49. True. $dy = f'(x)\,dx = \frac{d}{dx}(ax+b)\,dx = a\,dx$. On the other hand, $\Delta y = f(x+\Delta x) - f(x) = [a(x+\Delta x)+b] - (ax+b) = a\,\Delta x = a\,dx$.

50. True. The percentage change in A is approximately $\dfrac{100\left[f(x+\Delta x) - f(x)\right]}{f(x)} \approx \dfrac{100 f'(x)\,dx}{f(x)}$.

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1. $dy = f'(3)\,dx = 757.87\,(0.01) \approx 7.5787$.

2. $dy = f'(2)\,dx = -0.125639152666\,(-0.04) \approx -0.0050256$.

3. $dy = f'(1)\,dx = 1.04067285926\,(0.03) \approx 0.031220$.

4. $dy = f'(2)\,(-0.02) \approx 9.66379267622\,(-0.02) = -0.19328$.

5. $dy = f'(4)\,(0.1) \approx -0.198761598\,(0.1) = -0.01988$.

6. $dy = f'(3)\,(-0.05) \approx 12.3113248654\,(-0.05) = -0.6155662$.

7. If the interest rate changes from 5% to 5.3% per year, the monthly payment will increase by $dP = f'(0.05)\,(0.003) \approx 44.00$, or approximately \$44.00 per month. If the rate changes from 5% to 5.4% per year, the payment will increase by \$58.67 per month, and if it changes from 5% to 5.5% per year, the payment will increase by \$73.34 per month.

8. $A = \pi r^2$, so $dA = 2\pi r\,dr$. The area of the ring is approximately $dA = 2\pi\,(53{,}200)\,(15)$, or 5,013,982 km^2.

9. $dx = f'(40)\,(2) \approx -0.625$. That is, the quantity demanded will decrease by 625 watches per week.

10. $T'(22{,}000) = 0.0000570472$, so $\Delta T \approx T'(22{,}000)\,\Delta d \approx -0.0285236$. The period changes by $(-0.0285236)\,(24) \approx -0.6845664$, a decrease of approximately 0.69 hours.

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1. a. 0 **b.** nx^{n-1} **c.** $cf'(x)$. **d.** $f'(x) \pm g'(x)$

2. a. $f(x)\,g'(x) + g(x)\,f'(x)$ **b.** $\dfrac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$

3. a. $g'(f(x))\,f'(x)$ **b.** $n\left[f(x)\right]^{n-1} f'(x)$

4. Marginal cost, marginal revenue, marginal profit, marginal average cost

5. a. $-\dfrac{pf'(p)}{f(p)}$ **b.** Elastic, unitary, inelastic

6. Both sides, dy/dx

7. y, dy/dt, a

8. $-\dfrac{f(t)\,f'(t)}{g(t)}$, $-\dfrac{f(t)\,g'(t)}{g(t)}$

9. a. $x_2 - x_1$ **b.** $f(x+\Delta x) - f(x)$ **10.** $\Delta x, \Delta x, x, f'(x)\,dx$

CHAPTER 3 Review Exercises page 247

1. $f'(x) = \frac{d}{dx}\left(3x^5 - 2x^4 + 3x^2 - 2x + 1\right) = 15x^4 - 8x^3 + 6x - 2.$

2. $f'(x) = \frac{d}{dx}\left(4x^6 + 2x^4 + 3x^2 - 2\right) = 24x^5 + 8x^3 + 6x.$

3. $g'(x) = \frac{d}{dx}\left(-2x^{-3} + 3x^{-1} + 2\right) = 6x^{-4} - 3x^{-2}.$

4. $f'(t) = \frac{d}{dt}\left(2t^2 - 3t^3 - t^{-1/2}\right) = 4t - 9t^2 + \frac{1}{2}t^{-3/2}.$

5. $g'(t) = \frac{d}{dt}\left(2t^{-1/2} + 4t^{-3/2} + 2\right) = -t^{-3/2} - 6t^{-5/2}.$

6. $h'(x) = \frac{d}{dx}\left(x^2 + \frac{2}{x}\right) = 2x - \frac{2}{x^2}.$

7. $f'(t) = \frac{d}{dt}\left(t + 2t^{-1} + 3t^{-2}\right) = 1 - 2t^{-2} - 6t^{-3} = 1 - \frac{2}{t^2} - \frac{6}{t^3}.$

8. $g'(s) = \frac{d}{ds}\left(2s^2 - 4s^{-1} + 2s^{-1/2}\right) = 4s + 4s^{-2} - s^{-3/2} = 4s + \frac{4}{s^2} - \frac{1}{s^{3/2}}.$

9. $h'(x) = \frac{d}{dx}\left(x^2 - 2x^{-3/2}\right) = 2x + 3x^{-5/2} = 2x + \frac{3}{x^{5/2}}.$

10. $f(x) = \frac{x+1}{2x-1}$, so $f'(x) = \frac{(2x-1)(1) - (x+1)(2)}{(2x-1)^2} = -\frac{3}{(2x-1)^2}.$

11. $g(t) = \frac{t^2}{2t^2+1}$, so $g'(t) = \frac{(2t^2+1)\frac{d}{dt}(t^2) - t^2\frac{d}{dt}(2t^2+1)}{(2t^2+1)^2} = \frac{(2t^2+1)(2t) - t^2(4t)}{(2t^2+1)^2} = \frac{2t}{(2t^2+1)^2}.$

12. $h(t) = \frac{t^{1/2}}{t^{1/2}+1}$, so $h'(t) = \frac{(t^{1/2}+1)\frac{1}{2}t^{-1/2} - t^{1/2}\left(\frac{1}{2}t^{-1/2}\right)}{(t^{1/2}+1)^2} = \frac{1}{2\sqrt{t}\left(\sqrt{t}+1\right)^2}.$

