Section 6.5 Chap. 6 Review below

b. The number of passengers in 2030 is projected to be $N(20) \approx 146.711e^{0.0456(20)} - 0.72 \approx 364.49$, or approximately 364.5 million.

c. The average growth rate from 2012 to 2030 is approximately $\dfrac{1}{20-2}\displaystyle\int_2^{20} R(t)\,dt \approx \dfrac{364.49-160}{18} \approx 11.36$, or approximately 11.4 million per year.

66. $\frac{1}{5}\int_0^5 p\,dt = \frac{1}{5}\int_0^5 \left(18 - 3e^{-2t} - 6e^{-t/3}\right)dt = \frac{1}{5}\left(18t + \frac{3}{2}e^{-2t} + 18e^{-t/3}\right)\Big|_0^5$
$= \frac{1}{5}\left[18(5) + \frac{3}{2}e^{-10} + 18e^{-5/3} - \frac{3}{2} - 18\right] = 14.78$, or \$14.78.

67. $\frac{1}{h}\int_0^h (2gx)^{1/2}\,dx = \frac{1}{3h}(2gx)^{3/2}\Big|_0^h = \frac{2}{3}\sqrt{2gh}$; that is, $\frac{2}{3}\sqrt{2gh}$ ft/sec.

68. The average content of oxygen in the pond over the first 10 days is

$$A = \frac{1}{10-0}\int_0^{10} 100\left(\frac{t^2+10t+100}{t^2+20t+100}\right)dt = \frac{100}{10}\int_0^{10}\left[1 - \frac{10}{t+10} + \frac{100}{(t+10)^2}\right]dt$$
$$= 10\int_0^{10}\left[1 - 10(t+10)^{-1} + 100(t+10)^{-2}\right]dt.$$

Using the substitution $u = t + 10$ for the third integral, we have

$$A = 10\left[t - 10\ln(t+10) - \frac{100}{t+10}\right]\Big|_0^{10} = 10\left\{\left[10 - 10\ln 20 - \frac{100}{2p}\right] - \left[-10\ln 10 - 10\right]\right\}$$
$$= 10(10 - 10\ln 20 - 5 + 10\ln 10 + 10) \approx 80.6853, \text{ or approximately } 80.7\%.$$

69. $\int_a^a f(x)\,dx = F(x)|_a^a = F(a) - F(a) = 0$, where $F'(x) = f(x)$.

70. $\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a) = -[F(a) - F(b)] = -F(x)|_b^a = -\int_b^a f(x)\,dx$.

71. $\int_1^3 x^2\,dx = \frac{1}{3}x^3\Big|_1^3 = 9 - \frac{1}{3} = \frac{26}{3} = -\int_3^1 x^2\,dx = -\frac{1}{3}x^3\Big|_3^1 = -\frac{1}{3} + 9 = \frac{26}{3}$.

72. $\int_a^b cf(x)\,dx = xF(x)|_a^b = c[F(b) - F(a)] = c\int_a^b f(x)\,dx$.

73. $\int_1^9 2\sqrt{x}\,dx = \frac{4}{3}x^{3/2}\Big|_1^9 = \frac{4}{3}(27-1) = \frac{104}{3}$ and $2\int_1^9 \sqrt{x}\,dx = 2\left(\frac{2}{3}x^{3/2}\right)\Big|_1^9 = \frac{104}{3}$.

74. $\int_0^1 (1 + x - e^x)\,dx = \left(x + \frac{1}{2}x^2 - e^x\right)\Big|_0^1 = \left(1 + \frac{1}{2} - e\right) + 1 = \frac{5}{2} - e$ and
$\int_0^1 dx + \int_0^1 x\,dx - \int_0^1 e^x\,dx = x|_0^1 + \left(\frac{1}{2}x^2\right)\Big|_0^1 - (e^x)|_0^1 = (1-0) + \left(\frac{1}{2} - 0\right) - (e-1) = \frac{5}{2} - e$, demonstrating Property 4.

75. $\int_0^3 \left(1 + x^3\right)dx = x + \frac{1}{4}x^4\Big|_0^3 = 3 + \frac{81}{4} = \frac{93}{4}$ and
$\int_0^1 \left(1 + x^3\right)dx + \int_1^3 \left(1 + x^3\right)dx = \left(x + \frac{1}{4}x^4\right)\Big|_0^1 + \left(x + \frac{1}{4}x^4\right)\Big|_1^3 = \left(1 + \frac{1}{4}\right) + \left(3 + \frac{81}{4}\right) - \left(1 + \frac{1}{4}\right) = \frac{93}{4}$, demonstrating Property 5.

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1. The consumer's surplus is \$18,000,000 and the producer's surplus is \$11,700,000.

2. The consumer's surplus is \$13,333 and the producer's surplus is \$11,667.

3. The consumer's surplus is \$33,120 and the producer's surplus is \$2880.

4. The consumer's surplus is \$55,104 and the producer's surplus is \$141,669.

5. Investment A will generate a higher net income.

6. Investment B will generate a higher net income.

CHAPTER 6 Concept Review Questions page 496

1. a. $F'(x) = f(x)$ b. $F(x) + C$

2. a. $c\int f(x)\,dx$ b. $\int f(x)\,dx \pm \int g(x)\,dx$

3. a. unknown b. function

4. $g'(x)\,dx$, $\int f(u)\,du$

5. a. $\int_a^b f(x)\,dx$ b. minus

6. a. $F(b) - F(a)$, antiderivative b. $\int_a^b f'(x)\,dx$

7. a. $\dfrac{1}{b-a}\int_a^b f(x)\,dx$ b. area, area

8. $\int_a^b [f(x) - g(x)]\,dx$

9. a. $\int_0^{\bar{x}} D(x)\,dx - \bar{p}\bar{x}$ b. $\bar{p}\bar{x} - \int_0^{\bar{x}} S(x)\,dx$

10. a. $e^{rT}\int_0^T R(t)\,e^{-rt}\,dt$ b. $\int_0^T R(t)\,e^{-rt}\,dt$

11. $\dfrac{mP}{r}\left(e^{rT} - 1\right)$

12. $2\int_0^1 [x - f(x)]\,dx$

CHAPTER 6 Review Exercises page 496

1. $\int \left(x^3 + 2x^2 - x\right) dx = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 + C.$

