mr. Konichek
Hmwk 3.6

3.6 Pg 231 (1, 5, 9, 15, 31, 33, 50, 64, 65)

1. $x + 2y = 5$

a.) $2y = 5 - x$

$y = \frac{5}{2} - \frac{x}{2}$

$f(x) = \frac{5}{2} - \frac{x}{2}$

$\boxed{f'(x) = -\frac{1}{2}}$

b.) $\frac{d}{dx}x + \frac{d}{dx}(2y) = \frac{d}{dx}5$

$1 + 2\frac{dy}{dx} = 0$

$2\frac{dy}{dx} = -1$

$\boxed{\frac{dy}{dx} = -\frac{1}{2}}$

5. $x^3 - x^2 - xy = 4$

a.) $-xy = 4 - x^3 + x^2$

$y = -\frac{4}{x} + x^2 - x$

$f(x) = -\frac{4}{x} + x^2 - x$

$\boxed{f'(x) = \frac{4}{x^2} + 2x - 1}$

OR $f'(x) = \frac{2x^3 - x^2 + 4}{x^2}$

b.) $\frac{d}{dx}x^3 - \frac{d}{dx}x^2 - \frac{d}{dx}(xy) = 4\frac{d}{dx}$

$= 3x^2 - 2x - \left(y(1) + x\frac{dy}{dx}\right)$

$= 0$

$-3x^2 + 2x + y + x\frac{dy}{dx} = 0$

$x\frac{dy}{dx} = 3x^2 - 2x - y$

$\frac{dy}{dx} = \frac{3x^2 - 2x - y}{x}$

$\boxed{\frac{dy}{dx} = 3x - 2 - \frac{y}{x}}$

3.6 Pg 231 (9, 15, 31, 33, 50, 64, 65)

9. $x^2+y^2=16$

$$\frac{d}{dy}x^2+\frac{d}{dy}y^2=\frac{d}{dy}16$$

$$2x+2y\frac{dy}{dx}=0$$

$$2y\frac{dy}{dx}=-2x \quad \text{so} \quad \frac{dy}{dx}=\frac{-2x}{2y}=\boxed{-\frac{x}{y}}$$

15. $x^2y^2-xy=8$

$$\frac{d}{dy}(x^2y^2)-\frac{d}{dy}(xy)=\frac{d}{dy}(8)$$

$$y^2(2x)+x^2\left(2y\frac{dy}{dx}\right)-\left[\cancel{\frac{dx}{dx}(1)+y(1)}+x\frac{dy}{dx}\right]=0$$

$$2xy^2+x^2\left(2y\frac{dy}{dx}\right)-y-x\frac{dy}{dx}=0$$

$$x^2\left(2y\frac{dy}{dx}\right)-x\left(\frac{dy}{dx}\right)=(y-2xy^2)$$

$$\frac{dy}{dx}(2x^2y-x)=y(1-2xy)$$

$$\frac{dy}{dx}=\frac{y(1-2xy)}{x(2xy-1)}=-\frac{y(\cancel{1-2xy})}{x(\cancel{1-2xy})}$$

$$\boxed{\frac{dy}{dx}=-\frac{y}{x}}$$

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Hmwk 3.6 (cont.)

3.6 Pg 231 (31, 33, 50, 64, 65)

31. $4x^2+9y^2=36$; $(0,2)$

$$\frac{d}{dx}(4x^2)+\frac{d}{dx}(9y^2)=\frac{d}{dx}36$$

$$8x+18y\frac{dy}{dx}=0$$

$$18y\frac{dy}{dx}=-8x$$

$$\frac{dy}{dx}=-\frac{8x}{18y}=-\frac{4x}{9y}$$

$$\left.\frac{dy}{dx}\right|_{(0,2)}=\left.-\frac{4x}{9y}\right|_{(0,2)}=\frac{-4(0)}{9(2)}=\frac{0}{18}=0$$

Note: point-slope form $y-y_1=m(x-x_1)$ given pt. (x_1,y_1)