13. $f(x) = \frac{\sqrt{x}-1}{\sqrt{x}+1} = \frac{x^{1/2}-1}{x^{1/2}+1}$, so

$$f'(x) = \frac{(x^{1/2}+1)\left(\frac{1}{2}x^{-1/2}\right) - (x^{1/2}-1)\left(\frac{1}{2}x^{-1/2}\right)}{(x^{1/2}+1)^2} = \frac{\frac{1}{2} + \frac{1}{2}x^{-1/2} - \frac{1}{2} + \frac{1}{2}x^{-1/2}}{(x^{1/2}+1)^2} = \frac{x^{-1/2}}{(x^{1/2}+1)^2}$$

$$= \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}.$$

14. $f(t) = \dfrac{t}{2t^2+1}$, so $f'(t) = \dfrac{(2t^2+1)(1) - t(4t)}{(2t^2+1)^2} = \dfrac{1-2t^2}{(2t^2+1)^2}$.

15. $f(x) = \dfrac{x^2(x^2+1)}{x^2-1}$, so

$$f'(x) = \frac{(x^2-1)\frac{d}{dx}(x^4+x^2) - (x^4+x^2)\frac{d}{dx}(x^2-1)}{(x^2-1)^2} = \frac{(x^2-1)(4x^3+2x) - (x^4+x^2)(2x)}{(x^2-1)^2}$$
$$= \frac{4x^5+2x^3-4x^3-2x-2x^5-2x^3}{(x^2-1)^2} = \frac{2x^5-4x^3-2x}{(x^2-1)^2} = \frac{2x(x^4-2x^2-1)}{(x^2-1)^2}.$$

16. $f(x) = (2x^2+x)^3$, so $f'(x) = 3(2x^2+x)^2 \dfrac{d}{dx}(2x^2+x) = 3(4x+1)(2x^2+x)^2$.

17. $f(x) = (3x^3-2)^8$, so $f'(x) = 8(3x^3-2)^7(9x^2) = 72x^2(3x^3-2)^7$.

18. $h(x) = (\sqrt{x}+2)^5$, so $h'(x) = 5(x^{1/2}+2)^4 \dfrac{d}{dx}x^{1/2} = 5(x^{1/2}+2)^4 \cdot \frac{1}{2}x^{-1/2} = \dfrac{5(\sqrt{x}+2)^4}{2\sqrt{x}}$.

19. $f'(t) = \dfrac{d}{dt}(2t^2+1)^{1/2} = \frac{1}{2}(2t^2+1)^{-1/2}\dfrac{d}{dt}(2t^2+1) = \frac{1}{2}(2t^2+1)^{-1/2}(4t) = \dfrac{2t}{\sqrt{2t^2+1}}$.

20. $g(t) = \sqrt[3]{1-2t^3} = (1-2t^3)^{1/3}$, so $g'(t) = \frac{1}{3}(1-2t^3)^{-2/3}(-6t^2) = -2t^2(1-2t^3)^{-2/3}$.

21. $s(t) = (3t^2-2t+5)^{-2}$, so

$$s'(t) = -2(3t^2-2t+5)^{-3}(6t-2) = -4(3t^2-2t+5)^{-3}(3t-1) = -\frac{4(3t-1)}{(3t^2-2t+5)^3}.$$

22. $f(x) = (2x^3-3x^2+1)^{-3/2}$, so

$$f'(x) = -\tfrac{3}{2}(2x^3-3x^2+1)^{-5/2}(6x^2-6x) = -9x(x-1)(2x^3-3x^2+1)^{-5/2}.$$

23. $h(x) = \left(x+\dfrac{1}{x}\right)^2 = (x+x^{-1})^2$, so

$$h'(x) = 2(x+x^{-1})(1-x^{-2}) = 2\left(x+\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right) = 2\left(\frac{x^2+1}{x}\right)\left(\frac{x^2-1}{x^2}\right) = \frac{2(x^2+1)(x^2-1)}{x^3}.$$

24. $h(x) = \dfrac{1+x}{(2x^2+1)^2}$, so

$$h'(x) = \frac{(2x^2+1)^2(1) - (1+x)2(2x^2+1)(4x)}{(2x^2+1)^4} = \frac{(2x^2+1)\left[(2x^2+1) - 8x - 8x^2\right]}{(2x^2+1)^4} = -\frac{6x^2+8x-1}{(2x^2+1)^3}.$$

25. $h(t) = (t^2+t)^4(2t^2)$, so

$$h'(t) = (t^2+t)^4\frac{d}{dt}(2t^2) + 2t^2\frac{d}{dt}(t^2+t)^4 = (t^2+t)^4(4t) + 2t^2 \cdot 4(t^2+t)^3(2t+1)$$
$$= 4t(t^2+t)^3\left[(t^2+t) + 4t^2 + 2t\right] = 4t^2(5t+3)(t^2+t)^3.$$

26. $f(x) = (2x+1)^3 (x^2+x)^2$, so

$$f'(x) = (2x+1)^3 \cdot 2(x^2+x)(2x+1) + (x^2+x)^2\, 3(2x+1)^2(2)$$
$$= 2(2x+1)^2(x^2+x)\left[(2x+1)^2 + 3(x^2+x)\right] = 2(2x+1)^2(x^2+x)(7x^2+7x+1).$$

