

Summary of Chapter 6 Integral Calculus Math 111

Section 6:1

Rule 1: $\int k \, dx = kx + C$ (where k & C are constants) example: $\int 14 \, dx = 14x + C$

Rule 2: $\int x^n \, dx = \frac{1}{(n+1)} x^{n+1} + C$ (where $n \neq -1$) example: $\int x^3 \, dx = \frac{1}{(3+1)} x^{3+1} + C = \frac{1}{4} x^4 + C$

Rule 3: $\int c f(x) \, dx = c \int f(x) \, dx$ (where c is a constant) example: $\int 4x \, dx = 4 \int x \, dx = 4(\frac{1}{2})x^2 + C = 2x^2 + C$

Rule 4: $\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$ example: $\int (3x^2 + 2x) \, dx = 3(\frac{1}{3})x^3 + 2(\frac{1}{2})x^2 + C = x^3 + x^2 + C$

Rule 5: $\int e^x \, dx = e^x + C$ example: $\int 9e^x \, dx = 9 \int e^x \, dx = 9e^x + C$

Rule 6: $\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + C$ (where $x \neq 0$) example: $\int (\frac{5}{x}) \, dx = 5 \int \frac{1}{x} \, dx = 5 \ln|x| + C$

Section 6.2

Example 1: $\int 4x(2x^2 + 1)^7 \, dx$ simplify by letting $u = (2x^2 + 1)$ then $du/dx = 4x$ so $du = 4x \, dx$

Substituting above gives a much easier problem: $\int u^7 \, du = (\frac{1}{8})u^8 + C = (\frac{1}{8})(2x^2 + 1)^8 + C$

Example 2: $\int \frac{2}{(x-2)} \, dx = \int 2(x-2)^{-1} \, dx$ simplify by letting $u = (x-2)$ then $du/dx = 1$ so $du = dx$

Substituting above gives $2 \int u^{-1} \, du = 2 \int (\frac{1}{u}) \, du = 2 \ln|u| + C = \ln u^2 + C = \ln(x-2)^2 + C$

Example 3: $\int 3x^2 \sqrt{(x^3 + 2)} \, dx = \int 3x^2 (x^3 + 2)^{1/2} \, dx$ let $u = (x^3 + 2)$ then $du/dx = 3x^2$
 $du = (3x^2) \, dx$

gives $\int u^{1/2} \, du = (\frac{1}{\frac{1}{2}+1}) u^{3/2} + C = \frac{1}{\frac{3}{2}} u^{3/2} + C = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (x^3 + 2)^{3/2} + C$

Example 4: $\int x^2 (2x^3 + 3)^4 \, dx$ let $u = (2x^3 + 3)$ so $du = 6x^2 \, dx$ so $(\frac{1}{6}) \, du = x^2 \, dx$

Substituting simplifies to: $\frac{1}{6} \int u^4 \, du = \frac{1}{6} (\frac{1}{5}) u^5 + C = \frac{1}{30} (2x^3 + 3)^5 + C$