3.6 Exercises

In Exercises 1–8, find the derivative dy/dx (a) by solving each of the implicit equations for y explicitly in terms of x and (b) by differentiating each of the equations implicitly. Show that in each case, the results are equivalent.

1. $x + 2y = 5$
2. $3x + 4y = 6$
3. $xy = 1$
4. $xy - y - 1 = 0$
5. $x^3 - x^2 - xy = 4$
6. $x^2y - x^2 + y - 1 = 0$
7. $\frac{x}{y} - x^2 = 1$
8. $\frac{y}{x} - 2x^3 = 4$

In Exercises 9–30, find dy/dx by implicit differentiation.

9. $x^2 + y^2 = 16$
10. $2x^2 + y^2 = 16$
11. $x^2 - 2y^2 = 16$
12. $x^3 + y^3 + y - 4 = 0$
13. $x^2 - 2xy = 6$
14. $x^2 + 5xy + y^2 = 10$
15. $x^2y^2 - xy = 8$
16. $x^2y^3 - 2xy^2 = 5$
17. $x^{1/2} + y^{1/2} = 1$
18. $x^{1/3} + y^{1/3} = 1$
19. $\sqrt{x + y} = x$
20. $(2x + 3y)^{1/3} = x^2$
21. $\frac{1}{x^2} + \frac{1}{y^2} = 1$
22. $\frac{1}{x^3} + \frac{1}{y^3} = 5$
23. $\sqrt{xy} = x + y$
24. $\sqrt{xy} = 2x + y^2$
25. $\frac{x + y}{x - y} = 3x$
26. $\frac{x - y}{2x + 3y} = 2x$
27. $xy^{3/2} = x^2 + y^2$
28. $x^2y^{1/2} = x + 2y^3$
29. $(x + y)^3 + x^3 + y^3 = 0$
30. $(x + y^2)^{10} = x^2 + 25$

In Exercises 31–34, find an equation of the tangent line to the graph of the function f defined by the equation at the indicated point.

31. $4x^2 + 9y^2 = 36$; $(0, 2)$
32. $y^2 - x^2 = 16$; $(2, 2\sqrt{5})$
33. $x^2y^3 - y^2 + xy - 1 = 0$; $(1, 1)$
34. $(x - y - 1)^3 = x$; $(1, -1)$

In Exercises 35–38, find the second derivative d^2y/dx^2 of each function defined implicitly by the equation.

35. $xy = 1$
36. $x^3 + y^3 = 28$
37. $y^2 - xy = 8$
38. $x^{1/3} + y^{1/3} = 1$

39. **Volume of a Cylinder** The volume of a right-circular cylinder of radius r and height h is $V = \pi r^2 h$. Suppose the radius and height of the cylinder are changing with respect to time t.
    a. Find a relationship between dV/dt, dr/dt, and dh/dt.
    b. At a certain instant of time, the radius and height of the cylinder are 2 and 6 in. and are increasing at the rate of 0.1 in./sec and 0.3 in./sec, respectively. How fast is the volume of the cylinder increasing?

40. **Distance Between Two Cars** A car leaves an intersection traveling west. Its position 4 sec later is 20 ft from the intersection. At the same time, another car leaves the same intersection heading north so that its position 4 sec later is 28 ft from the intersection. If the speeds of the cars at that instant of time are 9 ft/sec and 11 ft/sec, respectively, find the rate at which the distance between the two cars is changing.

41. **Effect of Price on Demand for Tires** Suppose the quantity demanded weekly of the Super Titan radial tires is related to its unit price by the equation

$$p + x^2 = 144$$

where p is measured in dollars and x is measured in units of a thousand. How fast is the quantity demanded weekly changing when $x = 9$, $p = 63$, and the price per tire is increasing at the rate of \$2/week?

42. **Effect of Price on Supply of Tires** Suppose the quantity x of Super Titan radial tires made available each week in the marketplace is related to the unit selling price by the equation

$$p - \frac{1}{2}x^2 = 48$$

where x is measured in units of a thousand and p is in dollars. How fast is the weekly supply of Super Titan radial tires being introduced into the marketplace changing when $x = 6$, $p = 66$, and the price per tire is decreasing at the rate of \$3/week?

43. **Effect of Price on Demand for Headphones** The demand equation for a certain brand of two-way headphones is

$$100x^2 + 9p^2 = 3600$$

where x represents the number (in thousands) of headphones demanded each week when the unit price is \$$p$. How fast is the quantity demanded increasing when the unit price per headphone is \$14 and the price is dropping at the rate of \$0.15/headphone/week?
Hint: To find the value of x when $p = 14$, solve the equation $100x^2 + 9p^2 = 3600$ for x when $p = 14$.

44. **Effect of Price on Supply of Eggs** Suppose the wholesale price of a certain brand of medium-sized eggs p (in dollars per carton) is related to the weekly supply x (in thousands of cartons) by the equation

$$625p^2 - x^2 = 100$$

If 25,000 cartons of eggs are available at the beginning of a certain week and the price is falling at the rate of 2¢/carton/week, at what rate is the weekly supply falling?
Hint: To find the value of p when $x = 25$, solve the supply equation for p when $x = 25$.