27. $g(x) = x^{1/2}(x^2-1)^3$, so

$$g'(x) = \frac{d}{dx}\left[x^{1/2}(x^2-1)^3\right] = x^{1/2} \cdot 3(x^2-1)^2(2x) + (x^2-1)^3 \cdot \tfrac{1}{2}x^{-1/2}$$
$$= \tfrac{1}{2}x^{-1/2}(x^2-1)^2\left[12x^2 + (x^2-1)\right] = \frac{(13x^2-1)(x^2-1)^2}{2\sqrt{x}}.$$

28. $f(x) = \dfrac{x}{(x^3+2)^{1/2}}$, so

$$f'(x) = \frac{(x^3+2)^{1/2}(1) - x \cdot \frac{1}{2}(x^3+2)^{-1/2} \cdot 3x^2}{x^3+2} = \frac{\frac{1}{2}(x^3+2)^{-1/2}\left[2(x^3+2) - 3x^3\right]}{x^3+2} = \frac{4-x^3}{2(x^3+2)^{3/2}}.$$

29. $h(x) = \dfrac{(3x+2)^{1/2}}{4x-3}$, so

$$h'(x) = \frac{(4x-3)\frac{1}{2}(3x+2)^{-1/2}(3) - (3x+2)^{1/2}(4)}{(4x-3)^2} = \frac{\frac{1}{2}(3x+2)^{-1/2}\left[3(4x-3) - 8(3x+2)\right]}{(4x-3)^2}$$
$$= -\frac{12x+25}{2\sqrt{3x+2}\,(4x-3)^2}.$$

30. $f(t) = \dfrac{(2t+1)^{1/2}}{(t+1)^3}$, so

$$f'(t) = \frac{(t+1)^3 \frac{1}{2}(2t+1)^{-1/2}(2) - (2t+1)^{1/2} \cdot 3(t+1)^2(1)}{(t+1)^6}$$
$$= \frac{(2t+1)^{-1/2}(t+1)^2\left[(t+1) - 3(2t+1)\right]}{(t+1)^6} = -\frac{5t+2}{\sqrt{2t+1}\,(t+1)^4}.$$

31. $f(x) = 2x^4 - 3x^3 + 2x^2 + x + 4$, so $f'(x) = \dfrac{d}{dx}\left(2x^4 - 3x^3 + 2x^2 + x + 4\right) = 8x^3 - 9x^2 + 4x + 1$ and

$$f''(x) = \frac{d}{dx}\left(8x^3 - 9x^2 + 4x + 1\right) = 24x^2 - 18x + 4 = 2\left(12x^2 - 9x + 2\right).$$

32. $g(x) = x^{1/2} + x^{-1/2}$, so $g'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$ and $g''(x) = -\frac{1}{4}x^{-3/2} + \frac{3}{4}x^{-5/2} = -\dfrac{1}{4x^{3/2}} + \dfrac{3}{4x^{5/2}}$.

33. $h(t) = \dfrac{t}{t^2+4}$, so $h'(t) = \dfrac{(t^2+4)(1) - t(2t)}{(t^2+4)^2} = \dfrac{4-t^2}{(t^2+4)^2}$ and

$$h''(t) = \frac{(t^2+4)^2(-2t) - (4-t^2)\,2(t^2+4)(2t)}{(t^2+4)^4} = \frac{-2t(t^2+4)\left[(t^2+4) + 2(4-t^2)\right]}{(t^2+4)^4} = \frac{2t(t^2-12)}{(t^2+4)^3}.$$

34. $f(x) = (x^3+x+1)^2$, so

$$f'(x) = 2(x^3+x+1)(3x^2+1) = 2(3x^5+3x^3+3x^2+x^3+x+1) = 2(3x^5+4x^3+3x^2+x+1)$$

and $f''(x) = 2(15x^4 + 12x^2 + 6x + 1)$.

35. $f'(x) = \frac{d}{dx}\left(2x^2+1\right)^{1/2} = \frac{1}{2}\left(2x^2+1\right)^{-1/2}(4x) = 2x\left(2x^2+1\right)^{-1/2}$, so
$f''(x) = 2\left(2x^2+1\right)^{-1/2} + 2x \cdot \left(-\frac{1}{2}\right)\left(2x^2+1\right)^{-3/2}(4x) = 2\left(2x^2+1\right)^{-3/2}\left[\left(2x^2+1\right) - 2x^2\right] = \dfrac{2}{\left(2x^2+1\right)^{3/2}}$.

36. $f(t) = t\left(t^2+1\right)^3$, so
$f'(t) = \left(t^2+1\right)^3 + t \cdot 3\left(t^2+1\right)^2(2t) = \left(t^2+1\right)^2\left[\left(t^2+1\right)+6t^2\right] = \left(t^2+1\right)^2\left(7t^2+1\right)$ and
$f''(t) = \left(t^2+1\right)^2(14t) + \left(7t^2+1\right)(2)\left(t^2+1\right)(2t) = 2t\left(t^2+1\right)\left[7\left(t^2+1\right)+2\left(7t^2+1\right)\right]$
$= 6t\left(t^2+1\right)\left(7t^2+3\right)$.

37. $6x^2 - 3y^2 = 9$. Differentiating this equation implicitly, we have $12x - 6y\dfrac{dy}{dx} = 0$ and $-6y\dfrac{dy}{dx} = -12x$. Therefore, $\dfrac{dy}{dx} = \dfrac{-12x}{-6y} = \dfrac{2x}{y}$.

38. $2x^3 - 3xy = 4$. Differentiating this equation implicitly, we have $6x^2 - 3y - 3x\dfrac{dy}{dx} = 0$, so $-3x\dfrac{dy}{dx} = -6x^2 + 3y$. Thus, $\dfrac{dy}{dx} = \dfrac{2x^2 - y}{x}$.

