5.4 Concept Questions

1. State the rule for differentiating (a) $f(x) = e^x$ and (b) $g(x) = e^{f(x)}$, where f is a differentiable function.

2. Let $f(x) = e^{kx}$.
   a. Compute $f'(x)$.
   b. Use the result to deduce the behavior of f for the case $k > 0$ and the case $k < 0$.

5.4 Exercises

In Exercises 1–28, find the derivative of the function.

1. $f(x) = e^{3x}$
2. $f(x) = 3e^x$
3. $g(t) = e^{-t}$
4. $f(x) = e^{-2x}$
5. $f(x) = e^x + x^2$
6. $f(x) = 2e^x - x^2$
7. $f(x) = x^3e^x$
8. $f(u) = u^2e^{-u}$
9. $f(x) = \dfrac{e^x}{x}$
10. $f(x) = \dfrac{x}{e^x}$
11. $f(x) = 3(e^x + e^{-x})$
12. $f(x) = \dfrac{e^x + e^{-x}}{2}$
13. $f(w) = \dfrac{e^w + 2}{e^w}$
14. $f(x) = \dfrac{e^x}{e^x + 1}$
15. $f(x) = 2e^{3x-1}$
16. $f(t) = 4e^{3t+2}$
17. $h(x) = e^{-x^2}$
18. $f(x) = e^{x^2-1}$
19. $f(x) = 3e^{-1/x}$
20. $f(x) = e^{1/(2x)}$
21. $f(x) = (e^x + 1)^{25}$
22. $f(x) = (4 - e^{-3x})^3$
23. $f(x) = e^{\sqrt{x}}$
24. $f(t) = -e^{-\sqrt{2t}}$
25. $f(x) = (x - 1)e^{3x+2}$
26. $f(s) = (s^2 + 1)e^{-s^2}$
27. $f(x) = \dfrac{e^x - 1}{e^x + 1}$
28. $g(t) = \dfrac{e^{-t}}{1 + t^2}$

In Exercises 29–32, find the second derivative of the function.

29. $f(x) = e^{-4x} + e^{3x}$
30. $f(t) = 3e^{-2t} - 5e^{-t}$
31. $f(x) = 2xe^{3x}$
32. $f(t) = t^2e^{-2t}$

33. Find an equation of the tangent line to the graph of $y = e^{2x-3}$ at the point $(\frac{3}{2}, 1)$.

34. Find an equation of the tangent line to the graph of $y = e^{-x^2}$ at the point $(1, 1/e)$.

35. Determine the intervals where the function $f(x) = e^{-x^2/2}$ is increasing and where it is decreasing.

36. Determine the intervals where the function $f(x) = x^2e^{-x}$ is increasing and where it is decreasing.

37. Determine the intervals of concavity for the graph of the function $f(x) = \dfrac{e^x - e^{-x}}{2}$.

38. Determine the intervals of concavity for the graph of the function $f(x) = xe^x$.

39. Find the inflection point of the function $f(x) = xe^{-2x}$.

40. Find the inflection point(s) of the function $f(x) = 2e^{-x^2}$.

41. Find the equations of the tangent lines to the graph of $f(x) = e^{-x^2}$ at its inflection points.

42. Find an equation of the tangent line to the graph of $f(x) = xe^{-x}$ at its inflection point.

In Exercises 43–46, find the absolute extrema of the function.

43. $f(x) = e^{-x^2}$ on $[-1, 1]$
44. $h(x) = e^{x^2-4}$ on $[-2, 2]$
45. $g(x) = (2x - 1)e^{-x}$ on $[0, \infty)$
46. $f(x) = xe^{-x^2}$ on $[0, 2]$

In Exercises 47–50, use the curve-sketching guidelines of Chapter 4, page 295, to sketch the graph of the function.

47. $f(t) = e^t - t$
48. $h(x) = \dfrac{e^x + e^{-x}}{2}$
49. $f(x) = 2 - e^{-x}$
50. $f(x) = \dfrac{3}{1 + e^{-x}}$

In Exercises 51 and 52, find dy/dx by implicit differentiation.

51. $x^2 + y^3 = 2e^{2y}$
52. $xy^2 + \sqrt{x}e^y = 10$

In Exercises 53 and 54, find d^2y/dx^2 by implicit differentiation.

53. $x = y + e^y$
54. $e^x - e^y = y - x$

55. Find dy/dx at the point $(0, 1)$ using implicit differentiation if $xy + e^y = e$.

56. Find an equation of the tangent line to the graph of the equation $x + y - e^{x-y} = 1$ at the point $(1, 1)$.

57. **Online Video Viewers** As broadband Internet grows more popular, video services such as YouTube will continue to expand. The number of online video viewers (in millions) is projected to grow from 2008 through 2013 according to the rule

$$N(t) = 135e^{0.067t} \qquad (1 \le t \le 6)$$

where $t = 1$ corresponds to 2008.