2. $\int \left(\frac{1}{3}x^3 - 2x^2 + 8\right) dx = \frac{1}{12}x^4 - \frac{2}{3}x^3 + 8x + C.$

3. $\displaystyle\int\left(x^4-2x^3+\frac{1}{x^2}\right)dx=\frac{x^5}{5}-\frac{x^4}{2}-\frac{1}{x}+C.$

4. $\int\left(x^{1/3}-x^{1/2}+4\right)dx=\frac{3}{4}x^{4/3}-\frac{2}{3}x^{3/2}+4x+C.$

5. $\int x\left(2x^2+x^{1/2}\right)dx=\int\left(2x^3+x^{3/2}\right)dx=\frac{1}{2}x^4+\frac{2}{5}x^{5/2}+C.$

6. $\int\left(x^2+1\right)\left(\sqrt{x}-1\right)dx=\int\left(x^{5/2}-x^2+x^{1/2}-1\right)dx=\frac{2}{7}x^{7/2}-\frac{1}{3}x^3+\frac{2}{3}x^{3/2}-x+C.$

7. $\displaystyle\int\left(x^2-x+\frac{2}{x}+5\right)dx=\int x^2\,dx-\int x\,dx+2\int\frac{dx}{x}+5\int dx=\frac{1}{3}x^3-\frac{1}{2}x^2+2\ln|x|+5x+C.$

8. Let $u=2x+1$, so $du=2\,dx$ and $dx=\frac{1}{2}\,du$. Then $\int\sqrt{2x+1}\,dx=\frac{1}{2}\int u^{1/2}\,du=\frac{1}{3}u^{3/2}=\frac{1}{3}\left(2x+1\right)^{3/2}+C.$

9. Let $u=3x^2-2x+1$, so $du=(6x-2)\,dx=2\,(3x-1)\,dx$ or $(3x-1)\,dx=\frac{1}{2}du$. So $\int(3x-1)\left(3x^2-2x+1\right)^{1/3}dx=\frac{1}{2}\int u^{1/3}du=\frac{3}{8}u^{4/3}+C=\frac{3}{8}\left(3x^2-2x+1\right)^{4/3}+C.$

10. Put $u=x^3+2$, so $du=3x^2\,dx$ and $x^2\,dx=\frac{1}{3}\,du$. Then $\displaystyle\int x^2\left(x^3+2\right)^{10}dx=\frac{1}{3}\int u^{10}\,du=\frac{1}{33}u^{11}+C=\frac{\left(x^3+2\right)^{11}}{33}+C.$

11. Let $u=x^2-2x+5$, so $du=2\,(x-1)\,dx$ and $(x-1)\,dx=\frac{1}{2}\,du$. Then $\displaystyle\int\frac{x-1}{x^2-2x+5}\,dx=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln|u|+C=\frac{1}{2}\ln\left(x^2-2x+5\right)+C.$

12. Let $u=-2x$, so $du=-2\,dx$. Then $\int 2e^{-2x}\,dx=-\int e^u\,du=-e^u+C=-e^{-2x}+C.$

13. Put $u=x^2+x+1$, so $du=(2x+1)\,dx=2\left(x+\frac{1}{2}\right)dx$ and $\left(x+\frac{1}{2}\right)dx=\frac{1}{2}du$. Then $\int\left(x+\frac{1}{2}\right)e^{x^2+x+1}\,dx=\frac{1}{2}\int e^u\,du=\frac{1}{2}e^u+C=\frac{1}{2}e^{x^2+x+1}+C.$

14. Let $u=e^{-x}+x$, so $du=\left(-e^{-x}+1\right)dx$ and $\left(e^{-x}-1\right)dx=-du$. Then $\displaystyle\int\frac{e^{-x}-1}{\left(e^{-x}+x\right)^2}\,dx=-\int\frac{du}{u^2}=\frac{1}{u}+C=\frac{1}{e^{-x}+x}+C.$

15. Let $u=\ln x$, so $du=\frac{1}{x}\,dx$. Then $\displaystyle\int\frac{(\ln x)^5}{x}\,dx=\int u^5\,du=\frac{1}{6}u^6+C=\frac{1}{6}\left(\ln x\right)^6+C.$

16. $\displaystyle\int\frac{\ln x^2}{x}\,dx=2\int\frac{\ln x}{x}\,dx.$ Now put $u=\ln x$, so $du=\frac{1}{x}\,dx$. Then $\displaystyle\int\frac{\ln x^2}{x}\,dx=2\int u\,du=u^2+C=(\ln x)^2+C.$

17. Let $u=x^2+1$, so $x^2=u-1$, $du=2x\,dx$, $x\,dx=\frac{1}{2}\,du$. Then

$$\int x^3\left(x^2+1\right)^{10}dx=\frac{1}{2}\int(u-1)\,u^{10}\,du=\frac{1}{2}\int\left(u^{11}-u^{10}\right)du=\frac{1}{2}\left(\frac{1}{12}u^{12}-\frac{1}{11}u^{11}\right)+C$$
$$=\frac{1}{264}u^{11}\left(11u-12\right)+C=\frac{1}{264}\left(x^2+1\right)^{11}\left(11x^2-1\right)+C.$$

18. Let $u = x + 1$, so $du = dx$. Then $x = u - 1$, so

$$\int x\sqrt{x+1}\,dx = \int (u-1)\,u^{1/2}\,du = \int \left(u^{3/2} - u^{1/2}\right) du = \tfrac{2}{5}u^{5/2} - \tfrac{2}{3}u^{3/2} + C$$
$$= \tfrac{2}{15}u^{3/2}(3u-5) + C = \tfrac{2}{15}(3x-2)(x+1)^{3/2} + C.$$

19. Put $u = x - 2$, so $du = dx$. Then $x = u + 2$ and

$$\int \frac{x}{\sqrt{x-2}}\,dx = \int \frac{u+2}{\sqrt{u}}\,du = \int \left(u^{1/2} + 2u^{-1/2}\right) du = \int u^{1/2}\,du + 2\int u^{-1/2}\,du = \tfrac{2}{3}u^{3/2} + 4u^{1/2} + C$$
$$= \tfrac{2}{3}u^{1/2}(u+6) + C = \tfrac{2}{3}\sqrt{x-2}\,(x-2+6) + C = \tfrac{2}{3}(x+4)\sqrt{x-2} + C.$$

