c. The rate of change of the amount of radium present after t years is given by

$$\begin{aligned} Q'(t) &= \frac{d}{dt}\left[200e^{-0.0004332t}\right] \\ &= (200)(-0.0004332)e^{-0.0004332t} \\ &= -0.08664e^{-0.0004332t} \end{aligned}$$

d. After 800 years, the amount of radium present is changing at the rate of

$$\begin{aligned} Q'(800) &= -0.08664e^{(-0.0004332)(800)} \\ &\approx -0.06126 \end{aligned}$$

or decreasing at the rate of approximately 0.06126 milligrams per year.

APPLIED EXAMPLE 3 Carbon-14 Decay Carbon 14, a radioactive isotope of carbon, has a half-life of 5730 years. What is its decay constant?

Solution We have $Q(t) = Q_0e^{-kt}$. Since the half-life of the element is 5730 years, half of the substance is left at the end of that period; that is,

$$\begin{aligned} Q(5730) = Q_0e^{-5730k} &= \frac{1}{2}Q_0 \\ e^{-5730k} &= \frac{1}{2} \end{aligned}$$

Taking the natural logarithm on both sides of this equation, we have

$$\begin{aligned} \ln e^{-5730k} &= \ln\frac{1}{2} \\ -5730k &= -0.693147 \\ k &\approx 0.000121 \end{aligned}$$

Carbon-14 dating is a well-known method used by anthropologists to establish the age of animal and plant fossils. This method assumes that the proportion of carbon 14 (C-14) present in the atmosphere has remained fairly constant over the past 50,000 years. Professor Willard Libby, recipient of the Nobel Prize in chemistry in 1960, proposed this theory.

The amount of C-14 in the tissues of a living plant or animal is fairly constant. However, when an organism dies, it stops absorbing new quantities of C-14, and the amount of C-14 in the remains diminishes because of the natural decay of the radioactive substance. Therefore, the approximate age of a plant or animal fossil can be determined by measuring the amount of C-14 present in the remains.

APPLIED EXAMPLE 4 Carbon-14 Dating A skull from an archeological site has one tenth the amount of C-14 that it originally contained. Determine the approximate age of the skull.

Solution Here,

$$\begin{aligned} Q(t) &= Q_0e^{-kt} \\ &= Q_0e^{-0.000121t} \end{aligned}$$