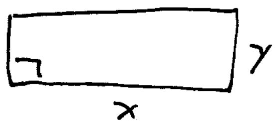
Mr. Konichek
Hmwk 4.5

4.5 Pg 327 (1, 2, 4, 6, 9)

1. 

$p = 100\,ft$ largest area = ?

$p = 2x + 2y = 100$ so $2(x+y) = 100$

$x + y = 50$

$\boxed{y = 50 - x}$

$A = f(x) = xy = x(50-x) = -x^2 + 50x$

$f'(x) = -2x + 50$

$-2x + 50 = 0$ $\quad f(25) = 25(50-25) = 225$ $\quad 225\,ft^2$

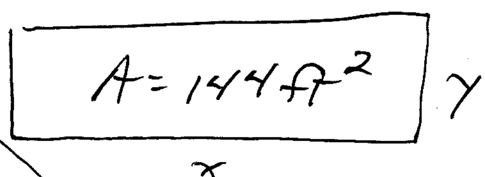
$-2(x-25) = 0$ $\quad f(0) = 0$

$x = 25$ $\quad f(50) = 50(50-50) = 0$

note: $x > 0$ and $y > 0$ or $50 - x > 0$ so $-x > -50$ means $x < 50$

closed, so interval $[0, 50]$

so the largest area occurs when the length is 25 ft and the width is 25 ft

2. 

smallest possible perimeter

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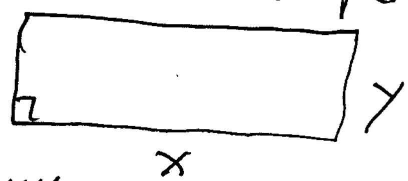
4.5 Pg. 327 (2)

2. $A = 144\ ft^2$ smallest perimeter?



$A = xy = 144$ so $y = \frac{144}{x}$

$p = 2x + 2y$ substituting gives

$$p = f(x) = 2x + 2\left(\frac{144}{x}\right) = \frac{2x^2 + 288}{x}$$

$$p' = f'(x) = \frac{x(4x) - (2x^2 + 288)(1)}{x^2}$$

$$f'(x) = \frac{4x^2 - 2x^2 - 288}{x^2} = \frac{2x^2 - 288}{x^2}$$

$$f'(x) = 2 - \frac{288}{x^2}$$

interval

$x > 0$

& $y > 0$

so $\frac{144}{x} > 0$

$x = \infty$

$[0, \infty)$

$$2 - \frac{288}{x^2} = 0$$

$$-\frac{288}{x^2} = -2$$

$$x^2 = 144$$

$$x = \pm 12$$

$f(12) = \frac{2(12)^2 + 288}{12} = \frac{288 + 288}{12} = \frac{576}{12} = 48$ smallest #

$f(-12) = -48$ ← smallest # but $x > 0$

$f(0)$ = undefined

$f(\infty) = \infty$

so dimensions $x = 48$ ft & $y = \frac{144}{48} = \frac{38}{12}$

$y = 3\frac{1}{6}$ ft

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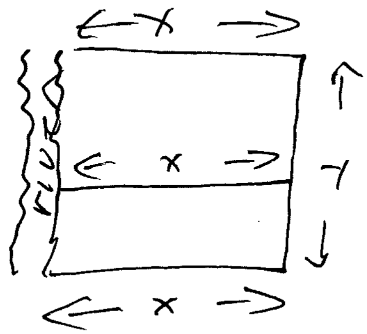
Hmwk 4.5 CONT

4.5 Pg 327 (2 cont., 4, 6, 9)

~~2. $f(0) = \frac{2(0)+288}{0}$ undefined at 0~~

~~$f(\infty) = 2\infty$ $\lim_{x \to \infty} \frac{2x+288}{x} = 2$~~

4.



largest area

$p = 3000$ yd

$p = 3x + 2y = 3000$

$2y = -3x + 3000$

$y = -\frac{3}{2}x + 1500$

$A = xy$

$A = f(x) = x\left(-\frac{3}{2}x + 1500\right) = -\frac{3}{2}x^2 + 1500x$

$f'(x) = -3x + 1500 = 0$

$-3x = -1500$

$x = 500$

$x > 0$

& $y \geq 0$ so

$-\frac{3}{2}x + 1500 > 0$

$-\frac{3}{2}x > -1500$

$-3x > -3000$

$x < 1000$

interval $[0, 1000]$

$f(0) = 0$

$f(1000) = -\frac{3}{2}(1000)^2 + 1500(1000) = \frac{-3(\overset{500{,}000}{1{,}000{,}000})}{\cancel{2}} + 1{,}500{,}000$

