6.4 Exercises

In Exercises 1–4, find the area of the region under the graph of the function f on the interval $[a, b]$, using the Fundamental Theorem of Calculus. Then verify your result using geometry.

1. $f(x) = 2; [1, 4]$
2. $f(x) = 4; [-1, 2]$
3. $f(x) = 2x; [1, 3]$
4. $f(x) = -\frac{1}{4}x + 1; [1, 4]$

In Exercises 5–16, find the area of the region under the graph of the function f on the interval $[a, b]$.

5. $f(x) = 2x + 3; [-1, 2]$
6. $f(x) = 4x - 1; [2, 4]$
7. $f(x) = -x^2 + 4; [-1, 2]$
8. $f(x) = 4x - x^2; [0, 4]$
9. $f(x) = \frac{1}{x}; [1, 2]$
10. $f(x) = \frac{1}{x^2}; [2, 4]$
11. $f(x) = \sqrt{x}; [1, 9]$
12. $f(x) = x^3; [1, 3]$
13. $f(x) = 1 - \sqrt[3]{x}; [-8, -1]$
14. $f(x) = \frac{1}{\sqrt{x}}; [1, 9]$
15. $f(x) = e^x; [0, 2]$
16. $f(x) = e^x - x; [1, 2]$

In Exercises 17–40, evaluate the definite integral.

17. $\int_2^4 3\,dx$
18. $\int_{-1}^2 -2\,dx$
19. $\int_1^4 (2x + 3)\,dx$
20. $\int_{-1}^0 (4 - x)\,dx$
21. $\int_{-1}^3 2x^2\,dx$
22. $\int_0^2 8x^3\,dx$
23. $\int_{-2}^2 (x^2 - 1)\,dx$
24. $\int_1^4 \sqrt{u}\,du$
25. $\int_1^8 2x^{1/3}\,dx$
26. $\int_1^4 2x^{-3/2}\,dx$
27. $\int_0^1 (x^3 - 2x^2 + 1)\,dx$
28. $\int_1^2 (t^5 - t^3 + 1)\,dt$
29. $\int_1^4 \frac{1}{x}\,dx$
30. $\int_1^3 \frac{2}{x}\,dx$
31. $\int_0^4 x(x^2 - 1)\,dx$
32. $\int_0^2 (x - 4)(x - 1)\,dx$
33. $\int_1^3 (t^2 - t)^2\,dt$
34. $\int_{-1}^1 (x^2 - 1)^2\,dx$
35. $\int_{-3}^{-1} \frac{1}{x^2}\,dx$
36. $\int_1^2 \frac{2}{x^3}\,dx$
37. $\int_1^4 \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)dx$
38. $\int_0^1 \sqrt{2x}(\sqrt{x} + \sqrt{2})\,dx$
39. $\int_1^4 \frac{3x^3 - 2x^2 + 4}{x^2}\,dx$
40. $\int_1^2 \left(1 + \frac{1}{u} + \frac{1}{u^2}\right)du$

41. **Personal Bankruptcy** The number of personal bankruptcy filings by fiscal years ending September 30 between 2010 and 2012 was declining at the rate of

$$R(t) = 0.077t + 0.0825 \qquad (0 \le t \le 2)$$

million cases/year, t years after September 30, 2010. The number of filings as of September 30, 2010, stood at approximately 1.538 million cases.

a. Estimate the change in the number of personal bankruptcy cases filed between September 30, 2010, and September 30, 2012.

b. What was the approximate number of personal bankruptcy cases filed in 2012?

Hint: If $N(t)$ denotes the number of bankruptcy filings in year t, then $N'(t) = -R(t)$.

Source: Administrative Office of the U.S. Courts.

42. **Health Care Costs** According to a study conducted by the Centers for Medicare & Medicaid Services in 2010, the national spending for out-of-pocket health-care costs is projected to increase over the next several years. The amount spent annually from 2010 ($t = 0$) is expected to grow at the rate of

$$R(t) = 1.0952t + 17.357 \qquad (0 \le t \le 6)$$

billion dollars/year in year t. The national spending in 2010 was \$317 billion. What is the projected national spending in 2016?

Source: Centers for Medicare & Medicaid Services.

43. **Marginal Cost** A division of Ditton Industries manufactures a deluxe toaster oven. Management has determined that the daily marginal cost function associated with producing these toaster ovens is given by

$$C'(x) = 0.0003x^2 - 0.12x + 20$$

where $C'(x)$ is measured in dollars per unit and x denotes the number of units produced. Management has also determined that the daily fixed cost incurred in the production is \$800.

a. Find the total cost incurred by Ditton in producing the first 300 units of these toaster ovens per day.

b. What is the total cost incurred by Ditton in producing the 201st through 300th units/day?

44. **Marginal Revenue** The management of Ditton Industries has determined that the daily marginal revenue function associated with selling x units of their deluxe toaster ovens is given by

$$R'(x) = -0.1x + 40$$

where $R'(x)$ is measured in dollars per unit.

a. Find the daily total revenue realized from the sale of 200 units of the toaster oven.

b. Find the additional revenue realized when the production (and sales) level is increased from 200 to 300 units.