Chapter 2.1 – Functions (Review)

Definition:

A function is a rule or "formula" that assigns one element of a set to one and only one element of another set. The most common function notation we will use is:

$$y = f(x)$$

That is, y is a function of x. Here a single x-value (input) is assigned to a single y-value (output). Note that for a function, the reverse does not have to be true.

Domain:

The domain of a function refers to the set of possible input values (x-values). To find the domain of a function, you should keep in mind that:

a) The denominator of a function can NEVER be equal to zero.

b) The even root of a negative number is not real.

Find the domain of the following:

1. $f(x) = \dfrac{1}{x^2 - 9} = \dfrac{1}{(x-3)(x+3)}$ $\boxed{\text{x cannot be +3 or -3 because you cannot divide by zero}}$

2. $f(x) = x^2 - 7x + 6$ $\boxed{\text{x can be all real \#'s}}$

3. $f(x) = \sqrt{x+2}$ $\boxed{\{x : x \geq -2\}}$ because when $x < -2$ you have the $\sqrt{\ }$ of a negative #

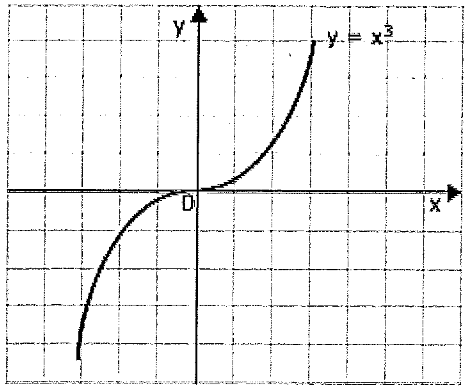
Range:

The range of a function refers to the set of possible output values (y-values).

The Vertical Line Test:

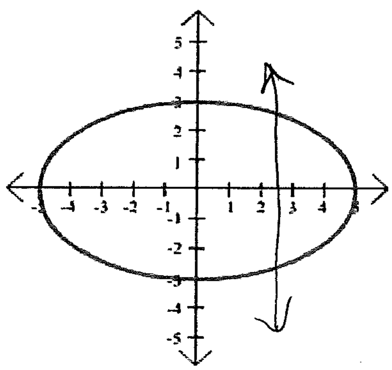
A graph is that of a function iff every vertical line intersects the graph no more than once.

8.



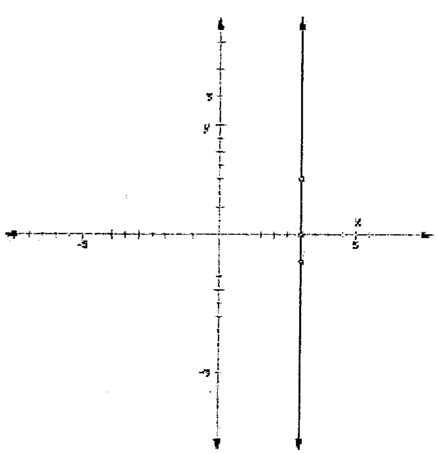
yes

9.



No

10.



No

11. The equation of a circle.

14. $(f \cdot g)(x) = f(x) \cdot g(x) = (x-1)(x^2+1)$
$= x^3 + x - x^2 - 1$
$= \boxed{x^3 - x^2 + x - 1}$

15. $(g \circ f)(x) = g(f(x)) = g(x-1) = (x-1)^2+1$
$= x^2 - 2x + 1 + 1$
$= \boxed{x^2 - 2x + 2}$

16. $(f \circ g)(x) = f(g(x)) = f(x^2+1) = (x^2+1) - 1 = \boxed{x^2}$

For $f(x) = x + 2,\ g(x) = x^2 - x - 6$

17. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x+2}{x^2 - x - 6} = \dfrac{\cancel{(x+2)}}{(x-3)\cancel{(x+2)}} = \boxed{\dfrac{1}{(x-3)}}$

$\left(\dfrac{g}{f}\right)(x) = \dfrac{g(x)}{f(x)} = \dfrac{x^2 - x - 6}{x+2} = \dfrac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)}} = \boxed{x-3}$

18.

Example 1:

A company has a fixed cost of $30,000 and a production cost of $6 for each unit it manufactures. A unit sells for $10.

a. What is the cost function?
b. What is the revenue function?
c. What is the profit function?
d. Find the profit (loss) corresponding to production levels of 6000, 8000 and 12000 units.