39. $y^3 + 3x^2 = 3y$. Differentiating this equation implicitly, we have $3y^2y' + 6x = 3y'$, $3y^2y' - 3y' = -6x$, and $y'\left(3y^2 - 3\right) = -6x$. Therefore, $y' = -\dfrac{6x}{3\left(y^2-1\right)} = -\dfrac{2x}{y^2-1}$.

40. $x^2 + 2x^2y^2 + y^2 = 10$. Differentiating this equation implicitly, we have $2x + 4xy^2 + 2x^2\left(2yy'\right) + 2yy' = 0$, $2yy'\left(2x^2+1\right) = -2x\left(1+2y^2\right)$, and thus $y' = -\dfrac{x\left(1+2y^2\right)}{y\left(2x^2+1\right)}$.

41. $x^2 - 4xy - y^2 = 12$. Differentiating this equation implicitly, we have $2x - 4xy' - 4y - 2yy' = 0$ and $y'(-4x-2y) = -2x+4y$. Therefore, $y' = \dfrac{-2(x-2y)}{-2(2x+y)} = \dfrac{x-2y}{2x+y}$.

42. $3x^2y - 4xy + x - 2y = 6$. Differentiating this equation implicitly, we have $6xy + 3x^2y' - 4y - 4xy' + 1 - 2y' = 0$, $y'\left(3x^2 - 4x - 2\right) = 4y - 6xy - 1$, and thus $y' = \dfrac{4y - 6xy - 1}{3x^2 - 4x - 2}$.

43. $f(x) = x^2 + \dfrac{1}{x^2}$, so $df = f'(x)\,dx = \left(2x - 2x^{-3}\right)dx = \left(2x - \dfrac{2}{x^3}\right)dx = \dfrac{2\left(x^4-1\right)}{x^3}\,dx$.

44. $f(x) = \dfrac{1}{\sqrt{x^3+1}}$, so $df = f'(x)\,dx = \dfrac{d}{dx}\left(x^3+1\right)^{-1/2}dx = -\frac{1}{2}\left(x^3+1\right)^{-3/2}\left(3x^2\right)dx = -\dfrac{3x^2}{2\left(x^3+1\right)^{3/2}}\,dx$.

45. a. $df = f'(x)\,dx = \dfrac{d}{dx}\left(2x^2+4\right)^{1/2}dx = \frac{1}{2}\left(2x^2+4\right)^{-1/2}(4x) = \dfrac{2x}{\sqrt{2x^2+4}}\,dx$.

b. Setting $x = 4$ and $dx = 0.1$, we find $\Delta f \approx df = \dfrac{2(4)(0.1)}{\sqrt{2(16)+4}} = \dfrac{0.8}{6} = \dfrac{8}{60} = \dfrac{2}{15}$.

c. $\Delta f = f(4.1) - f(4) = \sqrt{2(4.1)^2+4} - \sqrt{2(16)+4} \approx 0.1335$. From part (b), $\Delta f \approx \dfrac{2}{15} \approx 0.1333$.

46. Take $y = f(x) = x^{1/3}$ and $x = 27$. Then $\Delta x = dx = 26.8 - 27 = -0.2$, so $\Delta y \approx dy = f'(x)\,\Delta x = \frac{1}{3}x^{-2/3}\Big|_{x=27} \cdot (-0.2) = \frac{1}{3(9)}(-0.2) = -\frac{2}{270} = -\frac{1}{135}$. Therefore, $\sqrt[3]{26.8} - \sqrt[3]{27} = \Delta y = -\frac{1}{135}$, so $\sqrt[3]{26.8} = \sqrt[3]{27} - \frac{1}{135} = 3 - \frac{1}{135} \approx 2.9926$.

47. $f(x) = 2x^3 - 3x^2 - 16x + 3$ and $f'(x) = 6x^2 - 6x - 16$.

a. To find the point(s) on the graph of f where the slope of the tangent line is equal to -4, we solve $6x^2 - 6x - 16 = -4$, obtaining $6x^2 - 6x - 12 = 0$, $6\left(x^2 - x - 2\right) = 0$, and $6(x-2)(x+1) = 0$. Thus, $x = 2$ or $x = -1$. Now $f(2) = 2(2)^3 - 3(2)^2 - 16(2) + 3 = -25$ and $f(-1) = 2(-1)^3 - 3(-1)^2 - 16(-1) + 3 = 14$, so the points are $(2, -25)$ and $(-1, 14)$.

b. Using the point-slope form of the equation of a line, we find that the equation of the tangent line at $(2, -25)$ is $y - (-25) = -4(x-2)$, $y + 25 = -4x + 8$, or $y = -4x - 17$, and the equation of the tangent line at $(-1, 14)$ is $y - 14 = -4(x+1)$, or $y = -4x + 10$.

48. $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 4x + 1$, so $f'(x) = x^2 + x - 4$.

a. Set $x^2 + x - 4 = -2$, so $x^2 + x - 2 = (x+2)(x-1) = 0$. Therefore, $x = -2$ or 1. The points are $\left(-2, \frac{25}{3}\right)$ and $\left(1, -\frac{13}{6}\right)$.

b. Equations are $y - \frac{25}{3} = -2(x+2)$, or $y = -2x + \frac{13}{3}$; and $y + \frac{13}{6} = -2(x-1)$, or $y = -2x - \frac{1}{6}$.

