Finally, solving for y', we have

$$y' = y\left(\frac{2}{x} + \frac{1}{x-1} + \frac{6x}{x^2+4}\right)$$
$$= x^2(x-1)(x^2+4)^3\left(\frac{2}{x} + \frac{1}{x-1} + \frac{6x}{x^2+4}\right)$$

Recall from Section 3.4 that the relative rate of change of a differentiable function Q of x is $Q'(x)/Q(x)$. In view of Rule 4, we see that the relative rate of change of Q at x can also be obtained by finding the derivative of $\ln Q$. We exploit this fact in Example 7.

APPLIED EXAMPLE 7 Population Growth The population of a town t months after the opening of an auto assembly plant in the surrounding area is given by the function

$$P(t) = 18000e^{-(\ln 9)e^{-0.1t}}$$

What is the relative rate of growth of the population 6 months after the opening of the auto assembly plant?

Solution We could find the required relative rate by computing $P'(t)/P(t)$ directly. Alternatively, we can proceed as follows:

$$\ln P(t) = \ln 18000e^{-(\ln 9)e^{-0.1t}}$$
$$= \ln 18000 + \ln e^{-(\ln 9)e^{-0.1t}}$$
$$= \ln 18000 - (\ln 9)e^{-0.1t} \qquad \ln e^x = x$$

$$\frac{P'(t)}{P(t)} = \frac{d}{dt}[\ln P(t)] = \frac{d}{dt}\ln 18000 - \frac{d}{dt}(\ln 9)e^{-0.1t}$$
$$= 0 - (\ln 9)(-0.1)e^{-0.1t} = (0.1)(\ln 9)e^{-0.1t}$$

$$\left.\frac{P'(t)}{P(t)}\right|_{t=6} = (0.1)(\ln 9)e^{-(0.1)(6)} \approx 0.121$$

... after the opening of the auto assembly plant, the relative rate of growth of the population is approximately 12.1% per month.

Solutions to Self-Check Exercises 5.5 can be found on

... logarithmic differentiation.

5.5 Exercises

In Exercises 1–34, find the derivative of the function.

1. $f(x) = 5\ln x$
2. $f(x) = \ln 5x$
3. $f(x) = \ln(x+1)$
4. $g(x) = \ln(2x+1)$
5. $f(x) = \ln x^8$
6. $h(t) = 2\ln t^5$
7. $f(x) = \ln\sqrt{x}$
8. $f(x) = \ln(\sqrt{x}+1)$
9. $f(x) = \ln\dfrac{1}{x^2}$
10. $f(x) = \ln\dfrac{1}{2x^3}$
11. $f(x) = \ln(4x^2 - 5x + 3)$
12. $f(x) = \ln(3x^2 - 2x + 1)$
13. $f(x) = \ln\dfrac{2x}{x+1}$
14. $f(x) = \ln\dfrac{x+1}{x-1}$
15. $f(x) = x^2\ln x$
16. $f(x) = 3x^2\ln 2x$
17. $f(x) = \dfrac{2\ln x}{x}$
18. $f(x) = \dfrac{3\ln x}{x^2}$
19. $f(u) = \ln(u-2)^3$
20. $f(x) = \ln(x^3-3)^4$
21. $f(x) = \sqrt{\ln x}$
22. $f(x) = \sqrt{\ln x + x}$
23. $f(x) = (\ln x)^2$
24. $f(x) = 2(\ln x)^{3/2}$
25. $f(x) = \ln(x^3+1)$
26. $f(x) = \ln\sqrt{x^2-4}$
27. $f(x) = e^x\ln x$
28. $f(x) = e^x\ln\sqrt{x+3}$
29. $f(t) = e^{2t}\ln(t+1)$
30. $g(t) = t^2\ln(e^{2t}+1)$
31. $f(x) = \dfrac{\ln x}{x^2}$
32. $g(t) = \dfrac{t}{\ln t}$
33. $f(x) = \ln(\ln x)$
34. $g(x) = \ln(e^x + \ln x)$

In Exercises 35–40, find the second derivative of the function.

35. $f(x) = \ln 2x$
36. $f(x) = \ln(x+5)$
37. $f(x) = \ln(x^2+2)$
38. $f(x) = (\ln x)^2$
39. $f(x) = x^2\ln x$
40. $g(x) = e^{2x}\ln x$

In Exercises 41–50, use logarithmic differentiation to find the derivative of the function.

41. $y = (x+1)^2(x+2)^3$
42. $y = (3x+2)^4(5x-1)^2$
43. $y = (x-1)^2(x+1)^3(x+3)^4$
44. $y = \sqrt{3x+5}(2x-3)^4$
45. $y = \dfrac{(2x^2-1)^5}{\sqrt{x+1}}$
46. $y = \dfrac{\sqrt{4+3x^2}}{\sqrt[3]{x^2+1}}$
47. $y = 3^x$
48. $y = x^{x+2}$
49. $y = (x^2+1)^x$
50. $y = x^{\ln x}$

In Exercises 51 and 52, use implicit differentiation to find dy/dx.

51. $\ln y - x\ln x = -1$
52. $\ln xy - y^2 = 5$

53. Find an equation of the tangent line to the graph of $y = x\ln x$ at the point $(1, 0)$.

54. Find an equation of the tangent line to the graph of $y = \ln x^2$ at the point $(2, \ln 4)$.

55. Determine the intervals where the function $f(x) = \ln x^2$ is increasing and where it is decreasing.

56. Determine the intervals where the function $f(x) = \dfrac{\ln x}{x}$ is increasing and where it is decreasing.

57. Determine the intervals of concavity for the graph of the function $f(x) = x^2 + \ln x^2$.

58. Determine the intervals of concavity for the graph of the function $f(x) = \dfrac{\ln x}{x}$.

59. Find the inflection points of the function $f(x) = \ln(x^2+1)$.

60. Find the inflection points of the function $f(x) = x^2\ln x$.

61. Find an equation of the tangent line to the graph of $f(x) = x^2 + 2\ln x$ at its inflection point.

62. Find an equation of the tangent line to the graph of $f(x) = e^{x/2}\ln x$ at its inflection point.
Hint: Show that $(1, 0)$ is the only inflection point of f.

63. Find the absolute extrema of the function $f(x) = x - \ln x$ on $[\frac{1}{2}, 3]$.

64. Find the absolute extrema of the function $g(x) = \dfrac{x}{\ln x}$ on $[2, 5]$.

In Exercises 65 and 66, find dy/dx by implicit differentiation.

65. $\ln(xy) = x + y$
66. $\ln x + e^{-y/x} = 10$

In Exercises 67 and 68, find d^2y/dx^2 by implicit differentiation.

67. $\ln x + xy = 5$
68. $\ln y + y = x$

69. Find dy/dx at the point $(1, 1)$ using implicit differentiation if $\ln y + xy = 1$.

70. Find an equation of the tangent line to the graph of the equation $\ln x + xe^y = 1$ at the point $(1, 0)$.

71. **Strain on Vertebrae** The strain (percentage of compression) on the lumbar vertebral disks in an adult human as a function of the load x (in kilograms) is given by

$$f(x) = 7.2956\ln(0.0645012x^{0.95} + 1)$$

What is the rate of change of the strain with respect to the load when the load is 100 kg? When the load is 500 kg?
Source: Benedek and Villars, *Physics with Illustrative Examples from Medicine and Biology.*