0.473(4) + 15.0 = 17.916$, or \$17.196 trillion.

31. a. The GDP per capita in 2000 was $f(10) = 1.86251(10)^2 - 28.08043(10) + 884 = 789.4467$, or \$789.45.

b. The GDP per capita in 2030 is projected to be $f(40) = 1.86251(40)^2 - 28.08043(40) + 884 = 2740.7988$, or \$2740.80.

32. **a.** The number of enterprise IM accounts in 2006 is given by $N(0) = 59.7$, or 59.7 million.

b. The number of enterprise IM accounts in 2010, assuming a continuing trend, is given by $N(4) = 2.96(4)^2 + 11.37(4) + 59.7 = 152.54$ million.

33. $S(6) = 0.73(6)^2 + 15.8(6) + 2.7 = 123.78$ million kilowatt-hr.
$S(8) = 0.73(8)^2 + 15.8(8) + 2.7 = 175.82$ million kilowatt-hr.

34. The U.S. public debt in 2005 was $f(0) = 8.246$, or \$8.246 trillion. The public debt in 2008 was $f(3) = -0.03817(3)^3 + 0.4571(3)^2 - 0.1976(3) + 8.246 = 10.73651$, or approximately \$10.74 trillion.

35. The percentage who expected to work past age 65 in 1991 was $f(0) = 11$, or 11%. The percentage in 2013 was $f(22) = 0.004545(22)^3 - 0.1113(22)^2 + 1.385(22) + 11 = 35.99596$, or approximately 36%.

36. $N(0) = 0.7$ per 100 million vehicle miles driven. $N(7) = 0.0336(7)^3 - 0.118(7)^2 + 0.215(7) + 0.7 = 7.9478$ per 100 million vehicle miles driven.

37. **a.** Total global mobile data traffic in 2009 was $f(0) = 0.06$, or 60,000 terabytes.

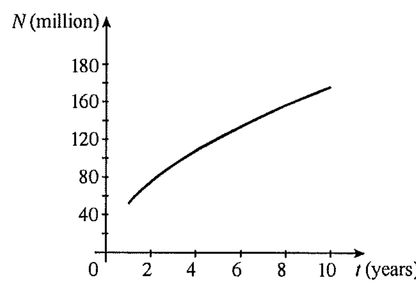
b. The total in 2014 will be $f(5) = 0.021(5)^3 + 0.015(5)^2 + 0.12(5) + 0.06 = 3.66$, or 3.66 million terabytes.

38. Here $Y = 0.06$, $D = 0.2$, and $R = 0.05$, so the leveraged return is $L = \dfrac{0.06 - (1 - 0.2)(0.05)}{0.2} = 0.1$, or 10%.

39. **a.** We first construct a table.

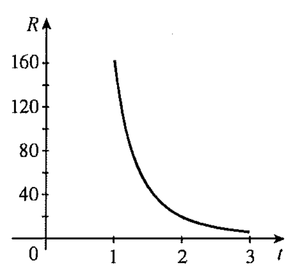
t	$N(t)$
1	52
2	75
3	93
4	109
5	122

t	$N(t)$
6	135
7	146
8	157
9	167
10	177



b. The number of viewers in 2012 is given by $N(10) = 52(10)^{0.531} \approx 176.61$, or approximately 177 million viewers.

40. **a.**



$R(1) = 162.8(1)^{-3.025} = 162.8$, $R(2) = 162.8(2)^{-3.025} \approx 20.0$, and $R(3) = 162.8(3)^{-3.025} \approx 5.9$.

b. The infant mortality rates in 1900, 1950, and 2000 are 162.8, 20.0, and 5.9 per 1000 live births, respectively.

41. $N(5) = 0.0018425(10)^{2.5} \approx 0.58265$, or approximately 0.583 million. $N(13) = 0.0018425(18)^{2.5} \approx 2.5327$, or approximately 2.5327 million.

42. **a.** $S(0) = 4.3(0 + 2)^{0.94} \approx 8.24967$, or approximately \$8.25 billion.

b. $S(8) = 4.3(8 + 2)^{0.94} \approx 37.45$, or approximately \$37.45 billion.

86. True. If $P(x)$ is a polynomial function, then $P(x) = \frac{P(x)}{1}$ and so it is a rational function. The converse is false. For example, $R(x) = \frac{x+1}{x-1}$ is a rational function that is not a polynomial.

87. False. $f(x) = x^{1/2}$ is not defined for negative values of x.

88. False. A power function has the form x^r, where r is a real number.

Using Technology

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1. $(-3.0414, 0.1503)$, $(3.0414, 7.4497)$.