$y-2=0(x-0)$

$y-2=0$

$\boxed{y=2}$

33. $x^2y^3-y^2+xy-1=0$; $(1,1)$

point-slope form

cont. from bottom of page

$y-1=\frac{-3}{2}(x-1)$

$y=\frac{-3}{2}(x-1)+1$

$\boxed{y=\frac{-3}{2}x+\frac{5}{2}}$

$$\frac{d}{dy}(x^2y^3)-\frac{d}{dy}y^2+\frac{d}{dy}(xy)-\frac{d}{dy}(1)=\frac{d}{dy}0$$

$$y^3(2x)+x^2(3y^2)\frac{dy}{dx}-2y\frac{dy}{dx}+y(1)+x\frac{dy}{dx}=0$$

$$3x^2y^2\frac{dy}{dx}-2y\frac{dy}{dx}+x\frac{dy}{dx}=-2xy^3-y$$

$$\frac{dy}{dx}=\frac{(-2xy^3-y)}{(3x^2y^2-2y+x)} \text{ at } (1,1)=\frac{-2-1}{3-2+1}=\frac{-3}{2}=m \text{ slope}$$

cont. on side

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Hmwk 3.6
CONT.

3.6 Pg 231 (50, 64, 65)

50. $V = x^3$ $\quad x = 5$ inches

$\frac{dx}{dt}(x) = 0.1 \frac{\text{inches}}{\text{sec}}$

$V' = 3x^2$

$V'' = 6x$

$125 \text{ in}^3 (6(0.1)) = 125(.6)$

$= 75.0 \frac{\text{in}^3}{\text{sec}}$

64. $x^2 + y^2 = 400$ $\quad \frac{dx}{dt} = 5 \frac{\text{ft}}{\text{sec}}$

$\frac{d}{dy} x^2 + \frac{d}{dy} y^2 = \frac{d}{dy}(400)$

$2x + 2y \frac{dx}{dy} = 0$

$\frac{dx}{dy} = \frac{2x}{2y} = \frac{x}{y}$ when $x = 12$

$x^2 = -y^2 + 400$ need y

when $x = 12$

$24 = -y^2 + 400$

$y^2 = 400 - 24$

$y^2 = 376$

$y = \sqrt{376}$

$2x \frac{dx}{dt} = -2y \frac{dy}{dt}$

$2(12)(5) = -2(\sqrt{376}) \frac{dy}{dt}$

$\frac{dy}{dt} = -\frac{120}{2\sqrt{376}} = -\frac{120\sqrt{376}}{752} = -\frac{60\sqrt{376}}{376} = -\frac{30\sqrt{376}}{188}$

$\frac{dy}{dt} = -\frac{15\sqrt{376}}{94} \approx (-.77 \text{ ft/sec})$

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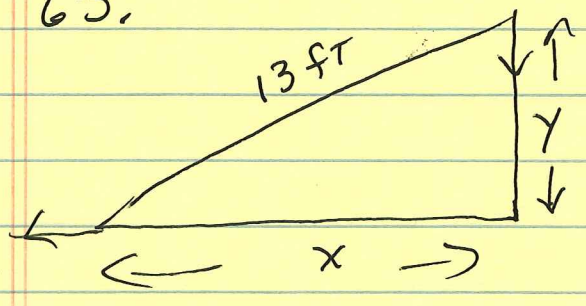
cont.

3.6 Pg 233 (65)

65.



when $x = 12$ ft $\quad \frac{dx}{dt} = 8 \frac{ft}{sec}$

$\frac{dy}{dt} = ?$

$x^2 + y^2 = 169$ when $x = 12$

$144 + y^2 = 169$
$y^2 = 169 - 144$
$y^2 = 25$
$y = 5$

$\frac{d}{dy} x^2 + \frac{d}{dy} y^2 = \frac{d}{dy} 169$

$2x + 2y \frac{dx}{dy} = 0$

$\frac{dx}{dy} = \frac{2x}{2y} = \frac{x}{y}$

$x^2 = -y^2 + 169$

$2x \frac{dx}{dt} = -2y \frac{dy}{dt}$

$2(12)(8) = -2(5) \frac{dy}{dt}$

$192 = -10 \frac{dy}{dt}$

$\frac{dy}{dt} = -19.2 \frac{ft}{sec}$

↑ means falling