a. Make a sketch of the graph of P. (Your answer will *not* be unique.)
b. Where is the function increasing?
c. Does the graph of P have a horizontal asymptote? If so, what is it?
d. Discuss the concavity of the graph of P. Explain its significance.
e. Is there an inflection point on the graph of P? If so, explain its significance.

In Exercises 33–36, use the information summarized in the table to sketch the graph of f.

33. $f(x) = x^3 - 3x^2 + 1$

Domain: $(-\infty, \infty)$
Intercept: y-intercept: 1
Asymptotes: None
Intervals where f is ↗ and ↘: ↗ on $(-\infty, 0)$ and $(2, \infty)$; ↘ on $(0, 2)$
Relative extrema: Rel. max. at $(0, 1)$; rel. min. at $(2, -3)$
Concavity: Downward on $(-\infty, 1)$; upward on $(1, \infty)$
Point of inflection: $(1, -1)$

34. $f(x) = \frac{1}{9}(x^4 - 4x^3)$

Domain: $(-\infty, \infty)$
Intercepts: x-intercepts: 0, 4; y-intercept: 0
Asymptotes: None
Intervals where f is ↗ and ↘: ↗ on $(3, \infty)$; ↘ on $(-\infty, 3)$
Relative extrema: Rel. min. at $(3, -3)$
Concavity: Downward on $(0, 2)$; upward on $(-\infty, 0)$ and $(2, \infty)$
Points of inflection: $(0, 0)$ and $(2, -\frac{16}{9})$

35. $f(x) = \dfrac{4x - 4}{x^2}$

Domain: $(-\infty, 0) \cup (0, \infty)$
Intercept: x-intercept: 1
Asymptotes: x-axis and y-axis
Intervals where f is ↗ and ↘: ↗ on $(0, 2)$; ↘ on $(-\infty, 0)$ and $(2, \infty)$
Relative extrema: Rel. max. at $(2, 1)$
Concavity: Downward on $(-\infty, 0)$ and $(0, 3)$ upward on $(3, \infty)$
Point of inflection: $(3, \frac{8}{9})$

36. $f(x) = x - 3x^{1/3}$

Domain: $(-\infty, \infty)$
Intercepts: x-intercepts: $\pm 3\sqrt{3}, 0$; y-intercept: 0
Asymptotes: None
Intervals where f is ↗ and ↘: ↗ on $(-\infty, -1)$ and $(1, \infty)$; ↘ on $(-1, 1)$
Relative extrema: Rel. max. at $(-1, 2)$; rel. min. at $(1, -2)$
Concavity: Downward on $(-\infty, 0)$; upward on $(0, \infty)$
Point of inflection: $(0, 0)$

In Exercises 37–60, sketch the graph of the function, using the curve-sketching guide of this section.

37. $g(x) = 4 - 3x - 2x^3$ **38.** $f(x) = x^2 - 2x + 3$

39. $h(x) = x^3 - 3x + 1$ **40.** $f(x) = 2x^3 + 1$

41. $f(x) = -2x^3 + 3x^2 + 12x + 2$

42. $f(t) = 2t^3 - 15t^2 + 36t - 20$

43. $h(x) = \frac{3}{2}x^4 - 2x^3 - 6x^2 + 8$

44. $f(t) = 3t^4 + 4t^3$

45. $f(t) = \sqrt{t^2 - 4}$ **46.** $f(x) = \sqrt{x^2 + 5}$

47. $g(x) = \frac{1}{2}x - \sqrt{x}$ **48.** $f(x) = \sqrt[3]{x^2}$

49. $g(x) = \dfrac{2}{x - 1}$ **50.** $f(x) = \dfrac{1}{x + 1}$

51. $h(x) = \dfrac{x + 2}{x - 2}$ **52.** $g(x) = \dfrac{x}{x - 1}$

53. $f(t) = \dfrac{t^2}{1 + t^2}$ **54.** $g(x) = \dfrac{x}{x^2 - 4}$

55. $g(t) = -\dfrac{t^2 - 2}{t - 1}$ **56.** $f(x) = \dfrac{x^2 - 9}{x^2 - 4}$

57. $g(t) = \dfrac{t^2}{t^2 - 1}$ **58.** $h(x) = \dfrac{1}{x^2 - x - 2}$

59. $h(x) = (x - 1)^{2/3} + 1$ **60.** $g(x) = (x + 2)^{3/2} + 1$

61. Cost of Removing Toxic Pollutants A city's main well was recently found to be contaminated with trichloroethylene (a cancer-causing chemical) as a result of an abandoned chemical dump that leached chemicals into the water. A proposal submitted to the city council indicated that the cost, measured in millions of dollars, of removing x% of the toxic pollutants is given by

$$C(x) = \frac{0.5x}{100 - x}$$

a. Find the vertical asymptote of the graph of C.
b. Is it possible to remove 100% of the toxic pollutant from the water?

62. Average Cost of Producing DVDs The average cost per disc (in dollars) incurred by Herald Media Corporation in pressing x DVDs is given by the average cost function

$$\overline{C}(x) = 2.2 + \frac{2500}{x}$$

a. Find the horizontal asymptote of the graph of $\overline{C}$.
b. What is the limiting value of the average cost?