5.1 Concept Questions

1. Define the exponential function f with base b and exponent x. What restrictions, if any, are placed on b?

2. For the exponential function $y = b^x$ ($b > 0, b \neq 1$), state (a) its domain and range, (b) its y-intercept, (c) where it is continuous, and (d) where it is increasing and where it is decreasing for the case $b > 1$ and the case $b < 1$.

5.1 Exercises

In Exercises 1–8, evaluate the expression.

1. **a.** $4^{-3} \cdot 4^5$ **b.** $3^{-3} \cdot 3^6$

2. **a.** $(2^{-1})^3$ **b.** $(3^{-2})^3$

3. **a.** $9(9)^{-1/2}$ **b.** $5(5)^{-1/2}$

4. **a.** $\left[\left(-\frac{1}{2}\right)^3\right]^{-2}$ **b.** $\left[\left(-\frac{1}{3}\right)^2\right]^{-3}$

5. **a.** $\dfrac{(-3)^4(-3)^5}{(-3)^8}$ **b.** $\dfrac{(2^{-4})(2^6)}{2^{-1}}$

6. **a.** $3^{1/4} \cdot 9^{-5/8}$ **b.** $2^{3/4} \cdot 4^{-3/2}$

7. **a.** $\dfrac{5^{3.3} \cdot 5^{-1.6}}{5^{-0.3}}$ **b.** $\dfrac{4^{2.7} \cdot 4^{-1.3}}{4^{-0.4}}$

8. **a.** $\left(\frac{1}{16}\right)^{-1/4}\left(\frac{27}{64}\right)^{-1/3}$ **b.** $\left(\frac{8}{27}\right)^{-1/3}\left(\frac{81}{256}\right)^{-1/4}$

In Exercises 9–16, simplify the expression.

9. **a.** $(64x^9)^{1/3}$ **b.** $(25x^3y^4)^{1/2}$

10. **a.** $(2x^3)(-4x^{-2})$ **b.** $(4x^{-2})(-3x^5)$

11. **a.** $\dfrac{6a^{-4}}{3a^{-3}}$ **b.** $\dfrac{4b^{-4}}{12b^{-6}}$

12. **a.** $y^{-3/2}y^{5/3}$ **b.** $x^{-3/5}x^{8/3}$

13. **a.** $(2x^3y^2)^3$ **b.** $(4x^2y^2z^3)^2$

14. **a.** $(x^{r/s})^{s/r}$ **b.** $(x^{-b/a})^{-a/b}$

15. **a.** $\dfrac{5^0}{(2^{-3}x^{-3}y^2)^2}$ **b.** $\dfrac{(x+y)(x-y)}{(x-y)^0}$

16. **a.** $\dfrac{(a^m \cdot a^{-n})^{-2}}{(a^{m+n})^2}$ **b.** $\left(\dfrac{x^{2n-2}y^{2n}}{x^{5n+1}y^{-n}}\right)^{1/3}$

In Exercises 17–26, solve the equation for x.

17. $6^{2x} = 6^6$

18. $5^{-x} = 5^3$

19. $3^{3x-4} = 3^5$

20. $10^{2x-1} = 10^{x+3}$

21. $(2.1)^{x+2} = (2.1)^5$

22. $(-1.3)^{x-2} = (-1.3)^{2x+1}$

23. $8^x = \left(\frac{1}{32}\right)^{x-2}$

24. $3^{x-x^2} = \dfrac{1}{9^x}$

25. $3^{2x} - 12 \cdot 3^x + 27 = 0$

26. $2^{2x} - 4 \cdot 2^x + 4 = 0$

In Exercises 27–36, sketch the graphs of the given functions on the same axes.

27. $y = 2^x$, $y = 3^x$, and $y = 4^x$

28. $y = \left(\frac{1}{2}\right)^x$, $y = \left(\frac{1}{3}\right)^x$, and $y = \left(\frac{1}{4}\right)^x$

29. $y = 2^{-x}$, $y = 3^{-x}$, and $y = 4^{-x}$

30. $y = 4^{0.5x}$ and $y = 4^{-0.5x}$

31. $y = 4^{0.5x}$, $y = 4^x$, and $y = 4^{2x}$

32. $y = e^x$, $y = 2e^x$, and $y = 3e^x$

33. $y = e^{0.5x}$, $y = e^x$, and $y = e^{1.5x}$

34. $y = e^{-0.5x}$, $y = e^{-x}$, and $y = e^{-1.5x}$

35. $y = 0.5e^{-x}$, $y = e^{-x}$, and $y = 2e^{-x}$

36. $y = 1 - e^{-x}$ and $y = 1 - e^{-0.5x}$

37. A function f has the form $f(x) = Ae^{kx}$. Find f if it is known that $f(0) = 100$ and $f(1) = 120$.
Hint: $e^{kx} = (e^k)^x$

38. If $f(x) = Axe^{-kx}$, find $f(3)$ if $f(1) = 5$ and $f(2) = 7$.
Hint: $e^{kx} = (e^k)^x$

39. If

$$f(t) = \frac{1000}{1 + Be^{-kt}}$$

find $f(5)$ given that $f(0) = 20$ and $f(2) = 30$.
Hint: $e^{kx} = (e^k)^x$

40. **Decline of *American Idol*** After having been on the air for more than a decade, Fox's *American Idol* seemed to be suffering from viewer fatigue. The average number of viewers from the 2011 season through the 2013 season is approximated by

$$f(t) = 32.744e^{-0.252t} \qquad (1 \le t \le 4)$$

where $f(t)$ is measured in millions, with $t = 1$ corresponding to the 2011 season.