20. Let $u = x + 1$, so $x = u - 1$ and $du = dx$. Then

$$\int \frac{3x}{\sqrt{x+1}}\,dx = 3\int \frac{u-1}{\sqrt{u}}\,du = 3\int \left(u^{1/2} - u^{-1/2}\right) du = 3\left(\tfrac{2}{3}u^{3/2} - 2u^{1/2}\right) + C = 2u^{1/2}(u-3) + C$$
$$= 2(x-2)\sqrt{x+1} + C.$$

21. $\int_0^1 \left(2x^3 - 3x^2 + 1\right) dx = \left(\tfrac{1}{2}x^4 - x^3 + x\right)\Big|_0^1 = \tfrac{1}{2} - 1 + 1 = \tfrac{1}{2}.$

22. $\int_0^2 \left(4x^3 - 9x^2 + 2x - 1\right) dx = \left(x^4 - 3x^3 + x^2 - x\right)\Big|_0^2 = 16 - 24 + 4 - 2 = -6.$

23. $\int_1^4 \left(x^{1/2} + x^{-3/2}\right) dx = \left(\tfrac{2}{3}x^{3/2} - 2x^{-1/2}\right)\Big|_1^4 = \left(\tfrac{2}{3}x^{3/2} - \tfrac{2}{\sqrt{x}}\right)\Big|_1^4 = \left(\tfrac{16}{3} - 1\right) - \left(\tfrac{2}{3} - 2\right) = \tfrac{17}{3}.$

24. Let $u = 2x^2 + 1$, so $du = 4x\,dx$ and $x\,dx = \tfrac{1}{4}du$. If $x = 0$, then $u = 1$ and if $x = 1$, then $u = 3$, so $\int_0^1 20x\left(2x^2+1\right)^4 dx = \tfrac{20}{4}\int_1^3 u^4\,du = u^5\Big|_1^3 = 243 - 1 = 242.$

25. Put $u = x^3 - 3x^2 + 1$, so $du = \left(3x^2 - 6x\right) dx = 3\left(x^2 - 2x\right) dx$ and $\left(x^2 - 2x\right) dx = \tfrac{1}{3}\,du$. If $x = -1$, $u = -3$ and if $x = 0$, $u = 1$, so $\int_{-1}^0 12\left(x^2 - 2x\right)\left(x^3 - 3x^2 + 1\right)^3 dx = (12)\left(\tfrac{1}{3}\right)\int_{-3}^1 u^3\,du = 4\left(\tfrac{1}{4}\right)u^4\Big|_{-3}^1 = 1 - 81 = -80.$

26. Let $u = x - 3$, so $du = dx$. If $x = 4$, then $u = 1$ and if $x = 7$, then $u = 4$, so

$$\int_4^7 x\sqrt{x-3}\,dx = \int_1^4 (u+3)\sqrt{u}\,du = \int_1^4 \left(u^{3/2} + 3u^{1/2}\right) du = \left(\tfrac{2}{5}u^{5/2} + 2u^{3/2}\right)\Big|_1^4$$
$$= \left(\tfrac{64}{5} + 16\right) - \left(\tfrac{2}{5} + 2\right) = \tfrac{132}{5}.$$

27. Let $u = x^2 + 1$, so $du = 2x\,dx$ and $x\,dx = \tfrac{1}{2}du$. If $x = 0$, then $u = 1$, and if $x = 2$, then $u = 5$, so

$$\int_0^2 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_1^5 \frac{du}{u} = \tfrac{1}{2}\ln u\Big|_1^5 = \tfrac{1}{2}\ln 5.$$

28. Let $u = 5 - 2x$, so $du = -2\,dx$, or $dx = -\tfrac{1}{2}du$. If $x = 0$, then $u = 5$ and if $x = 1$, then $u = 3$, so

$$\int_0^1 (5-2x)^{-2}\,dx = \int_5^3 \left(-\frac{1}{2}\right)\frac{du}{u^2} = \tfrac{1}{2}u^{-1}\Big|_5^3 = \tfrac{1}{6} - \tfrac{1}{10} = \tfrac{1}{15}.$$

29. Let $u = 1 + 2x^2$, so $du = 4x\,dx$ and $x\,dx = \tfrac{1}{4}\,du$. If $x = 0$, then $u = 1$ and if $x = 2$, then $u = 9$, so

$$\int_0^2 \frac{4x}{\sqrt{1+2x^2}}\,dx = \int_1^9 \frac{du}{u^{1/2}} = 2u^{1/2}\Big|_1^9 = 2(3-1) = 4.$$

30. Let $u = -\frac{1}{2}x^2$, so $du = -x\,dx$ and $x\,dx = -du$. If $x = 0$, then $u = 0$ and if $x = 2$, then $u = -2$, so $\int_0^2 xe^{-x^2/2}\,dx = -\int_0^{-2} e^u\,du = -e^u\big|_0^{-2} = -e^{-2} + 1 = 1 - \frac{1}{e^2}$.

31. Let $u = 1 + e^{-x}$, so $du = -e^{-x}\,dx$ and $e^{-x}\,dx = -du$. Then

$$\int_{-1}^{0} \frac{e^{-x}}{\left(1+e^{-x}\right)^2}\,dx = -\int_{1+e}^{2} \frac{du}{u^2} = \frac{1}{u}\bigg|_{1+e}^{2} = \frac{1}{2} - \frac{1}{1+e} = \frac{e-1}{2\,(1+e)}.$$

32. Let $u = \ln x$, so $du = \dfrac{dx}{x}$. If $x = 1$, then $u = 0$, and if $x = e$, then $u = \ln e = 1$, so

$$\int_1^e \frac{\ln x}{x}\,dx = \int_0^1 u\,du = \tfrac{1}{2}u^2\Big|_0^1 = \tfrac{1}{2}.$$

33. $f(x) = \int f'(x)\,dx = \int \left(3x^2 - 4x + 1\right)dx = 3\int x^2\,dx - 4\int x\,dx + \int dx = x^3 - 2x^2 + x + C$. The given condition implies that $f(1) = 1$, so $1 - 2 + 1 + C = 1$, and thus $C = 1$. Therefore, the required function is $f(x) = x^3 - 2x^2 + x + 1$.