49. $y = \left(4 - x^2\right)^{1/2}$, so $y' = \frac{1}{2}\left(4 - x^2\right)^{-1/2}(-2x) = -\dfrac{x}{\sqrt{4 - x^2}}$. The slope of the tangent line is obtained by letting $x = 1$, giving $m = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$. Therefore, an equation of the tangent line at $x = 1$ is $y - \sqrt{3} = -\frac{\sqrt{3}}{3}(x-1)$, or $y = -\frac{\sqrt{3}}{3}x + \frac{4\sqrt{3}}{3}$.

50. $y = x(x+1)^5$, so $y' = (x+1)^5 + x \cdot 5(x+1)^4(1) = (x+1)^4[(x+1) + 5x] = (6x+1)(x+1)^4$. The slope of the tangent line is obtained by letting $x = 1$. Then $m = (6+1)(2)^4 = 112$. An equation of the tangent line is $y - 32 = 112(x-1)$, or $y = 112x - 80$.

51. $f(x) = (2x-1)^{-1}$, so $f'(x) = -2(2x-1)^{-2}$, $f''(x) = 8(2x-1)^{-3} = \dfrac{8}{(2x-1)^3}$, and $f'''(x) = -48(2x-1)^4 = -\dfrac{48}{(2x-1)^4}$. Because $(2x-1)^4 = 0$ when $x = \frac{1}{2}$, we see that the domain of f''' is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

52. a. $S(0) = 3.1$, or \$3.1 billion. $S(5) = 0.14(5)^2 + 0.68(5) + 3.1 = 10$, or \$10 billion.

b. $S'(t) = 0.28t + 0.68$, so $S'(0) = 0.28(0) + 0.68 = 0.68$, or \$0.68 billion, and $S'(5) = 0.28(5) + 0.68 = 2.08$, or \$2.08 billion/yr.

53. a. The number of UK digital viewers in 2015 is projected to be $N(t) = 65.71(5)^{0.085} \approx 75.3$, or 75.3 million.

b. $N'(t) = 65.71(0.085)t^{-0.915}$, so $N'(5) = 65.71(0.085)(5)^{-0.915} \approx 1.28$. Thus, the number of viewers is expected to be increasing at the rate of approximately 1.3 million per year.

54. $P(t) = 0.01484t^2 + 0.446t + 15$.

a. $P(0) = 15$, or 15%, and $P(22) = 0.01484(22)^2 + 0.446(22) + 15 \approx 31.99$, or approximately 31.99%.

b. $P'(t) = 2(0.01484)t + 0.446 = 0.02968t + 0.446$, so $P'(2) = 0.02968(2) + 0.446 = 0.50536$, or approximately 0.51%/yr, and $P'(20) = 0.02968(20) + 0.446 = 1.0396$, or approximately 1.04%/yr.

55. a. The number of cameras that will be shipped after 2 years is given by $N(2) = 6(2^2) + 200(2) + 4\sqrt{2} + 20{,}000 \approx 20{,}429.7$, or approximately 20,430 cameras.

b. The rate of change in the number of cameras shipped after 2 years is given by $N'(2) = \left(12t + 200 + 2t^{-1/2}\right)\big|_2 = 12(2) + 200 + \frac{2}{\sqrt{2}} \approx 225.4$, or approximately 225 cameras/yr.

56. a. The GDP in 2013 is given by $f(3) = 0.1(3)^3 + 0.5(3)^2 + 2(3) + 20 = 33.2$, or \$33.2 billion.

b. The rate of change of the GDP in 2013 is given by $f'(3) = \left(0.3t^2 + t + 2\right)\big|_{t=3} = 0.3(3)^2 + 3 + 2 = 7.7$, or \$7.7 billion/yr.

57. a. The population after 3 years is given by $P(3) = 30 - \dfrac{20}{2(3)+3} \approx 27.7778$, or approximately 27,778. The current population is $P(0) = 30 - \frac{20}{3} \approx 23.333$, or approximately 23,333. So the population will have changed by $27{,}778 - 23{,}333 = 4445$; that is, it would have increased by 4445.

b. $P'(t) = \dfrac{d}{dt}\left[30 - 20(2t+3)^{-1}\right] = \dfrac{40}{(2t+3)^2}$, so the rate of change after 3 years is $P'(3) = \dfrac{40}{[2(3)+3]^2} \approx 0.4938$; that is, it will be increasing at the rate of approximately 494 people/yr.

58. a. The number of copies sold after 12 weeks is given by $N(12) = [4 + 5(12)]^{5/3} = 1024$, or 1,024,000.

b. The rate of change after 12 weeks is given by $N'(12) = \frac{5}{3}(4+5t)^{2/3}(5)\Big|_{t=12} \approx 133.33$, or approximately 133,000 copies/wk.

59. $N(x) = 1000(1+2x)^{1/2}$, so $N'(x) = 1000\left(\frac{1}{2}\right)(1+2x)^{-1/2}(2) = \dfrac{1000}{\sqrt{1+2x}}$. The rate of increase at the end of the twelfth week is $N'(12) = \frac{1000}{\sqrt{25}} = 200$, or 200 subscribers/week.

60. $f(t) = 31.88(1+t)^{-0.45}$, so $f'(t) = 31.88(-0.45)(1+t)^{-1.45} = -14.346(1+t)^{-1.45}$. It is changing at the rate of $f'(2) \approx -2.917$; that is, decreasing at the rate of approximately 2.9 cents/minute/yr. The average price per minute at the beginning of 2000 was $f(2) = 31.88(1+2)^{-0.45}$, or approximately 19.45 cents/minute.

61. He can expect to live $f(100) = 46.9[1 + 1.09(100)]^{0.1} \approx 75.0433$, or approximately 75.04 years. $f'(t) = 46.9(0.1)(1+1.09t)^{-0.9}(1.09) = 5.1121(1+1.09t)^{-0.9}$, so the required rate of change is $f'(100) = 5.1121[1 + 1.09(100)]^{-0.9} \approx 0.074$, or approximately 0.07 yr/yr.

