

Mech. 2.

You are to perform an experiment investigating the conservation of mechanical energy involving a transformation from initial gravitational potential energy to translational kinetic energy.

- (a) You are given the equipment listed below, all the supports required to hold the equipment, and a lab table. On the list below, indicate each piece of equipment you would use by checking the line next to each item.

| | | |
|---------------------------------|--|---|
| <input type="checkbox"/> Track | <input checked="" type="checkbox"/> Meterstick | <input type="checkbox"/> Set of objects of different masses |
| <input type="checkbox"/> Cart | <input type="checkbox"/> Electronic balance | <input type="checkbox"/> Lightweight low-friction pulley |
| <input type="checkbox"/> String | <input checked="" type="checkbox"/> Stopwatch | |

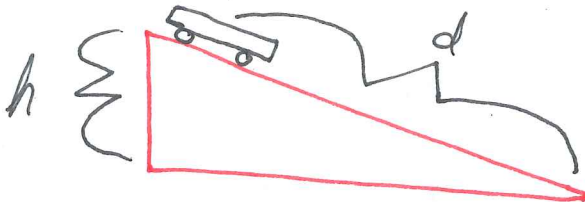
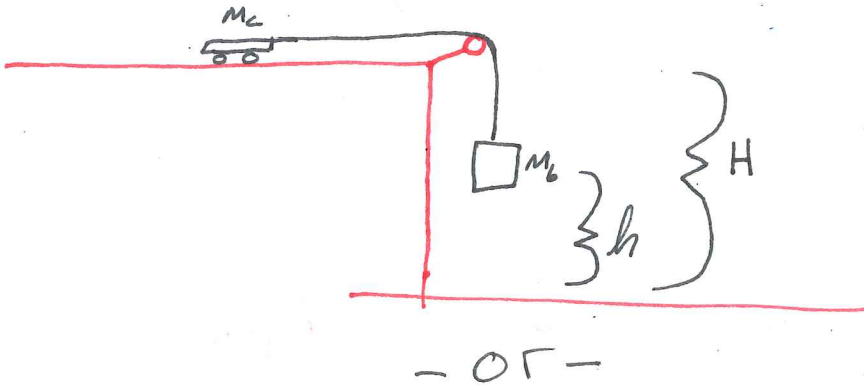
- (b) Outline a procedure for performing the experiment. Include a diagram of your experimental setup. Label the equipment in your diagram. Also include a description of the measurements you would make and a symbol for each measurement.

Height needed for GPE 1pt.

Distance & time to calc velocity 1pt.

Diagram → Height measurement 1pt.

Diagram → Distance measurement 1pt.



- (c) Give a detailed account of the calculations of gravitational potential energy and translational kinetic energy both before and after the transformation, in terms of the quantities measured in part (b).

Initial Potential $U_{g0} = m_c g h + m_b g h$ 1 pt.

Final Potential $U_{gf} = m_c g h$ 1 pt.

Initial Kinetic $KE_0 = \frac{1}{2} m v^2 = 0$ 1 pt.

Final Kinetic $KE_f = \frac{1}{2} (m_c + m_b) v_f^2$ 1 pt.

Instantaneous Velocity $d = \frac{1}{2} (v_0 + v_f) t$ 2 pts.

- (d) After your first trial, your calculations show that the energy increased during the experiment. Assuming you made no mathematical errors, give a reasonable explanation for this result.

Cause for increase in energy 1 pt.

Explanation related to cause 1 pt.

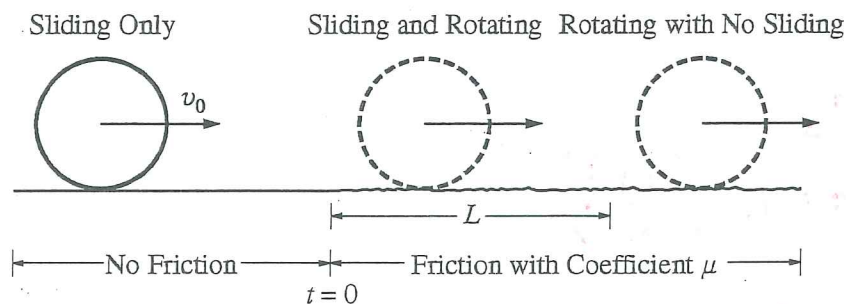
An unintentional push was applied to the cart, thus increasing the initial energy

- (e) On all other trials, your calculations show that the energy decreased during the experiment. Assuming you made no mathematical errors, give a reasonable physical explanation for the fact that the average energy you determined decreased. Include references to conservative and nonconservative forces, as appropriate.

Reasonable cause for decrease 1 pt.

Explanation related to cause 1 pt.

Friction is acting on the object causing the speed to decrease, thereby decreasing the energy.



Mech. 3.

A ring of mass M , radius R , and rotational inertia MR^2 is initially sliding on a frictionless surface at constant velocity v_0 to the right, as shown above. At time $t = 0$ it encounters a surface with coefficient of friction μ and begins sliding and rotating. After traveling a distance L , the ring begins rolling without sliding. Express all answers to the following in terms of M , R , v_0 , μ , and fundamental constants, as appropriate.

(a) Starting from Newton's second law in either translational or rotational form, as appropriate, derive a differential equation that can be used to solve for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

$$\begin{aligned} \sum F = -F_f = ma & \quad F_f = \mu N = \mu mg \\ -\mu mg = ma & \rightarrow -\mu g = a \rightarrow \frac{dv}{dt} = -\mu g \\ a = \frac{dv}{dt} & \end{aligned}$$

ii. The angular velocity ω of the ring as a function of time t

$$\begin{aligned} \tau = MR^2 a & \quad a = \frac{d\omega}{dt} \\ \tau = \mu MgR & \quad \frac{d\omega}{dt} = \frac{\mu g}{R} \end{aligned}$$

(b) Derive an expression for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

$$\int_{v_0}^v dv = \int_0^t -\mu g dt \Rightarrow v = v_0 - \mu g t$$

ii. The angular velocity ω of the ring as a function of time t

$$\int_0^\omega d\omega = \int_0^t \frac{\mu g}{R} dt \rightarrow \omega = \frac{\mu g t}{R}$$

(c) Derive an expression for the time it takes the ring to travel the distance L .

$$\begin{aligned}v &= d \dot{\theta} \\v &= R \omega \\v_0 - \mu g t &= R \left(\frac{\mu g t}{R} \right)\end{aligned}$$

$$t = \frac{v_0}{2\mu g}$$

(d) Derive an expression for the magnitude of the velocity of the ring immediately after it has traveled the distance L .

$$v = v_0 + a t$$

$$v = v_0 - \mu g \left(\frac{v_0}{2\mu g} \right)$$

$$v = \frac{v_0}{2}$$

(e) Derive an expression for the distance L .

$$L = \int_0^t (v_0 - \mu g t) dt$$

$$L = \int_0^{\frac{v_0}{2\mu g}} (v_0 - \mu g t) dt = \left[v_0 t - \frac{1}{2} \mu g t^2 \right]_0^{\frac{v_0}{2\mu g}}$$

$$L = \frac{3v_0^2}{8\mu g}$$