34. $f(x) = \displaystyle\int f'(x)\,dx = \int \frac{x}{\sqrt{x^2+1}}\,dx$. Let $u = x^2 + 1$, so $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. Then $f(x) = \dfrac{1}{2}\displaystyle\int \frac{du}{\sqrt{u}} = \sqrt{u} + C = \sqrt{x^2+1} + C$. Now $f(0) = 1$ implies $\sqrt{0+1} + C = 1$, so $C = 0$. Thus, $f(x) = \sqrt{x^2+1}$.

35. $f(x) = \int f'(x)\,dx = \int \left(1 - e^{-x}\right)dx = x + e^{-x} + C$. Now $f(0) = 2$ implies $0 + 1 + C = 2$, so $C = 1$ and the required function is $f(x) = x + e^{-x} + 1$.

36. $f(x) = \displaystyle\int f'(x)\,dx = \int \frac{\ln x}{x}\,dx$. Let $u = \ln x$, so $du = \dfrac{dx}{x}$. Then $f(x) = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$. $f(1) = 0 + C = -2$ implies that $C = -2$, so the required function is $f(x) = \frac{1}{2}(\ln x)^2 - 2$.

37. **a.** The integral $\int_0^T \left[f(t) - g(t)\right]dt$ represents the distance in feet between Car A and Car B at time T. If Car A is ahead, the integral is positive and if Car B is ahead, it is negative.

b. The distance is greatest at $t = 10$, at which point Car B's velocity exceeds that of Car A and Car B starts catching up. At that instant, the distance between the cars is $\int_0^{10} \left[f(t) - g(t)\right]dt$.

38. **a.** The integral $\int_0^T \left[f(t) - g(t)\right]dt$ represents the difference in the total revenues of Branch A and Branch B at time T. If Branch A has greater revenues, the integral is positive, and if Branch B has greater revenues, it is negative.

b. The difference is greatest at $t = 10$, at which point Branch B's revenues begin growing faster than those of Branch A. At that instant, the difference in revenues is $\int_0^{10} \left[f(t) - g(t)\right]dt$.

39. $\Delta x = \frac{2-1}{5} = \frac{1}{5}$, so $x_1 = \frac{6}{5}$, $x_2 = \frac{7}{5}$, $x_3 = \frac{8}{5}$, $x_4 = \frac{9}{5}$, $x_5 = \frac{10}{5}$. The Riemann sum is

$$\begin{aligned} f(x_1)\,\Delta x + \cdots + f(x_5)\,\Delta x &= \left\{\left[-2\left(\tfrac{6}{5}\right)^2 + 1\right] + \left[-2\left(\tfrac{7}{5}\right)^2 + 1\right] + \cdots + \left[-2\left(\tfrac{10}{5}\right)^2 + 1\right]\right\}\left(\tfrac{1}{5}\right) \\ &= \tfrac{1}{5}\left(-1.88 - 2.92 - 4.12 - 5.48 - 7\right) = -4.28. \end{aligned}$$

40. The percentage of mobile phone users with smartphones is $P(t) = \int R(t)\,dt = \int 10.8\,dt = 10.8t + C$. To find C, we use the condition $P(0) = 38.5$, obtaining $C = 38.5$. Thus, $P(t) = 10.8t + 38.5$. The projected percentage of mobile phone users with smartphones in October 2013 is $P(2) = 10.8(2) + 38.5 = 60.1$, or 60.1%.

41. $V(t) = \int V'(t)\,dt = 3800 \int (t - 10)\,dt = 1900(t - 10)^2 + C$. The initial condition implies that $V(0) = 200{,}000$, that is, $190{,}000 + C = 200{,}000$, and so $C = 10{,}000$. Therefore, $V(t) = 1900(t - 10)^2 + 10{,}000$. The resale value of the computer after 6 years is given by $V(6) = 1900(-4)^2 + 10{,}000 = 40{,}400$, or \$40,400.

42. $C(x) = \int C'(x)\,dx = \int \left(0.00003x^2 - 0.03x + 20\right) dx = 0.00001x^3 - 0.015x^2 + 20x + k$. $C(0) = k = 500$, so the required total cost function is $C(x) = 0.00001x^3 - 0.015x^2 + 20x + 500$. The total cost of producing the first 400 coffeemakers per day is $C(400) = 0.00001(400)^3 - 0.015(400)^2 + 20(400) + 500 = 6740$, or \$6740.

43. **a.** $R(x) = \int R'(x)\,dx = \int (-0.03x + 60)\,dx = -0.015x^2 + 60x + C$. $R(0) = 0$ implies that $C = 0$, so $R(x) = -0.015x^2 + 60x$.

b. From $R(x) = px$, we have $-0.015x^2 + 60x = px$, and so $p = -0.015x + 60$.

44. **a.** We have the initial-value problem $T'(t) = 0.15t^2 - 3.6t + 14.4$ with $T(0) = 24$. Integrating, we find $T(t) = \int T'(t)\,dt = \int \left(0.15t^2 - 3.6t + 14.4\right) dt = 0.05t^3 - 1.8t^2 + 14.4t + C$. Using the initial condition, we find $T(0) = 24 = 0 + C$, so $C = 24$. Therefore, $T(t) = 0.05t^3 - 1.8t^2 + 14.4t + 24$.

b. The temperature at 10 a.m. was $T(4) = 0.05(4)^3 - 1.8(4)^2 + 14.4(4) + 24 = 56$, or 56°F.

45. **a.** The total number of DVDs sold as of year t is $T(t) = \int R(t)\,dt = \int \left(-0.03t^2 + 0.218t - 0.032\right) dt = -0.01t^3 + 0.109t^2 - 0.032t + C$. Using the condition $T(0) = 0.1$, we find $T(0) = C = 0.1$. Therefore, $T(t) = -0.01t^3 + 0.109t^2 - 0.032t + 0.1$.

b. The total number of DVDs sold in 2003 is $T(4) = -0.01(4)^3 + 0.109(4)^2 - 0.032(4) + 0.1 = 1.076$, or 1.076 billion.

46. $C(t) = \int C'(t)\,dt = \int \left(0.003t^2 + 0.06t + 0.1\right) dt = 0.001t^3 + 0.03t^2 + 0.1t + k$. But $C(0) = 2$, so $C(0) = k = 2$. Therefore, $C(t) = 0.001t^3 + 0.03t^2 + 0.1t + 2$. The pollution five years from now will be $C(5) = 0.001(5)^3 + 0.03(5)^2 + 0.1(5) + 2 = 3.375$, or 3.375 parts per million.

