6.2 Practice Pg 430 (1, 3, 5, 7, 17, 19, 23, 27)

1. $\int 4(4x+3)^4 dx$

let $u = 4x+3$ Then $\frac{du}{dx} = 4$

substituting

so $du = 4dx$

$\int u^4 du = \frac{1}{5}u^5 + C = \boxed{\frac{1}{5}(4x+3)^5 + C}$

2. $\int 4x(2x^2+1)^7 dx$

let $u = 2x^2+1$ Then $\frac{du}{dx} = 4x$

substituting

So $du = 4x\,dx$

$\int u^7 du = \frac{1}{8}u^8 + C = \boxed{\frac{1}{8}(2x^2+1)^8 + C}$

3. $\int (x^3-2x)^2(3x^2-2)\,dx$

let $u = x^3 - 2x$

Then $\frac{du}{dx} = 3x^2 - 2$

Substitution

$du = (3x^2-2)\,dx$

$\int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}(x^3-2x)^3 + C$

check: $f'(x) = \left(\frac{1}{3}\right)(3)(x^3-2x)^2(3x^2-2)$

6.2 cont. practice

(5.) $\int \frac{4x}{(2x^2+3)^3}\,dx$ let $u = 2x^2+3$

$\frac{du}{dx} = 4x$ so $du = 4x\,dx$

substituting

$$\int \frac{du}{u^3} = \int u^{-3}\,du = -\frac{1}{2}u^{-2} + C = \boxed{-\frac{1}{2(2x^2+3)^2} + C}$$

(7.) $\int 3t^2\sqrt{t^3+2}\,dt$ let $u = t^3+2$

$\frac{du}{dt} = 3t^2$ so $du = 3t^2\,dt$

substituting

$$\int u^{+1/2}\,du = \frac{1}{\frac{3}{2}}u^{3/2} + C = \boxed{\frac{2}{3}(t^3+2)^{3/2} + C}$$

9. $\int 2(x^2-1)^9 x\,dx$ let $u = x^2-1$ then

$\frac{du}{dx} = 2x$

substituting

so $du = 2x\,dx$

$$\int u^9\,du = \frac{1}{10}u^{10} + C = \boxed{\frac{1}{10}(x^2-1)^{10} + C}$$

6.2 cont.

* 11. $\int \frac{x^4}{1-x^5}\,dx$ let $u = 1 - x^5$

$\frac{du}{dx} = -5x^4$ so $du = -5x^4 dx$

$-\frac{du}{5} = x^4 dx$

Substituting

$$\int -\frac{1}{5}\left(\frac{1}{u}\right)du = -\frac{1}{5}\int \frac{1}{u}\,du = -\frac{1}{5}\ln u + C$$

$$= \boxed{-\frac{1}{5}\ln(1-x^5) + C}$$

13. $\int \frac{2}{x-2}\,dx$ let $u = x - 2$

Then $\frac{du}{dx} = 1$ so $du = dx$

$$\int 2\left(\frac{1}{u}\right)du = 2\int \frac{1}{u}\,du = \boxed{2\ln u + C}$$

15. $\int \frac{(0.3x - 0.2)}{(0.3x^2 - 0.4x + 2)}\,dx$ let $u = .3x^2 - .4x + 2$

then $\frac{du}{dx} = .6x - .4$

so $\frac{du}{dx} = 2(.3x - .2)$ and $\frac{1}{2}du = (.3x - .2)dx$

Substituting

$$\int \frac{1}{2}\left(\frac{1}{u}\right)du = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln u + C = \boxed{\frac{1}{2}\ln(.3x^2 - .4x + 2) + C}$$

6.2

(17.) $\int \frac{2x}{3x^2-1}\,dx$ let $u = (3x^2-1)$

$\frac{du}{dx} = 6x$ so $\frac{1}{3}du = 2x\,dx$

substituting

$\int \frac{1}{3}\left(\frac{1}{u}\right)du = \frac{1}{3}\int \frac{1}{u}du = \frac{1}{3}\ln u + C$

$= \boxed{\frac{1}{3}\ln(3x^2-1) + C}$

(19.) $\int e^{-2x}\,dx$ $u = e^{-x}$

$\frac{dy}{dx} = -e^{-x}$ so $-du = e^{-x}dx$

substituting

$\int -u\,du = -\int u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}(e^{-x})^2 + C$