62. $C(x) = 2500 + 2.2x$.

a. The marginal cost is $C'(x) = 2.2$. The marginal cost when $x = 1000$ is $C'(1000) = 2.2$. The marginal cost when $x = 2000$ is $C'(2000) = 2.2$.

b. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{2500 + 2.2x}{x} = 2.2 + \dfrac{2500}{x}$, so $\overline{C}'(x) = -\dfrac{2500}{x^2}$.

c. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty}\left(2.2 + \frac{2500}{x}\right) = 2.2.$

63. $p'(x) = \frac{d}{dt}\left[\frac{1}{10}x^{3/2} + 10\right] = \frac{3}{20}x^{1/2} = \frac{3}{20}\sqrt{x}$, so $p'(40) = \frac{3}{20}\sqrt{40} \approx 0.9487$, or \$0.9487. When the number of units is 40,000, the price will increase \$0.9487 for each 1000 radios demanded.

64. $p'(x) = \frac{d}{dx}\left[20\left(-x^2+100\right)^{1/2}\right] = 10\left(-x^2+100\right)^{-1/2}(-2x) = \frac{-20x}{\sqrt{-x^2+100}}$, so $p'(6) = -\frac{20(6)}{\sqrt{-36+100}} = -15$. When the number of units is 6000, the price drops by \$15 for each thousand units demanded.

65. a. The actual cost incurred in the manufacturing of the 301st MP3 player is

$$C(301) - C(300) = \left[0.0001(301)^3 - 0.02(301)^2 + 24(301) + 2000\right] - \left[0.0001(300)^3 - 0.02(300)^2 + 24(300) + 2000\right] \approx 39.07,$$ or approximately \$39.07.

b. The marginal cost is $C'(300) = \left(0.0003x^2 - 0.04x + 24\right)\big|_{x=300} \approx 39$, or approximately \$39.

66. a. $R(x) = px = (-0.02x + 600)x = -0.02x^2 + 600x.$

b. $R'(x) = -0.04x + 600.$

c. $R'(10{,}000) = -0.04(10{,}000) + 600 = 200$. This says that the sale of the 10,001st phone will bring a revenue of \$200.

67. a. $R(x) = px = (2000 - 0.04x)x = 2000x - 0.04x^2$, so

$$P(x) = R(x) - C(x) = \left(2000x - 0.04x^2\right) - \left(0.000002x^3 - 0.02x^2 + 1000x + 120{,}000\right)$$
$$= -0.000002x^3 - 0.02x^2 + 1000x - 120{,}000.$$

Therefore,

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{0.000002x^3 - 0.02x^2 + 1000x + 120{,}000}{x} = 0.000002x^2 - 0.02x + 1000 + \frac{120{,}000}{x}.$$

b. $C'(x) = \frac{d}{dx}\left(0.000002x^3 - 0.02x^2 + 1000x + 120{,}000\right) = 0.000006x^2 - 0.04x + 1000,$

$R'(x) = \frac{d}{dx}\left(2000x - 0.04x^2\right) = 2000 - 0.08x,$

$P'(x) = \frac{d}{dx}\left(-0.000002x^3 - 0.02x^2 + 1000x - 120{,}000\right) = -0.000006x^2 - 0.04x + 1000$, and

$\overline{C}'(x) = \frac{d}{dx}\left(0.000002x^2 - 0.02x + 1000 + 120{,}000x^{-1}\right) = 0.000004x - 0.02 - 120{,}000x^{-2}.$

c. $C'(3000) = 0.000006(3000)^2 - 0.04(3000) + 1000 = 934$, $R'(3000) = 2000 - 0.08(3000) = 1760$, and $P'(3000) = -0.000006(3000)^2 - 0.04(3000) + 1000 = 826.$

d. $\overline{C}'(5000) = 0.000004(5000) - 0.02 - 120{,}000(5000)^{-2} = -0.0048$, and $\overline{C}'(8000) = 0.000004(8000) - 0.02 - 120{,}000(8000)^{-2} \approx 0.0101$. At a production level of 5000 machines, the average cost of each additional unit is decreasing at a rate of 0.48 cents. At a production level of 8000 machines, the average cost of each additional unit is increasing at a rate of approximately 1 cent per unit.

68. **a.** $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{80x + 150{,}000}{x} = 80 + \dfrac{150{,}000}{x}$.

b. $\overline{C}'(x) = -\dfrac{150{,}000}{x^2}$.

c. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty}\left(80 + \dfrac{150{,}000}{x}\right) = 80$. If the production level is very high, then the unit cost approaches \$80/unit.

69. $x = f(p) = -\frac{5}{2}p + 30$, so $f'(p) = -\frac{5}{2}$ and $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{p\left(-\frac{5}{2}\right)}{-\frac{5}{2}p + 30} = \dfrac{p}{12 - p}$.

a. $E(3) = \frac{3}{9} = \frac{1}{3}$, so demand is inelastic.

b. $E(6) = \frac{6}{12-6} = 1$, so demand is unitary.

c. $E(9) = \frac{9}{12-9} = 3$, so demand is elastic.