47. $C(x) = \int C'(x)\,dx = \int \left(0.00003x^2 - 0.03x + 10\right) dx = 0.00001x^3 - 0.015x^2 + 10x + k$. Now $C(0) = 600$ implies that $k = 600$, so $C(x) = 0.00001x^3 - 0.015x^2 + 10x + 600$. The total cost incurred in producing the first 500 corn poppers is $C(500) = 0.00001(500)^3 - 0.015(500)^2 + 10(500) + 600 = 3100$, or \$3100.

48. The number is $\int_0^{10} \left(0.00933t^3 + 0.019t^2 - 0.10833t + 1.3467\right) dt = \left(0.0023325t^4 + 0.0063333t^3 - 0.054165t^2 + 1.3467t\right)\Big|_0^{10} = 37.7$, or approximately 37.7 million Americans.

49. Using the substitution $u = 1 + 0.4t$, we find that $N(t) = \int 3000(1 + 0.4t)^{-1/2}\,dt = \frac{3000}{0.4} \cdot 2(1 + 0.4t)^{1/2} + C = 15{,}000\sqrt{1 + 0.4t} + C$. $N(0) = 100{,}000$ implies $15{,}000 + C = 100{,}000$, so $C = 85{,}000$. Therefore, $N(t) = 15{,}000\sqrt{1 + 0.4t} + 85{,}000$. The number using the subway six months from now will be $N(6) = 15{,}000\sqrt{1 + 2.4} + 85{,}000 \approx 112{,}659$.

50. Let $u = 5 - x$, so $du = -x\,dx$. Then

$$p(x) = \int \frac{240}{(5-x)^2}\,dx = 240\int (5-x)^{-2}\,dx = 240\int \left(-u^{-2}\right)du = 240u^{-1} + C = \frac{240}{5-x} + C.$$ Next, the condition $p(2) = 50$ gives $\frac{240}{3} + C = 80 + C = 50$, so $C = -30$. Therefore, $p(x) = \dfrac{240}{5-x} - 30$.

51. a. The online retail sales will be

$S(t) = \int R(t)\,dt = 15.82\int e^{-0.176t}\,dt = -\frac{15.82}{0.176}e^{-0.176t} + C = -89.89e^{-0.176t} + C$.

$S(0) = 116$ implies that $-89.89 + C = 116$, so $C = 205.89$. Therefore, $S(t) = 205.89 - 89.89e^{-0.176t}$.

b. The sales will be $S(4) = 205.89 - 89.89e^{-0.176(4)} \approx 161.43$, or \$161.43 billion.

52. The total number of systems that Vista may expect to sell t months from the time they are put on the market is given by $f(t) = 3000t - 50{,}000\left(1 - e^{-0.04t}\right)$. The number is

$$\int_0^{12} \left(3000 - 2000e^{-0.04t}\right)dt = \int \left(3000 - 2000e^{-0.04t}\right)dt = \left(3000t - \tfrac{2000}{-0.04}e^{-0.04t}\right)\Big|_0^{12}$$
$$= 3000(12) + 50{,}000e^{-0.48} - 50{,}000 = 16{,}939.$$

53. The number of speakers sold at the end of 5 years is

$f(t) = \int f'(t)\,dt = \int_0^5 2000\left(3 - 2e^{-t}\right)dt = 2000\left[3(5) - 2e^{-5}\right] - 2000\left[3 - 2(1)\right] = 26{,}027$.

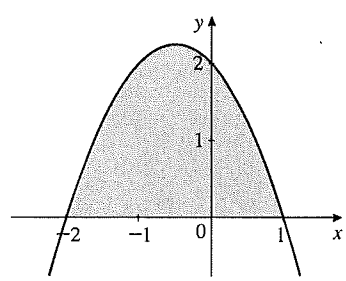
54. $A = \int_{-1}^{2} \left(3x^2 + 2x + 1\right)dx = \left(x^3 + x^2 + x\right)\Big|_{-1}^{2} = \left(2^3 + 2^2 + 2\right) - \left[(-1)^3 + 1 - 1\right] = 14 - (-1) = 15$.

55. $A = \int_0^2 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_0^2 = \frac{1}{2}\left(e^4 - 1\right)$.

56. $A = \displaystyle\int_1^3 \frac{1}{x^2}\,dx = \int_1^3 x^{-2}\,dx = -\frac{1}{x}\Big|_1^3 = -\frac{1}{3} + 1 = \frac{2}{3}$.

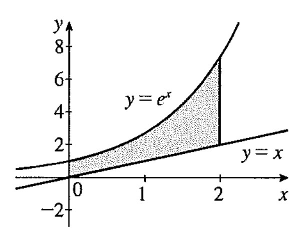
57. The graph of y intersects the x-axis at $x = -2$ and $x = 1$, so

$$A = \int_{-2}^{1} \left(-x^2 - x + 2\right)dx = \left(-\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 + 2x\right)\Big|_{-2}^{1}$$
$$= \left(-\tfrac{1}{3} - \tfrac{1}{2} + 2\right) - \left(\tfrac{8}{3} - \tfrac{4}{2} - 4\right) = \tfrac{7}{6} + \tfrac{10}{3} = \tfrac{9}{2}.$$



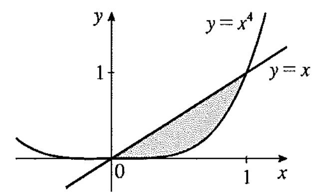
58. $A = \int_a^b \left[f(x) - g(x)\right]dx = \int_0^2 \left(e^x - x\right)dx = \left(e^x - \frac{1}{2}x^2\right)\Big|_0^2$

$= \left(e^2 - 2\right) - (1 - 0) = e^2 - 3$.



