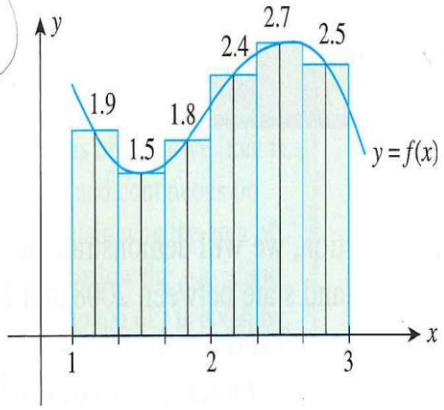
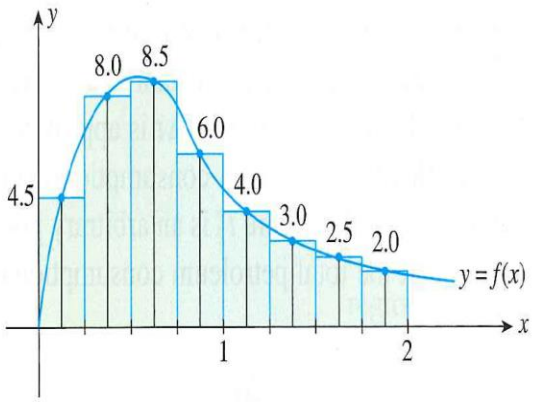
6.3 Exercises

In Exercises 1 and 2, find an approximation of the area of the region R under the graph of f by computing the Riemann sum of f corresponding to the partition of the interval into the subintervals shown in the accompanying figures. In each case, use the midpoints of the subintervals as the representative points.

1.



2.



3. Let $f(x) = 3x$.
   a. Sketch the region R under the graph of f on the interval $[0, 2]$, and find its exact area using geometry.
   b. Use a Riemann sum with four subintervals of equal length $(n = 4)$ to approximate the area of R. Choose the representative points to be the left endpoints of the subintervals.
   c. Repeat part (b) with eight subintervals of equal length $(n = 8)$.
   d. Compare the approximations obtained in parts (b) and (c) with the exact area found in part (a). Do the approximations improve with larger n?

4. Repeat Exercise 3, choosing the representative points to be the right endpoints of the subintervals.

5. Let $f(x) = 4 - 2x$.
   a. Sketch the region R under the graph of f on the interval $[0, 2]$, and find its exact area using geometry.
   b. Use a Riemann sum with five subintervals of equal length $(n = 5)$ to approximate the area of R. Choose the representative points to be the left endpoints of the subintervals.
   c. Repeat part (b) with ten subintervals of equal length $(n = 10)$.
   d. Compare the approximations obtained in parts (b) and (c) with the exact area found in part (a). Do the approximations improve with larger n?

6. Repeat Exercise 5, choosing the representative points to be the right endpoints of the subintervals.

7. Let $f(x) = x^2$, and compute the Riemann sum of f over the interval $[2, 4]$, choosing the representative points to be the midpoints of the subintervals and using:
   a. Two subintervals of equal length $(n = 2)$.
   b. Five subintervals of equal length $(n = 5)$.
   c. Ten subintervals of equal length $(n = 10)$.
   d. Can you guess at the area of the region under the graph of f on the interval $[2, 4]$?

8. Repeat Exercise 7, choosing the representative points to be the left endpoints of the subintervals.

9. Repeat Exercise 7, choosing the representative points to be the right endpoints of the subintervals.

10. Let $f(x) = x^3$, and compute the Riemann sum of f over the interval $[0, 1]$, choosing the representative points to be the midpoints of the subintervals and using:
    a. Two subintervals of equal length $(n = 2)$.
    b. Five subintervals of equal length $(n = 5)$.
    c. Ten subintervals of equal length $(n = 10)$.
    d. Can you guess at the area of the region under the graph of f on the interval $[0, 1]$?

11. Repeat Exercise 10, choosing the representative points to be the left endpoints of the subintervals.