skip 5.3

Chapter 5.2 – Logarithmic Functions

& E H

The following relationship is true:

$$y = \log_2 x \ \textit{if and only if} \ 2^y = x$$

Consider the logarithm,

$$\log_{10} 100 = 2$$

What relationship must be true according to this logarithm? $10^2 = 100$

Simply, $\textit{If } \log_a b = c, \textit{ then } a^c = b$. (Both "a" and "b" MUST be POSITIVE!!!) This is known as the equivalent exponential form. In fact, logarithmic and exponential functions are inverses of each other.

CONVERSIONS:

Convert to either exponential or logarithmic form:

1. $81 = 3^4$ $\boxed{\log_3 81 = 4}$

2. $\log_9 3 = \frac{1}{2}$ $\boxed{9^{1/2} = 3}$

3. $7 = \sqrt{49}$ $7 = 49^{1/2}$ $\boxed{\log_{49} 7 = 1/2}$

PROPERTIES OF LOGARITHMIC FUNCTIONS (Numerous, but not difficult to remember and apply)

(1) $\log_b 1 = 0$

$b^0 = 1$

(2) $\log_b b = 1$

$b^1 = b$

(3) $\log_b b^x = x$

$x \log_b b = x$; $x(1) = x$

(4) $b^{\log_b x} = x$

(5) $\log_b M \cdot N = \log_b M + \log_b N$

(6) $\log_b \dfrac{M}{N} = \log_b M - \log_b N$

(7) $\log_b M^p = p \log_b M$

(8) $\log_b M = \log_b N, \ \ iff \ \ M = N$

Rewrite the following using as many laws of logarithms as possible:

4. $\log_5 x^3 y^4$ $= \log_5 x^3 + \log_5 y^4$

$= \boxed{3\log_5 x + 4\log_5 y}$

5. $\log_3 \dfrac{x^2+1}{3^x}$ $= \log_3(x^2+1) - \log_3 3^x$

~~$= \log_3(x^2+1) - x\log_3 3 =$~~

$= \log_3(x^2+1) - x\log_3 3 = \boxed{\log_3(x^2+1) - x}$

6. $\log \dfrac{x^2\sqrt{x^2-1}}{10^x}$ $= \log x^2 + \log(x^2-1)^{1/2} - \log 10^x$

$= 2\log x + \frac{1}{2}\log(x^2-1) - x\log 10$

$= \boxed{2\log x + \frac{1}{2}\log(x^2-1) - x}$

NATURAL LOGARITHMS (ln) and Base "e".

The "natural logarithm", ln, is the inverse of base "e". Reviewing properties (3) and (4) above, we see that $\log_b b^x = x$. That is, taking the logarithm of an exponent with the same base, b, causes them to cancel each other out and we are left with just "x", the input value.

Also, $b^{\log_b x} = x$. That is, raising base b to the power of a logarithm with that same base, results in a cancellation, and again we are left with just "x".

Note $b^{\log_b x} = x$

in log form is $\log_b x = \log_b x$

Similarly, "ln" and "e" cancel each other out. Namely,

$$\ln(e^x) = x \quad \text{and}$$

$$e^{\ln x} = x$$

Solve for x to 4 decimal places. [Hint: Use ln]

7. $10^x = 3$ $\qquad x = \log 3 \doteq \boxed{.4771}$

8. $e^x = 2$ $\qquad x = \ln 2 \doteq \boxed{.6931}$

9. $4^x = 3$ $\qquad x = \log_4 3 = \dfrac{\ln 3}{\ln 4} = \dfrac{\text{LOG}\,3}{\text{LOG}\,4} \doteq \dfrac{1.09861}{1.386294} \doteq \boxed{.7925}$

10. $10^x = 7$ $\qquad x = \text{LOG}\,7 \doteq \boxed{.8451}$

11. $4^x = 5$ $\qquad x = \text{LOG}_4 5 = \dfrac{\ln 5}{\ln 4} = \dfrac{\text{LOG}\,5}{\text{LOG}\,4} \doteq \dfrac{.69897}{.6020599} \doteq \boxed{1.1610}$

12. $e^x = 6$ $\qquad x = \ln 6 = \boxed{1.7918}$

13. $\log_4 x = 2$ $\qquad 4^2 = x = \boxed{16}$

14. $\log_x 36 = 2$ $\qquad x^2 = 36 \;;\; \boxed{x = \pm 6}$

NOTE: Cannot have a negative base so $\boxed{x = +6}$

15. $\log_2 81 = x$ $\qquad \cancel{2^x = 81}$ $\quad x = \dfrac{\ln 81}{\ln 2} = \dfrac{\text{LOG}\,81}{\text{LOG}\,2} = \dfrac{\cancel{\text{LOG}}\; 1.908485}{.30102999}$

$\doteq \boxed{6.3399}$