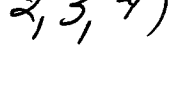
Area Between Two Curves

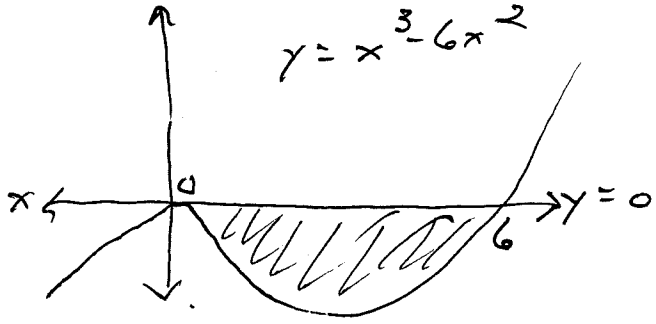
6.6 Pg 474 (1, 2, 3, 4)

Mr. Konichek
Hmwk 6.6
Last Hmwk. Assign.



(neg. 2nd derivative expression)

1.



under x-axis

$$= -\int_0^6 (x^3 - 6x^2)\,dx$$

$$= -\left[\frac{1}{4}x^4 - 6\left(\frac{1}{3}\right)x^3\right]\Big|_0^6$$

$$= -\left[\left(\frac{1}{4}(1296) - 2(216)\right) - 0\right]$$

$$= -(324 - 432) = -(-108) = 108 \text{ sq. units}$$

2. Note graph (again Area is below x-axis)

$$-\int_0^2 (x^4 - 2x^3)\,dx$$

$$= -\left(\frac{1}{5}x^5 - 2\left(\frac{1}{4}\right)x^4\right)\Big|_0^2$$

$$= -\left[\left(\frac{1}{5}(32) - \frac{1}{2}(16)\right) - 0\right]$$

$$= -\left(6\frac{2}{5} - 8\right) = -\left(-1\frac{3}{5}\right)$$

$$= +1\frac{3}{5} \text{ sq. units}$$

OR $= 1.6$ sq. units

6.6 Pg 474 (3, 4) these are last Two

3. Note graph
first region is below x-axis but second is above

$$-\int_{-1}^{0} x(1-x^2)^{1/2}\,dx + \int_{0}^{1} x(1-x^2)^{1/2}\,dx$$

by symmetry though we know the two areas are equal so to save work

$$2\int_{0}^{1} x(1-x^2)^{1/2}\,dx$$

let $u = (1-x^2)$ Then $du = -2x\,dx$

so $-\frac{1}{2}du = x\,dx$

if $x=0$ Then $u=1$ and if $x=1$ Then $u=0$

substituting above info. gives

$$2\int_{1}^{0}\left(-\tfrac{1}{2}\right)u^{1/2}\,du = -\int_{1}^{0} u^{1/2}\,du$$

$$= -\left[\frac{1}{\left(\frac{1}{2}+1\right)}u^{\left(\frac{1}{2}+1\right)}\right|_{1}^{0} = -\left(\frac{2}{3}u^{3/2}\right)\Big|_{1}^{0}$$

$$= -\left[\frac{2}{3}(0-\frac{2}{3})\right] = +\frac{2}{3}\text{ sq. units}$$

6.6 cont. #4 This is it!

4. Note graph on page 475

$$-\int_{-2}^{0}\left(\frac{2x}{(x^2+4)}\right)dx + \int_{0}^{2}\left(\frac{2x}{x^2+4}\right)dx$$

using symmetry again

$$= 2\int_{0}^{2}\frac{2x}{(x^2+4)}\,dx$$

let $u=(x^2+4)$ Then $du=(2x)dx$

If $x=0$ Then $u=4$ and if $x=2$ Then $u=8$

substituting

$$= 2\int_{4}^{8}\left(\frac{1}{u}\right)du = 2\left[\ln(u)\right]\Big|_{4}^{8}$$

$$= 2\left[\ln 8 - \ln 4\right] = 2\ln\frac{8}{4}$$

$= 2\ln 2$ sq. units

$\doteq 2(.693147)$

$\doteq 1.386$

OR $\doteq 1.4$ sq. units

Note:

Textbook answer ~~made~~ did not make a mistake on their last step

Textbook said $(\ln 8 - \ln 4)2 = \ln 4$ ~~NOT TRUE~~

$2(\ln 8 - \ln 4) = 2\ln\frac{8}{4} = 2\ln 2$

$= \ln 2^2 = \ln 4 \doteq 1.4$ sq. units