2. $(-5.3852, 9.8007)$, $(5.3852, -4.2007)$.

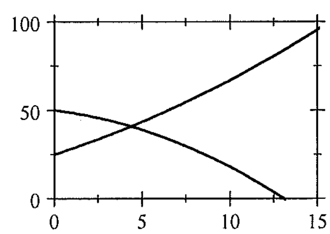
3. $(-2.3371, 2.4117)$, $(6.0514, -2.5015)$.

4. $(-2.5863, -0.3585)$, $(6.1863, -4.5694)$.

5. $(-1.0219, -6.3461)$, $(1.2414, -1.5931)$, and $(5.7805, 7.9391)$.

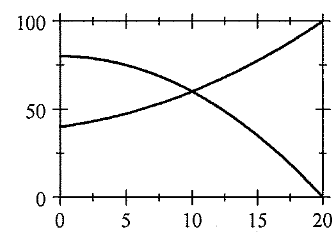
6. $(-0.0484, 2.0609)$, $(2.0823, 2.8986)$, and $(4.9661, 1.1405)$.

7. a.



b. 438 wall clocks; \$40.92.

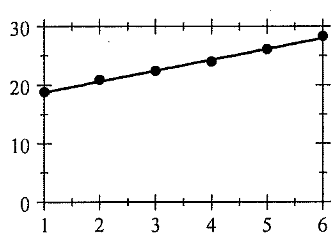
8. a.



b. 1000 cameras; \$60.00.

9. a. $f(t) = 1.85t + 16.9$.

b.



c.

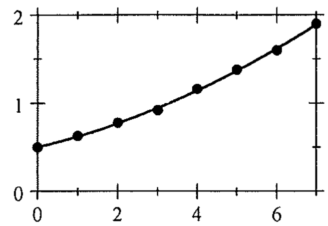
t	y
1	18.8
2	20.6
3	22.5
4	24.3
5	26.2
6	28.0

These values are close to the given data.

d. $f(8) = 1.85(8) + 16.9 = 31.7$ gallons.

10. **a.** $f(t) = 0.0128t^2 + 0.109t + 0.50.$

b.



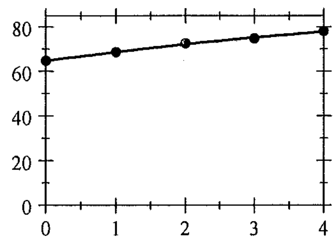
c.

t	y
0	0.50
3	0.94
6	1.61
7	1.89

These values are close to the given data.

11. **a.** $f(t) = -0.221t^2 + 4.14t + 64.8.$

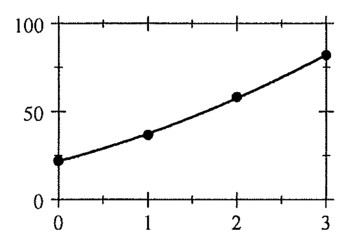
b.



c. 77.8 million

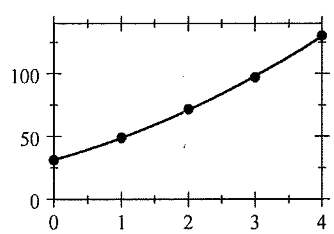
12. **a.** $f(t) = 2.25x^2 + 13.41x + 21.76.$

b.



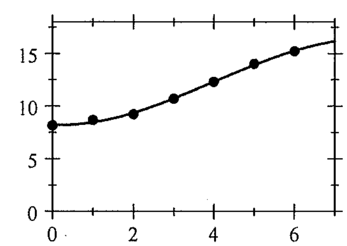
13. **a.** $f(t) = 2.4t^2 + 15t + 31.4.$

b.



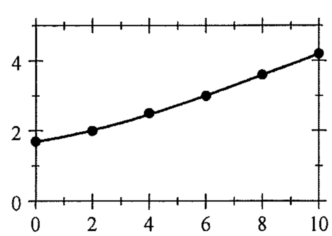
14. **a.** $f(t) = -0.038167t^3 + 0.45713t^2 - 0.19758t + 8.2457.$

b.



15. **a.** $f(t) = -0.00081t^3 + 0.0206t^2 + 0.125t + 1.69.$

b.



c.

t	y
1	1.8
5	2.7
10	4.2

The revenues were \$1.8 trillion in 2001, \$2.7 trillion in 2005, and \$4.2 trillion in 2010.