$f(1000) = -1{,}500{,}000 + 1{,}500{,}000 = 0$

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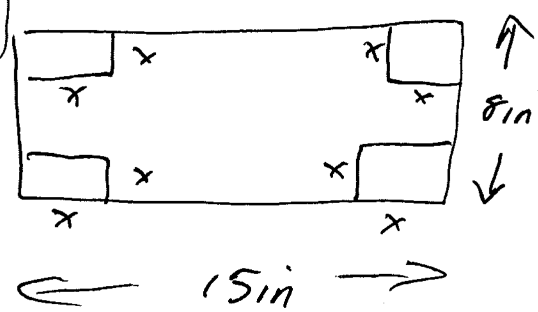
Hmwk 4.5 cont

4.5 Pg. 327 (6, 9)

#6



Note: Like Ex. 2

Max. Volume = ?

$$p = 2(15-2x) + 2(8-2x)$$
$$= 30 - 4x + 12 - 4x$$
$$P = -8x + 42 \quad \text{Note: } h = x$$

$$V = Bh = (15-2x)(8-2x)\,x$$
$$= (120 - 46x + 4x^2)\,x$$
$$V = 4x^3 - 46x^2 + 120x$$

$$V = f(x) = 4x^3 - 46x^2 + 120x$$
$$f'(x) = 12x^2 - 92x + 120$$
$$4(3x^2 - 23x + 30) = 0$$
$$4(3x-5)(x-5) = 0$$
$$x = 5/3 \qquad x = 5$$

$x > 0$ and $\quad 15 - 2x > 0 \qquad 8 - 2x > 0$

$$-2x > -15 \qquad -2x > -8$$
$$x < 7\tfrac{1}{2} \qquad x < 4$$

so $\quad V = f(x) = 4x^3 - 46x^2 + 120x$

$$= 2x(2x^2 - 23x + 60) \text{ on } [0, 4]$$

$$V(0) = 0$$
$$V(4) = 8(32 - 92 + 60) = 8(0) = 0$$
$$V(5) = 10(\cancel{50 - 115 + 60})^{-5} = -50$$
$$V(5/3) = \frac{10}{3}\left(\frac{50}{9} - \frac{115}{3} + 60\right) = \frac{10}{3}\left(\frac{50 - 345 + 540}{9}\right) = \frac{5900}{27}$$
$$\approx 218.52$$

#6 Answer The Volume will be $\boxed{\text{maximized when } x = 5/3 \text{ inches}}$;

$V = (15 - 2(5/3))(8 - 2(5/3))(5/3)$

$V = (15 - 3\tfrac{1}{3})(8 - 3\tfrac{1}{3})(5/3)$

$V = (11\tfrac{2}{3})(4\tfrac{2}{3})(5/3)$

$V \doteq 90.74 \text{ in}^3$

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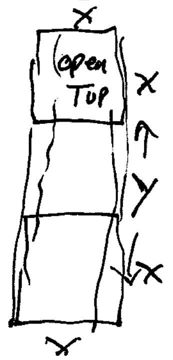
Hmwk 4.5 cont

Pg 327 (9)

9.



$V = 36 in^3$

sides $= .15/in^2$

bottom $= .40/in^2$

What should be the dimensions to minimize the construction cost?

$V = Bh = x^2 y = 36$

$y = \frac{36}{x^2}$

$Cost = 4xy(.15) + x^2(.40) = .6xy + .4x^2$

substituting $y = 36/x^2$

$Cost = f(x) = .6x\left(\frac{36}{x^2}\right) + .4x^2$

$f(x) = \frac{21.6}{x} + .4x^2$

$f'(x) = .8x - \frac{21.6}{x^2}$

$.8x - \frac{21.6}{x^2} = 0$

$.8x^3 - 21.6 = 0$ where $x \neq 0$

$8x^3 - 216 = 0$

$8(x^3 - 27) = 0$

$8(x-3)(x^2+3x+9) = 0$

$x = 3$ $\quad x = \frac{-3 \pm \sqrt{9 - 4(1)(9)}}{2}$ irrational Not real

interval?

$x > 0$

$y > 0$

means $\frac{36}{x^2} > 0$

$x = \infty$

so interval

$[0, \infty]$

$f(3) = \frac{21.6}{3} + .4(3)^2 =$

$7.2 + 3.6 = 10.8$ inches

smallest #

$f(0) =$ undefined

$f(8) = 8$

$\boxed{\text{so dimensions } x = 3 \text{ inches}; \ y = \frac{36}{3^2} = 4 \text{ inches}}$