More Practice

Practice:

① Simplify: $\sqrt{\frac{3}{5}}$

answer: $\frac{\sqrt{3}}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$

② Solve: $3 - 7x^2 = 0$

answer: $-7x^2 = -3$

$x^2 = \frac{3}{7}$

$x = \pm\sqrt{\frac{3}{7}} = \pm\frac{\sqrt{3}}{\sqrt{7}}\frac{\sqrt{7}}{\sqrt{7}} = \pm\frac{\sqrt{21}}{7}$

③ Solve: $\frac{(-10)(5-3x^2)}{(9+x^2)^5} = 0$

answer: $5 - 3x^2 = 0$

$-3x^2 = -5$

$x^2 = \frac{5}{3}$

$x = \pm\sqrt{\frac{5}{3}} = \pm\frac{\sqrt{5}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{\sqrt{15}}{3}$

④ factor

$$(2+x^2)^2(-2) - (-2x)(2)(2+x^2)(2x)$$

answer:

$$(-2)(2+x^2)\left((2+x^2) - (x)(2)(\cancel{2+x^2})(2x)\right)$$

$$(-2)(2+x^2)(2+x^2-4x^2)$$

$$= \boxed{(-2)(2+x^2)(2-3x^2)}$$

⑤ Determine where the graph of the function is concave upward and where it is concave downward.

$$f(x) = \frac{1}{(2+x^2)}$$

More Practice on Concavity.

#6. Determine where the graph of the function is concave upward and where it is concave downward.

Two ways to do same problem.

Product Rule Way

$$f(x) = \frac{1}{(2+x^2)}$$

$$f(x) = (2+x^2)^{-1}$$

$$f'(x) = (-1)(2+x^2)^{-2}(2x)$$

$$f'(x) = (-2x)(2+x^2)^{-2}$$

$$f''(x) = (2+x^2)^{-2}(-2) + (-2x)(-2)(2+x^2)^{-3}(2x)$$

$$f''(x) = (-2)(2+x^2)^{-3}\left((2+x^2)' + (-2x)(2x)\right)$$

$$f''(x) = \frac{(-2)(2-3x^2)}{(2+x^2)^3}$$

Quotient Rule Way

$$f(x) = \frac{1}{(2+x^2)}$$

$$f'(x) = \frac{(-2x)}{(2+x^2)^2} = \frac{(2+x^2)(0) - (1)(2x)}{(2+x^2)^2}$$

$$f''(x) = \frac{(2+x^2)^2(-2) - (-2x)(2)(2+x^2)(2x)}{(2+x^2)^4}$$

$$f''(x) = \frac{(-2)(2+x^2)\left[(2+x^2) - (2x)(2x)\right]}{(2+x^2)^3}$$

$$f''(x) = \frac{(-2)(2-3x^2)}{(2+x^2)^3}$$

$$2 - 3x^2 = 0$$

$$-3x^2 = -2$$

$$x^2 = \frac{2}{3}$$

$$x = \pm\sqrt{\frac{2}{3}} = \pm\frac{\sqrt{2}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{\sqrt{6}}{3}$$

$$f''(x) = \frac{(-2)(2-3x^2)}{(2+x^2)^3}$$

(-1)	$-\frac{\sqrt{6}}{3}$	(0)	$+\frac{\sqrt{6}}{3}$	(1)
$--$		$-+$		$--$
$\cup$		$\cap$		$\cup$

Concave upward

$\left[-\infty, -\frac{\sqrt{6}}{3}\right]$ & $\left[\frac{\sqrt{6}}{3}, \infty\right]$

Concave downward

$\left[-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right]$

(Chug & Plug Problem)

#7 Revenue Functions

The revenue (in dollars) realized by Apollo from the sale of its ink-jet printers is given by

$$R(x) = -0.1x^2 + 500x$$

where x denotes the number of units manufactured each month. What is Apollo's revenue when 10,000 units are produced?

$$R(10{,}000) = -0.1(10{,}000)^2 + 500(10{,}000)$$

$$R(10{,}000) = -(1\times10^{-1})(1\times10^{4})^2 + (5\times10^{2})(1\times10^{4})$$

$$R(10{,}000) = -(1\times10^{-1})(1\times10^{8}) + (5\times10^{6})$$

$$= -(1\times10^{7}) + (5\times10^{6})$$

$$= -(10\times10^{6}) + (5\times10^{6})$$

$$= -5\times10^{6}$$

$$= -\$5 \text{ million dollars} = -\$5{,}000{,}000$$

#8 What is Apollo's revenue when 1,000 units are produced?

$$R(1{,}000) = -0.1(1{,}000)^2 + 500(1{,}000)$$

$$= -(1\times10^{-1})(1\times10^{6}) + 5\times10^{5}$$

$$= (-1\times10^{5}) + (5\times10^{5})$$

$$= (4\times10^{5}) = \$400{,}000$$

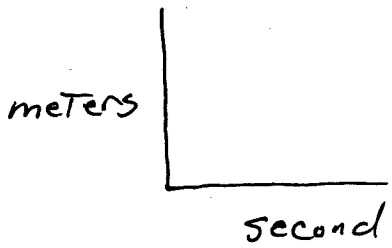
Having Trouble with Story Problems?

How would you Know whether you need To [integrate (anti-derivative)] or [differentiate (derivative)]

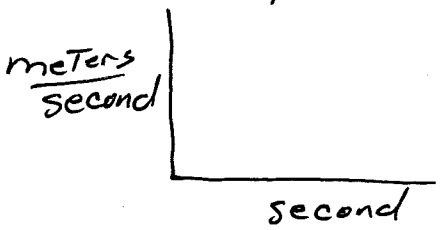
Let Units give it Away

examples

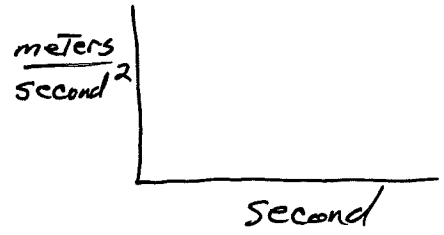
Distance vs Time



Velocity vs Time



Acceleration vs Time



Tell me whether you would integrate (I) (just say I) OR Take the derivative (D) (just say D)

	They give you an equation dealing with	And your answer will have These units	Answer
1.	meters	meters/second	
2.	meters/sec	meters/second²	
3.	meters/sec	meters	
4.	meters/second²	meters/sec	
5.	meters/second²	meters	
6.	meters	meters/second²	
7.	meters/second²	meters/second³	

Answers To Page 5

1. D because you want slope = $\frac{\text{rise}}{\text{run}} = \frac{\text{meters}}{\text{second}}$

2. D " " " " $\frac{\text{meters/sec}}{\text{sec}} = \frac{\text{meters}}{\text{sec}^2}$

3. I because you want Area $\frac{\text{meters}}{\text{second}} \times \text{second} = \text{meters}$

4. I " " " " $\frac{\text{meters}}{\text{second}^2} \times \text{second} = \frac{\text{meters}}{\text{sec}}$

5. I Twice $\frac{\text{meters}}{\text{second}^2} \times \text{second} \times \text{second} = \text{meters}$

6. D Twice $\frac{\text{meters}}{\text{second}}$ (1st derivative) $\frac{\text{meters}}{\text{second}^2}$ (2nd derivative)

7. D because you want slope = $\frac{\text{rise}}{\text{run}} = \frac{\text{meters/s}^2}{\text{s}}$

$= \text{m/s}^3$