70. $x = \dfrac{25}{\sqrt{p}} - 1$, so $f'(p) = -\dfrac{25}{2p^{3/2}}$ and $E(p) = -\dfrac{p\left(-\frac{25}{2p^{3/2}}\right)}{\frac{25}{p^{1/2}} - 1} = \dfrac{\frac{25}{2p^{1/2}}}{\frac{25-p^{1/2}}{p^{1/2}}} = \dfrac{25}{2\left(25 - p^{1/2}\right)}$. If $E(p) = 1$, then $2\left(25 - p^{1/2}\right) = 25$, so $25 - p^{1/2} = \frac{25}{2}$, $p^{1/2} = \frac{25}{2}$, and $p = \frac{625}{4}$. $E(p) > 1$ and demand is elastic if $p > 156.25$, $E(p) = 1$ and demand is unitary if $p = 156.25$, and $E(p) < 1$ and demand is inelastic if $p < 156.25$.

71. $x = 100 - 0.01p^2$, so $f'(p) = -0.02p$ and $E(p) = -\dfrac{p(-0.02p)}{100 - 0.01p^2} = \dfrac{p^2}{5000 - \frac{1}{2}p^2}$.

a. $E(40) = \dfrac{1600}{5000 - \frac{1}{2}(1600)} = \dfrac{1600}{4200} = \dfrac{8}{21} < 1$ and so demand is inelastic.

b. Because demand is inelastic, raising the unit price slightly causes revenue to increase.

72. **a.** $p = 9\sqrt[3]{1000 - x}$, so $\sqrt[3]{1000 - x} = \dfrac{p}{9}$, $1000 - x = \dfrac{p^3}{729}$, and $x = 1000 - \dfrac{p^3}{729}$. Therefore,

$x = f(p) = \dfrac{729{,}000 - p^3}{729}$ and $f'(p) = -\dfrac{3p^2}{729} = -\dfrac{p^2}{243}$. Then $E(p) = -\dfrac{p\left(-\frac{p^2}{243}\right)}{\frac{729{,}000 - p^3}{729}} = \dfrac{3p^3}{729{,}000 - p^3}$.

$E(60) = \dfrac{3(60)^3}{729{,}000 - 60^3} = \dfrac{648{,}000}{513{,}000} = \dfrac{648}{513} > 1$, and so demand is elastic.

b. From part (a), we see that raising the price slightly causes revenue to decrease.

73. $G'(t) = \dfrac{d}{dt}\left(-0.3t^3 + 1.2t^2 + 500\right) = -0.9t^2 + 2.4t$, so $G'(2) = -0.9(4) + 2.4(2) = 1.2$. Thus, the GDP is growing at the rate of \$1.2 billion/year. $G''(2) = (-1.8t + 2.4)|_{t=2} = -1.2$, so the rate of rate of change of the GDP is decreasing at the rate of \$1.2 billion/yr/yr.

74. $v = \frac{d}{dt}\left[t\left(2t^2+1\right)^{1/2}\right] = \left(2t^2+1\right)^{1/2} + t\left(\frac{1}{2}\right)\left(2t^2+1\right)^{-1/2}(4t) = \left(2t^2+1\right)^{1/2} + \frac{2t^2}{\left(2t^2+1\right)^{1/2}}$ and

$$a = \frac{d}{dt}\left[\left(2t^2+1\right)^{1/2} + 2t^2\left(2t^2+1\right)^{-1/2}\right]$$

$$= \tfrac{1}{2}\left(2t^2+1\right)^{-1/2}(4t) + 4t\left(2t^2+1\right)^{-1/2} + \left(2t^2\right)\left(-\tfrac{1}{2}\right)\left(2t^2+1\right)^{-3/2}(4t) = \frac{6t}{\left(2t^2+1\right)^{1/2}} - \frac{4t^3}{\left(2t^2+1\right)^{3/2}}.$$

Thus, the velocity after 2 seconds is $v(2) = 9^{1/2} + \frac{2(4)}{9^{1/2}} = \frac{17}{3}$ ft/sec and the acceleration after 2 seconds is $a(2) = \frac{12}{9^{1/2}} - \frac{4(8)}{9^{3/2}} = \frac{76}{27}$ ft/sec^2.

CHAPTER 3 Before Moving On... page 250

1. $f(x) = 2x^3 - 3x^{1/3} + 5x^{-2/3}$, so $f'(x) = 2\left(3x^2\right) - 3\left(\frac{1}{3}x^{-2/3}\right) + 5\left(-\frac{2}{3}x^{-5/3}\right) = 6x^2 - x^{-2/3} - \frac{10}{3}x^{-5/3}$.

2. $g'(x) = \frac{d}{dx}\left[x\left(2x^2-1\right)^{1/2}\right] = \left(2x^2-1\right)^{1/2} + x\left(\frac{1}{2}\right)\left(2x^2-1\right)^{-1/2}\frac{d}{dx}\left(2x^2-1\right)$

$$= \left(2x^2-1\right)^{1/2} + \tfrac{1}{2}x\left(2x^2-1\right)^{-1/2}(4x) = \left(2x^2-1\right)^{-1/2}\left[\left(2x^2-1\right) + 2x^2\right] = \frac{4x^2-1}{\sqrt{2x^2-1}}.$$

3. $y = f(x) = \frac{2x+1}{x^2+x+1}$, so

$$\frac{dy}{dx} = \frac{\left(x^2+x+1\right)(2) - (2x+1)(2x+1)}{\left(x^2+x+1\right)^2} = \frac{2x^2+2x+2-\left(4x^2+4x+1\right)}{\left(x^2+x+1\right)^2} = -\frac{2x^2+2x-1}{\left(x^2+x+1\right)^2}.$$

4. $f(x) = \frac{1}{\sqrt{x+1}} = (x+1)^{-1/2}$, so $f'(x) = \frac{d}{dx}(x+1)^{-1/2} = -\frac{1}{2}(x+1)^{-3/2} = -\frac{1}{2(x+1)^{3/2}}$.

