Chapter 2.6 – The Derivative

For linear equations, the slope is relatively simple:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

For non-linear equations (curves), we will use a more sophisticated yet similar formula to find the slope. In this class, deriving the formula from scratch is not key.

The slope at any point along a curve is equal to the slope of the tangent line to the curve at that point. The slope of the tangent line to the graph of f at a point P (x, y) is given by:

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The slope of the tangent line at a point on any curve is also known as the **derivative**. There are several notations for the derivative:

$$D_x f(x) \quad "d"-"sub"-"x"-"of"-"f"-"of"-"x"$$

$$\frac{dy}{dx} \quad "d"-"y"-"d"-"x"$$

$$y' \quad "y\ prime"$$

The most popular form we will use looks like: $f'(x)$ (Read: "f prime of x")

Thus the derivative (slope of a curve) at any point is:

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

This formula is commonly called the **"Limit Definition of the Derivative"** as it involves limits and the slope/derivative. Memorize it!

Example 1: Find the derivative of: $f(x) = x^2 - 2x$

(Continue in your notebook)

Find the derivative of the following:

1. $f(x) = 3x + 5$
2. $f(x) = 2x^2 + 3x + 4$
3. $f(x) = 3 - x$
4. $f(x) = 3x^2 - 3x$
5. $f(x) = x^3 - 5$

<u>Average Velocity versus Instantaneous Velocity</u>

Velocity is one representation of the slope since it is a measure of the change in distance over the change in time (think rise over run).

Average Velocity $\quad f(t) = t^2 - 4t, \quad t_1 = 6, \; t_2 = 6.1$

Average velocity is always *a comparison of velocities at two different times t_1 and t_2*. If the distance formula f(t) is given, then the average velocity, AV is:

$$AV = \frac{f(t_2) - f(t_1)}{t_2 - t_1} = \frac{change\ in\ dis\tan ce}{change\ in\ time}$$

Instantaneous Velocity $\quad f(t) = t^2 - 4t, \quad t = 6$

Instantaneous velocity is *the velocity at a single point/instance (in time).* Therefore, only 1 time value will be considered, t_1. If the distance formula f(t) is given, then instantaneous velocity, IV is:

$$IV = f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

Chapter 3.1 – Basic Rules of Differentiation

In the last chapter, we discussed some terms that were connected and/or synonymous. They were:

(i) Slope

(ii) Gradient

(iii) Tangent line

(iv) Rise over run

(v) "Delta" y over "delta" x

(vi) The change in y over the change in x

(vii) $\dfrac{y_2 - y_1}{x_2 - x_1}$

(viii) Derivative

(ix) $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(x) $f'(x)$

(xi) y'

(xii) $\dfrac{d(y)}{dx}, \ where \ y = f(x)$

One last term that indicates that we are finding the slope of a line/curve at a particular point is

(xiii) Differentiate – it simply means "find the derivative of"

Previously, we used the LIMIT DEFINITION (long method) to find the derivative, now we will use the Basic Rules of Differentiation (short method) to do the same.

Rule 1: Derivative of a Constant, c

$$\frac{d}{dx}(c) = 0$$

Rule 2: The Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1},\ if\ \ n = a\ real\ number$$

EXAMPLE 1:

$$f(x) = x^3$$
$$f'(x) = 3x^{3-1} = 3x^2$$

EXAMPLE 2:

$$y = \sqrt{x}$$

Rule 3: Derivative of a Constant Multiple of a Function

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

EXAMPLE 3:

$$f(x) = 4x^6$$

Rule 4: The Sum (and Difference) Rule

Remember that a difference is a <u>negative sum.</u>

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

In other words, the derivative of a sum of functions is equal to the sum of the derivatives of the functions.

NOTE: The same is NOT TRUE for products and quotients of functions!!!!!!!!!

EXAMPLE 4:

$$f(x) = 5x^3 - 3x^2 + 6x - 7$$

If you have time, try these:

$$f(x) = 3x^3 - 6x^2 + 5$$

$$f(x) = 12x^{\frac{1}{4}}$$

$$f(x) = \frac{1}{x^6}$$