59. To find the points of intersection of the two curves, we solve $x^4 = x$, obtaining $x(x^3 - 1) = 0$, and so $x = 0$ or 1. Thus,

$A = \int_0^1 (x - x^4)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{5}x^5\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{5} = \frac{3}{10}.$

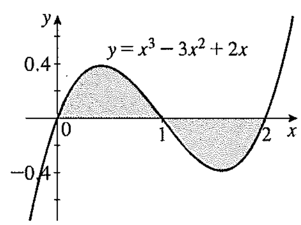


60. $A = \int_0^1 (x^3 - 3x^2 + 2x)\,dx - \int_1^2 (x^3 - 3x^2 + 2x)\,dx$

$= \left(\frac{1}{4}x^4 - x^3 + x^2\right)\Big|_0^1 - \left(\frac{1}{4}x^4 - x^3 + x^2\right)\Big|_1^2$

$= \frac{1}{4} - 1 + 1 - \left[(4 - 8 + 4) - \left(\frac{1}{4} - 1 + 1\right)\right]$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$



61. The amount of additional oil that will be produced over the next ten years is given by

$\int_0^{10} [R_2(t) - R_1(t)]\,dt = \int_0^{10} \left(100e^{0.08t} - 100e^{0.05t}\right) dt = 100\int_0^{10} \left(e^{0.08t} - e^{0.05t}\right) dt$

$= \left(\frac{100}{0.08}e^{0.08t} - \frac{100}{0.05}e^{0.05t}\right)\Big|_0^{10} = 1250e^{0.8} - 2000e^{0.5} - 1250 + 2000$

$= 2781.9 - 3297.4 - 1250 + 2000 = 234.5$, or 234,500 barrels.

62. $A = \frac{1}{3}\int_0^3 \frac{x}{\sqrt{x^2 + 16}}\,dx = \frac{1}{3} \cdot \frac{1}{2} \cdot 2\left(x^2 + 16\right)^{1/2}\Big|_0^3 = \frac{1}{3}\left(x^2 + 16\right)^{1/2}\Big|_0^3 = \frac{1}{3}(5 - 4) = \frac{1}{3}.$

63. The average temperature is

$\frac{1}{12}\int_0^{12} \left(-0.05t^3 + 0.4t^2 + 3.8t + 5.6\right) dt = \frac{1}{12}\left(-\frac{0.05}{4}t^4 + \frac{0.4}{3}t^3 + 1.9t^2 + 5.6t\right)\Big|_0^{12} = 26°\text{F}.$

64. $\overline{A} = \frac{1}{5}\int_0^5 \left(\frac{1}{12}t^2 + 2t + 44\right) dt = \frac{1}{5}\left(\frac{1}{36}t^3 + t^2 + 44t\right)\Big|_0^5 = \frac{1}{5}\left(\frac{125}{36} + 25 + 220\right) = \frac{125 + 900 + 7920}{5\,(36)}$

≈ 49.69, or 49.7 ft/sec.

65. The average rate of growth between $t = 0$ and $t = 9$ is

$\frac{1}{9-0}\int_0^9 R(t)\,dt = \frac{1}{9}\int_0^9 \left(-0.0039t^2 + 0.0374t + 0.0046\right) dt = \frac{1}{9}\left(-0.0013t^3 + 0.0187t^2 + 0.0046t\right)\Big|_0^9$

$= \frac{1}{9}\left[-0.0013\,(9^3) + 0.0187\,(9^2) + 0.0046\,(9)\right] = 0.0676$, or 67,600/yr.

66. Setting $p = 8$, we have $-0.01x^2 - 0.2x + 23 = 8$, $-0.01x^2 - 0.2x + 15 = 0$, and so $x^2 + 20x - 1500 = (x - 30)(x + 50) = 0$, giving $x = -50$ or 30. Thus,

$CS = \int_0^{30} \left(-0.01x^2 - 0.2x + 23\right) dx - 8\,(30) = \left(-\frac{0.01}{3}x^3 - 0.1x^2 + 23x\right)\Big|_0^{30} - 240$

$= -\frac{0.01}{3}(30)^3 - 0.1\,(900) + 23\,(30) - 240 = 270$, or \$270,000.

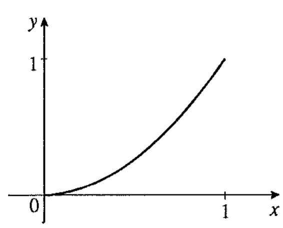
67. To find the equilibrium point, we solve $0.1x^2 + 2x + 20 = -0.1x^2 - x + 40$, obtaining $0.2x^2 + 3x - 20 = 0$, $x^2 + 15x - 100 = 0$, $(x + 20)(x - 5) = 0$, and so $x = 5$. Therefore, $p = -0.1(25) - 5 + 40 = 32.5$, and $CS = \int_0^5 (-0.1x^2 - x + 40)\,dx - (5)(32.5) = \left(-\frac{0.1}{3}x^3 - \frac{1}{2}x^2 + 40x\right)\Big|_0^5 - 162.5 = 20.833$, or \$2083. Also, $PS = 5(32.5) - \int_0^5 (0.1x^2 + 2x + 20)\,dx = 162.5 - \left(\frac{0.1}{3}x^3 + x^2 + 20x\right)\Big|_0^5 \approx 33.33$, or \$3333.

68. Use Equation (17) with $P = 4000$, $r = 0.08$, $T = 20$, and $m = 1$ to get $A = \dfrac{1 \cdot 4000}{0.08}\left(e^{1.6} - 1\right) \approx 197{,}651.62$. That is, Chi-Tai will have approximately \$197,652 in his account after 20 years.

69. Use Equation (18) with $P = 925$, $m = 12$, $T = 30$, and $r = 0.06$ to get $PV = \dfrac{mP}{r}\left(1 - e^{-rT}\right) = \dfrac{12 \cdot 925}{0.06}\left(1 - e^{-0.06 \cdot 30}\right) = 154{,}419.71$. We conclude that the present value of the purchase price of the house is $154{,}419.71 + 20{,}000$, or approximately \$174,420.

70. Here $P = 80{,}000$, $m = 1$, $T = 10$, and $r = 0.1$, so $PV = \dfrac{1 \cdot 80{,}000}{0.1}\left(1 - e^{-1}\right) \approx 505{,}696$, or approximately \$505,696.

71. a.



b. $f(0.3) = \frac{17}{18}(0.3)^2 + \frac{1}{18}(0.3) \approx 0.1$. Thus, 30% of the people receive 10% of the total income.
$f(0.6) = \frac{17}{18}(0.6)^2 + \frac{1}{18}(0.6) \approx 0.37$, so 60% of the people receive 37% of the total income.

c. The coefficient of inequality for this curve is
$L = 2\int_0^1 \left(x - \frac{17}{18}x^2 - \frac{1}{18}x\right) dx = \frac{17}{9}\int_0^1 (x - x^2)\,dx = \frac{17}{9}\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{17}{54} \approx 0.315$.