First, **let x = the number of units sold**. Remember that variables always represent numbers so it would be *incorrect* to let x equal just "units".

a. $C(x)$ = Variable cost + Fixed cost
$C(x) = 6x + 30{,}000$

b. $R(x) = 10x$

c. $P(x) = R(x) - C(x)$
$P(x) = 10x - (6x + 30{,}000)$
$P(x) = 10x - 6x - 30{,}000$
$P(x) = 4x - 30{,}000$

d. (i) $x = 6{,}000$
$P(x) = P(6{,}000) = 4(6{,}000) - 30{,}000 = -\$6{,}000$ (a loss)

(ii) $x = 8{,}000$
$P(x) = P(8{,}000) = 4(8{,}000) - 30{,}000 = \$2{,}000$ (a profit)

(iii) $x = 12{,}000$
$P(x) = P(12{,}000) = 4(12{,}000) - 30{,}000 = \$18{,}000$ (a profit)

1. A manufacturer has a production cost of $12 per unit produced and a fixed cost of $30,000. The product sells for $15/unit.
   a. What is the cost function? $C(x) = 12x + 30{,}000$
   b. What is the revenue function? $R(x) = 15x$
   c. What is the profit function? $P(x) = 15x - (12x + 30{,}000) = 3x - 30{,}000$
   d. What profit is expected if 8,000 units are sold? $P(8{,}000) = 24{,}000 - 30{,}000 = \boxed{-\$6{,}000}$
   e. What profit is expected if 12,000 units are sold? $P(12{,}000) = 36{,}000 - 30{,}000 = \boxed{+\$6{,}000}$
   f. How many units need to be sold to break even?

$\boxed{10{,}000 \text{ units}}$

as $3(10{,}000) - 30{,}000 = 0$

Example 2: Consider the piecewise function:

$$h(x)=\begin{cases} x & \text{if } x<1 \\ 0 & \text{if } x=1 \\ 2-x & \text{if } x>1 \end{cases}$$

Find $\lim_{x\to 1} h(x)$.

From the left: e.g. x = 0.999999999, h(0.999999999) = 0.999999999 which is close to 1.

From the right: e.g. x = 1.000000001, h(1.000000001) = 2 – 1.000000001 = 0.999999999 which is close to 1.

Therefore: $\lim_{x\to 1} h(x)=1$

Notice that when x = 1, the value of the function h(x) is actually 0. This value is not the same as the limit of 1, however, we know that the limit at x = 1 has nothing to do with what actually happens at x = 1.

Q: What happens if the left limit is not the same as the right limit?

A: The limit, L, Does Not Exist (DNE).

Q: What happens if the left limit and right limit both approach either positive or negative infinity?

A: The limit, L, again Does Not Exist (DNE).

Q: But even if they both approach, say, positive infinity?

A: Yes the limit still does not exist (DNE). Infinity is not a number you can quantify. If you cannot state the exact value of the limit, then the limit does not exist (DNE).

Step 2: Simplify the limit, if possible

$$\lim_{x\to\infty}\frac{1}{2x}$$

Steps 3: Substitute x.

$$\frac{1}{2(\infty)}=\frac{1}{\infty}=0$$

So, $\lim_{x\to\infty}\frac{x^2+3x+5}{2x^3-6x+7}=0$

Example 5. Find

$$\lim_{x\to-\infty}\frac{3x^5-9x+2}{2x^2+4x}$$

$$\lim_{x\to-\infty}\frac{3x^5-9x+2}{2x^2+4x}=\lim_{x\to-\infty}\frac{3x^5}{2x^2}=\lim_{x\to-\infty}\frac{3x^3}{2}=\lim_{x\to-\infty}\frac{3(-\infty)^3}{2}=-\infty=DNE$$

Find the corresponding limits:

2. $f(x)=x^3,\ \lim_{x\to3}f(x)$ $=\lim_{x\to3}x^3=3^3=\boxed{27}$

3. $k(x)=\begin{cases}2+x & if\ x\neq1\\ 1 & if\ x=1\end{cases}$ $\lim_{x\to1}k(x)=2+1=\boxed{3}$

$\lim_{x\to1}k(x)$

SPECIAL RULES: There are five special cases where direct substitution can be used to calculate limits. We have already encountered some of them.

a. Limits at infinity.
b. For functions that are polynomial.
c. For functions that are rational.
d. For functions that are trigonometric.
e. For functions that are radical.

Find the following limits:

1. $\lim_{x \to 2} 3x-5$ $= 3(2)-5 = 6-5 = \boxed{1}$

2. $\lim_{x \to 3} \frac{x^2-9}{x-3}$ $= \lim_{x \to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+3 = \boxed{6}$

3. $\lim_{x \to 6} \frac{x^2-4x-12}{x-6}$ $= \lim_{x \to 6} \frac{\cancel{(x-6)}(x+2)}{\cancel{(x-6)}} = 6+2 = \boxed{8}$

4. $\lim_{x \to 3} \sqrt{x^2+16}$ $= \sqrt{3^2+16} = \sqrt{9+16} = \sqrt{25} = \boxed{5}$

Note: Can only do the below if $x \to \pm\infty$

5. $\lim_{x \to 6} \frac{4x+12}{x-6}$ $=$ ~~$\lim_{x \to 6} \frac{4x}{x} = \lim_{x \to 6} 4 = \boxed{4}$~~ $\boxed{\text{D.N.E.}}$

6. $\lim_{x \to 2} \sqrt{5x^3-15}$ $= \sqrt{5(2)^3-15} = \sqrt{40-15} = \sqrt{25} = \boxed{5}$