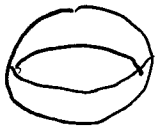
Notes Ch 3.6 Implicit Differentiation and Related Rates

(Problems on Notes)

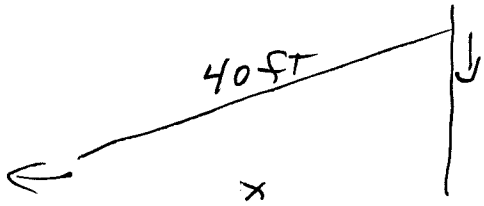
3. $\frac{dV}{dt} = 72\pi \frac{ft^3}{s}$  $V = \frac{4\pi r^3}{3}$

$d = 6ft$ so $r = 3ft$ find $\frac{dr}{dt}$

$V = \frac{4\pi r^3}{3}$ so $\frac{dV}{dt} = \frac{(3)4\pi r^2}{3}\frac{dr}{dt}$

so $\frac{dr}{dt} = \frac{\left(\frac{dV}{dt}\right)}{4\pi r^2} = \frac{72\pi \frac{ft^3}{s}}{4\pi (3ft)^2} = \frac{2ft}{s}$

$9ft^2$

4. 

$x^2 + y^2 = 1600 ft^2$

$24^2 + y^2 = 1600$

$y^2 = 1600 - 576 = 1024$

$y = 32 ft$

$\frac{dx}{dt} = ?$ $\frac{dy}{dt} = 9\frac{ft}{sec}$; $x = 24ft$

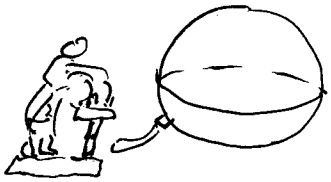
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ so $\frac{dx}{dt} = \frac{-2y\left(\frac{dy}{dt}\right)}{2x} = -\frac{4 \cancel{32ft}\left(9\frac{ft}{sec}\right)}{\cancel{24ft}_3}$

$\frac{dx}{dt} = -12\frac{ft}{sec}$

Note: $-$ means sliding to the left

mr. Konichek

3.6 Notes (cont.)

5. 

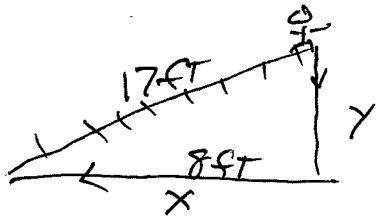
$V = \frac{4\pi r^3}{3}$

$\frac{dV}{dt} = 5\ \frac{cm^3}{min}$; $\frac{dr}{dt} = ?$; when $d = 20cm$, so $r = 10cm$

$\frac{dV}{dt} = (3)\frac{4\pi}{3} r^2 \frac{dr}{dt}$

so $\frac{dr}{dt} = \frac{\left(\frac{dV}{dt}\right)}{4\pi r^2} = \frac{5\ \frac{cm^3}{min}}{4\pi (10cm)^2} = \frac{1}{80\pi}\ \frac{cm}{min} \doteq .004\ \frac{cm}{min}$

$100cm^2$
20

6. 

$x^2 + y^2 = 17^2$

$8^2 + y^2 = 17^2$

$y^2 = 17^2 - 8^2$

when $x = 8ft$; $y = 15ft$ $\quad y^2 = 289 - 64 = 225$

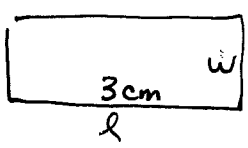
$y = 15ft$

$\frac{dx}{dt} = \frac{1}{4}\ \frac{ft}{sec}$ $\quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} =$ $\quad \frac{dy}{dt} = ?$

$\frac{dy}{dt} = -\frac{2x\frac{dx}{dt}}{2y} = -\frac{x}{y}\frac{dx}{dt} = -\frac{8ft}{15ft}\left(\frac{1}{4}\ \frac{ft}{sec}\right)$

$\frac{dy}{dt} = -\frac{2}{15}\ \frac{ft}{sec} \doteq -.13\ \frac{ft}{sec}$

Note: neg. sign means falling

7. 

$\frac{dw}{dt} = 4\ \frac{cm}{sec}$

$A = lw$ $\quad \frac{dw}{dt}\frac{dl}{dt} = \frac{2}{3}$ $\quad \frac{dA}{dt} = ?$ when $l = 12cm$, $w = 8cm$ $\quad$ so $\frac{4}{\frac{dl}{dt}} = \frac{2}{3}$

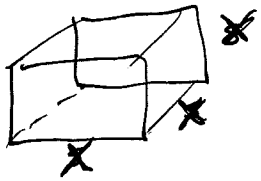
so $\frac{dl}{dt} = 6$

$\frac{dA}{dt} = w\frac{dl}{dt} + l\frac{dw}{dt} = (8cm)\left(6\ \frac{cm}{sec}\right) + (12cm)\left(4\ \frac{cm}{sec}\right)$

$\frac{dA}{dt} = 48\ cm^2/sec + 48\ cm^2/sec = 96\ \frac{cm^2}{sec}$

Mr. Konichek
Hmwk 3.6 #50

50. Volume of a Cube



$V = x^3$ $\quad \frac{dV}{dt} = ?$

when $x = 5\,in$; $V = 125\,in^3$; $\frac{dx}{dt} = 0.1 \frac{inches}{sec}$

find $\frac{dV}{dt}$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt} = 3\,(5\,in)^2\,(0.1\,\tfrac{in}{sec}) = \boxed{7.5\,\tfrac{in^3}{sec}}$$

$25\,in^2$