72. The average population will be $\frac{1}{5}\int 80{,}000e^{0.05t}\,dt = \frac{80{,}000}{5}\left(\frac{1}{0.05}\right)e^{0.05t}\Big|_0^5 = 320{,}000\left(e^{0.25} - 1\right) \approx 90{,}888$.

CHAPTER 6 Before Moving On... page 500

1. $$\int \left(2x^3 + \sqrt{x} + \frac{2}{x} - \frac{2}{\sqrt{x}}\right) dx = 2\int x^3\,dx + \int x^{1/2}\,dx + 2\int \frac{1}{x}\,dx - 2\int x^{-1/2}\,dx = \frac{1}{2}x^4 + \frac{2}{3}x^{3/2} + 2\ln|x| - 4x^{1/2} + C.$$

2. $f(x) = \int f'(x)\,dx = \int (e^x + x)\,dx = e^x + \frac{1}{2}x^2 + C$. $f(0) = 2$ implies $f(0) = e^0 + 0 + C = 2$, so $C = 1$. Therefore, $f(x) = e^x + \frac{1}{2}x^2 + 1$.

3. Let $u = x^2 + 1$, so $du = 2x\,dx$ or $x\,dx = \frac{1}{2}du$. Then
$$\int \frac{x}{\sqrt{x^2+1}}\,dx = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}\left(2u^{1/2}\right) + C = \sqrt{u} + C = \sqrt{x^2+1} + C.$$

4. Let $u = 2 - x^2$, so $du = -2x\,dx$ and $x\,dx = -\frac{1}{2}\,du$. If $x = 0$, then $u = 2$ and if $x = 1$, then $u = 1$. Therefore,

$\int_0^1 x\sqrt{2-x^2}\,dx = -\frac{1}{2}\int_2^1 u^{1/2}\,du = -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)\Big|_2^1 = -\frac{1}{3}u^{3/2}\Big|_2^1 = -\frac{1}{3}\left(1 - 2^{3/2}\right) = \frac{1}{3}\left(2\sqrt{2} - 1\right).$

5. To find the points of intersection, we solve $x^2 - 1 = 1 - x$, obtaining $x^2 + x - 2 = 0$, $(x+2)(x-1) = 0$, and so $x = -2$ or $x = 1$. The points of intersection are $(-2, 3)$ and $(1, 0)$. Thus, the required area is

$$A = \int_{-2}^1 \left[(1-x) - \left(x^2-1\right)\right] dx = \int_{-2}^1 \left(2 - x - x^2\right) dx = \left(2x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_{-2}^1$$
$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) = \frac{9}{2}.$$

CHAPTER 6 Explore & Discuss

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$F(2) = \frac{2}{15}(8+1)^{5/2} = \frac{162}{5}$. To find $F(2)$ using the function G, we have to compute $G(u)$, where $u = 2^3 + 1 = 9$, obtaining $G(9) = \frac{2}{15} \cdot 9^{5/2} = \frac{162}{5}$. We use the value 9 for u because $u = x^3 + 1$ and when $x = 2$, $u = 2^3 + 1 = 9$.

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1. Let $u = ax + b$, so that $du = a\,dx$ and $dx = \frac{1}{a}\,du$. Then

$\int f(ax+b)\,dx = \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}\int f(u)\,du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C.$

2. In order to evaluate $\int (2x+3)^5\,dx$, we write $f(u) = u^5$ so that $F(u) = \int f(u)\,du = \int u^5\,du = \frac{1}{6}u^6 + C$. Next, identifying $a = 2$ and $b = 3$, we obtain $\int (2x+3)^5\,dx = \frac{1}{2}F(2x+3)^6 + C = \frac{1}{2}\cdot\frac{1}{6}(2x+3)^6 + C = \frac{1}{12}(2x+3)^6 + C$. In order to evaluate $\int e^{3x-2}\,dx$, we let $f(x) = e^u$, $a = 3$, and $b = -2$. We find $\int e^{3x-2}\,dx = \frac{1}{3}e^{3x-2} + C$.

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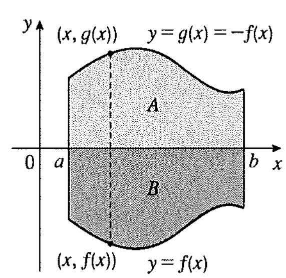
Observe that for each x in $[a, b]$, the point $(x, -f(x))$ is the mirror image of the point $(x, f(x))$ with respect to the x-axis. Therefore, the graph of the function $g = -f$ is symmetric to that of f with respect to the x-axis. Therefore, the area of A is equal to the area of B. But $g(x) = -f(x) \geq 0$ for all x in $[a, b]$, so that area is equal to $\int g(x)\,dx = \int [-f(x)]\,dx = -\int f(x)\,dx$, as was to be shown.



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1. A formal application of Equation (9) would seem to yield $\int_{-1}^1 \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_{-1}^1 = -1 - 1 = -2.$

2. The indicated observation would appear to follow.

3. Because $f(x) = 1/x^2$ is not continuous on the interval $[-1, 1]$, the fundamental theorem of calculus is not applicable. Thus, the result obtained in Exercise 1 is not valid. Furthermore, the fact that this result is suspect is suggested by the observation made in Exercise 2.

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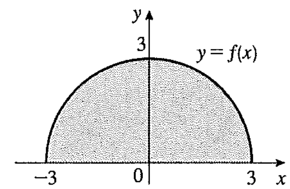
The graph of the integrand $y = f(x) = \sqrt{9 - x^2}$ is the upper semicircle with radius 3 centered at the origin. Interpreting the given integral as the area under the graph of f we find $\int_{-3}^{3} \sqrt{9 - x^2}\, dx = \frac{1}{2}(\pi)(3)^2 = \frac{9\pi}{2}$.