$= \boxed{-\frac{1}{2}e^{-2x} + C}$

21. $\int e^{2-x}\,dx$ let $u = 2-x$

$\frac{dy}{dx} = -1$ so $-du = dx$

substituting

$\int -e^u\,du = -\int e^u\,du = -e^u + C = \boxed{-e^{(2-x)} + C}$

#23. $\int x e^{-x^2} dx$

let $u = -x^2$

$\frac{du}{dx} = -2x$ so $-\frac{1}{2} du = x\,dx$

substituting

$$\int \left(-\frac{1}{2}\right) e^u du = -\frac{1}{2}\int e^u du = -\frac{1}{2} e^u + C$$

$$= \boxed{-\frac{1}{2} e^{-x^2} + C}$$

25. $\int (e^x - e^{-x})\, dx$

$u = x$

$\frac{du}{dx} = 1$ so $du = dx$

Note even use here why substitution? just do it!

substituting

$$\int (e^u - e^{-u})\, du = \int e^u du - \int e^{-u} du$$

$$= e^u + e^{-u} = \boxed{e^x + e^{-x} + C}$$

27. $\int \frac{2e^x}{1+e^x} dx$

let $u = 1 + e^x$

$\frac{du}{dx} = e^x$ so $du = e^x dx$

substituting

$$\int 2\left(\frac{1}{u}\right) du = 2\int \frac{1}{u} du = 2 \ln u + C$$

$$= \boxed{2 \ln(1+e^x) + C}$$

29. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

substituting

so $2du = \left(\frac{1}{\sqrt{x}}\right)dx$

$\int e^u(2)\,du = 2\int e^u\,du = 2e^u + C = \boxed{2e^{\sqrt{x}} + C}$

31. $\int \frac{e^{3x}+x^2}{(e^{3x}+x^3)^3}\,dx$

let $u = e^{3x}+x^3$

$\frac{du}{dx} = 3e^{3x}+3x^2$

$\frac{du}{dx} = 3(e^{3x}+x^2)$

substituting gives

$\frac{1}{3}du = (e^{3x}+x^2)\,dx$

$\int \frac{1}{3}\left(\frac{1}{u^3}\right)du = \frac{1}{3}\int u^{-3}\,du = \frac{1}{3}\left(\frac{1}{-2}\right)u^{-2} + C$

$= \boxed{-\frac{1}{6(e^{3x}+x^3)^2} + C}$

33. $\int e^{2x}(e^{2x}+1)^3\,dx$

let $u = (e^{2x}+1)$

$\frac{du}{dx} = e^{2x}$

substituting

so $du = e^{2x}\,dx$

$\int u^3\,du = \frac{1}{4}u^4 + C = \boxed{\frac{1}{4}(e^{2x}+1)^4 + C}$

35. $\int \frac{\ln 5x}{x}\,dx$

let $u = \ln 5x$

$\frac{du}{dx} = \frac{5}{5x} = \frac{1}{x}$

substituting

so $du = \frac{1}{x}\,dx$

$$\int u\,du = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}(\ln 5x)^2 + C}$$

37. $\int \frac{3}{x\ln x}\,dx$

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

substituting

$du = \frac{1}{x}\,dx$

$$\int \frac{3}{u}\,du = 3\int \frac{1}{u}\,du = 3\ln|u| + C$$

$$= \boxed{3\ln|\ln x| + C}$$

39. $\int \frac{\sqrt{\ln x}}{x}\,dx$

let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$ so $du = \frac{1}{x}\,dx$

substituting

$$\int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{1}{\frac{3}{2}}u^{3/2} + C = \frac{2}{3}(\ln x)^{3/2} + C$$

41. $\int \left(xe^{x^2} - \frac{x}{x^2+2}\right)dx$

for right half

let $u = x^2 + 2$

Then $\frac{du}{dx} = 2x$

So $du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

left half

let $u = x^2$

$\frac{du}{dx} = 2x$

so $\frac{1}{2}du = x\,dx$

$$= \int xe^{x^2}dx - \int \frac{x}{x^2+2}\,dx$$

$$= \int \frac{1}{2}e^u\,du - \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}e^u - \frac{1}{2}\ln u + C = \frac{1}{2}e^{x^2} - \frac{1}{2}\ln(x^2+2) + C$$