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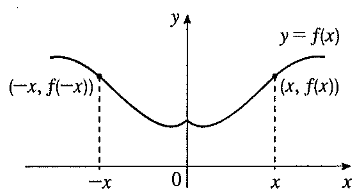
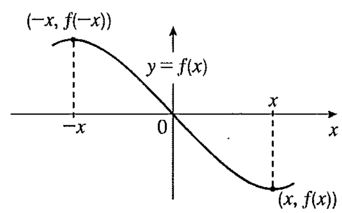
The required area is

$$\int_0^2 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx = \int_0^1 x^{1/2}\,dx + \int_1^2 (1/x)\,dx = \left(\tfrac{2}{3}x^{3/2}\right)\Big|_0^1 + (\ln x)|_1^2$$
$$= \tfrac{2}{3} + (\ln 2 - \ln 1) = \tfrac{2}{3} + \ln 2.$$

Page 472



f is even



f is odd

Suppose f is even, so that $f(-x) = f(x)$. If $(x, f(x))$ is any point lying on the graph of f, then $(-x, f(-x)) = (-x, f(x))$, and thus the graph of f is symmetric with respect to the y-axis. If f is odd, then $f(-x) = -f(x)$, and so f is symmetric with respect to the x-axis. Finally, if f is even, then $\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_0^a f(x)\,dx$.Let $u = -x$ in the first integral on the right-hand side. Then $du = -dx$, if $x = -a$, then $u = a$, and if $x = 0$, then $u = 0$. Using Property 2 of the definite integral, we have $\int_{-a}^{a} f(x)\,dx = \int_a^0 f(-u)(-du) + \int_0^a f(x)\,dx = \int_0^a f(u)\,du + \int_0^a f(x)\,dx$ because f is even. But u is a dummy variable and can be replaced with x, so the expression is equal to $2\int_0^a f(x)\,dx$. If f is odd, a similar argument gives

$$\int_{-a}^{a} f(x)\,dx = \int_a^0 f(x)\,dx + \int_0^a f(x)\,dx = -\int_0^a f(-u)(-du) + \int_0^a f(x)\,dx$$
$$= -\int_0^a f(u)\,du + \int_0^a f(x)\,dx = 0.$$

CHAPTER 6 Exploring with Technology

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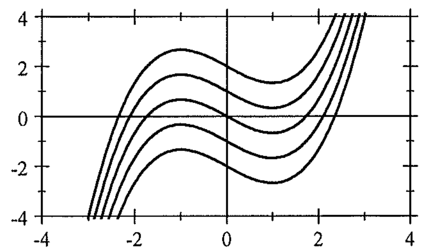
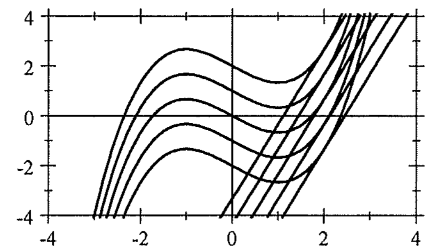
1. $F'(x) = \frac{1}{3}(3x^2) - 1 = x^2 - 1 = f(x)$, and so F is an antiderivative of f.

2.



3.



4. The slope of the tangent line is $f(2) = 4 - 1 = 3$.

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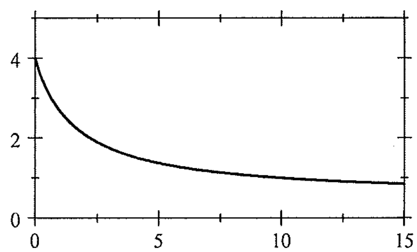
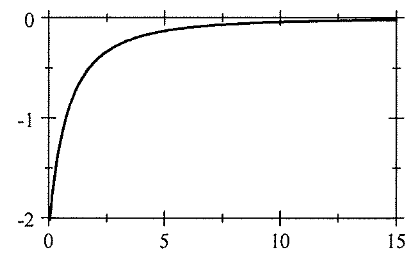
1.



$C'(12) \approx -0.0312314$.

2.



$C'(12) \approx -0.0312314$. Because C' is the derivative of C, we can find $C'(12)$ either by taking the derivative of C at $t = 12$ or by evaluating $C'(t)$ at $t = 12$.

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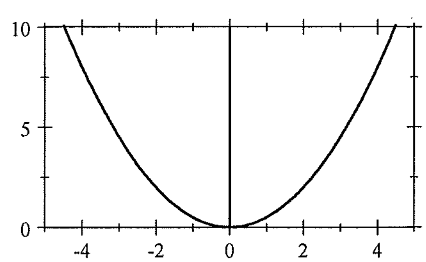
1.



2. The two graphs are identical.

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1. $\int_0^4 x\sqrt{9+x^2}\,dx \approx 32.6666666667$.

2. $\frac{1}{2}\int_9^{25} \sqrt{u}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_9^{25} = \frac{1}{3}\left(25^{3/2} - 9^{3/2}\right) = \frac{98}{3}$.

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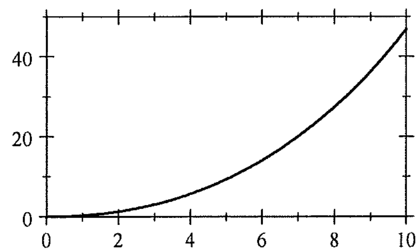
1. $F(x) = \int_0^x [R_1(t) - R(t)]\,dt = 20\left(\dfrac{e^{0.08t}}{0.08} - \dfrac{e^{0.05t}}{0.05}\right)\Big|_0^x = \left(250e^{0.08t} - 400e^{0.05t}\right)\Big|_0^x$

   $= 250e^{0.08x} - 400e^{0.05x} + 150.$

2. 

3. $F(5) = 250e^{0.08(5)} - 400e^{0.05(5)} + 150 = 9.346.$

4. The advantage of this model is that we can easily find the amount of oil saved by evaluating the function at the appropriate value of x.

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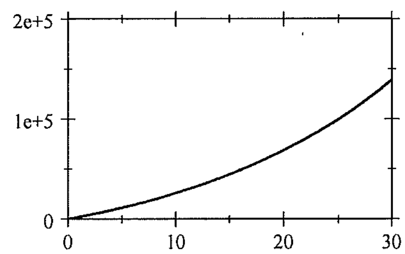
1. With $P = 2000$, $r = 0.05$, $m = 1$, and $T = x$, we obtain $A = f(x) = \frac{2000}{0.05}\left(e^{0.05x} - 1\right) = 40{,}000\left(e^{0.05x} - 1\right)$.

2. 

3. The advantage of the model is that we can compute the amount that Marcus will have in his IRA at any time T simply by evaluating the function f at $x = T$.