Thus, $f''(x) = -\frac{1}{2}\left(-\frac{3}{2}\right)(x+1)^{-5/2} = \frac{3}{4}(x+1)^{-5/2} = \frac{3}{4(x+1)^{5/2}}$ and

$f'''(x) = \frac{3}{4}\left(-\frac{5}{2}\right)(x+1)^{-7/2} = -\frac{15}{8}(x+1)^{-7/2} = -\frac{15}{8(x+1)^{7/2}}$.

5. $xy^2 - x^2y + x^3 = 4$. Differentiating both sides of the equation implicitly with respect to x gives $y^2 + x\left(2yy'\right) - 2xy - x^2y' + 3x^2 = 0$, so $\left(2xy - x^2\right)y' + \left(y^2 - 2xy + 3x^2\right) = 0$ and

$$y' = \frac{-y^2+2xy-3x^2}{2xy-x^2} = \frac{-y^2+2xy-3x^2}{x(2y-x)}.$$

6. a. $y = x\sqrt{x^2+5}$, so $dy = \frac{d}{dx}\left[x\left(x^2+5\right)^{1/2}\right]dx = \left[x\left(\frac{1}{2}\right)\left(x^2+5\right)^{-1/2}(2x)\right]dx + \left[\left(x^2+5\right)^{1/2}(1)\right]dx$

$$= \left(x^2+5\right)^{-1/2}\left[\left(x^2+5\right) + x^2\right]dx = \frac{2x^2+5}{\sqrt{x^2+5}}dx.$$

b. Here $dx = \Delta x = 2.01 - 2 = 0.01$. Therefore, $\Delta y \approx dy = \frac{2(4)+5}{\sqrt{4+5}}(0.01) = \frac{0.13}{3} \approx 0.043$.

CHAPTER 3 Explore & Discuss

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1. $R'(x) = p(x) + xp'(x)$. This says that the rate of change of the revenue (marginal revenue) is equal to the sum of the unit price of the product plus the product of the number of units sold and the rate of change of the unit price.

2. If $p(x)$ is a constant, say p, then $R'(x) = p$. In other words, the marginal revenue is equal to the unit price. This is expected because if the unit price is constant, then the revenue realized in selling one more unit (the marginal revenue) is p.

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1. The required expression is $\frac{dP}{dx} = g'(x)$.

2. The required expression is $\frac{dx}{dt} = f'(t)$.

3. $P = g(x) = g(f(x))$. Using the Chain Rule, we have $\frac{dP}{dt} = g'(f(x))\,f'(x)$.

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1. $\frac{dP}{dt}$ measures the rate of change of the population P with respect to the temperature of the medium.

2. $\frac{dT}{dt}$ measures the rate of change of the temperature of the medium with respect to time.

3. $\frac{dP}{dt} = \frac{dP}{dT} \cdot \frac{dT}{dt} = f'(T)\,g'(t)$ measures the rate of change of the population with respect to time.

4. $(f \circ g)(t) = f(g(t)) = P$ gives the population of bacteria at any time t.

5. $f'(g(t))\,g'(t) = \frac{dP}{dt}$ (by the Chain Rule), and this gives the rate of change of the population with respect to time (see part (c)).

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1. Thinking of x as a function of y and differentiating the given equation with respect to the independent variable y, we obtain $\frac{d}{dy}(y^3 - y + 2x^3 - x) = \frac{d}{dy}(8)$. Thus, $\frac{d}{dy}(y^3) - \frac{d}{dy}(y) + \frac{d}{dy}(2x^3) + \frac{d}{dy}(-x) = 0$, so $3y^2 - 1 + 2\left(3x^2\frac{dx}{dy}\right) - \frac{dx}{dy} = 0$, $(6x^2 - 1)\frac{dx}{dy} = 1 - 3y^2$, and $\frac{dx}{dy} = \frac{1 - 3y^2}{6x^2 - 1}$.

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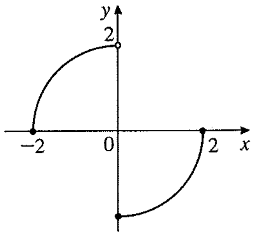
1. 

2. If $-2 \le x < 0$, then $h(x) = \left(4 - x^2\right)^{1/2}$, so
$h'(x) = \frac{1}{2}\left(4 - x^2\right)^{-1/2}(-2x) = -\frac{x}{\sqrt{4-x^2}} = -\frac{x}{y}$. If $0 \le x \le 2$,
then $h(x) = -\left(4 - x^2\right)^{1/2}$, and
$h'(x) = \frac{1}{2}\left(4 - x^2\right)^{-1/2}(-2x) = -\frac{x}{\sqrt{4-x^2}} = -\frac{x}{y}$ for $0 \le x < 2$.

3. At the point $\left(1, -\sqrt{3}\right)$, $h'(x) = -\frac{x}{y} = \frac{1}{\sqrt{3}}$, so $y + \sqrt{3} = \frac{\sqrt{3}}{3}(x - 1)$, $y + \sqrt{3} = \frac{\sqrt{3}}{3}x - \frac{\sqrt{3}}{3}$, and so $y = \frac{\sqrt{3}}{3}x - \frac{4\sqrt{3}}{3}$.

CHAPTER 3 Exploring with Technology

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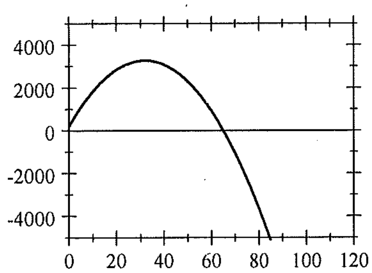
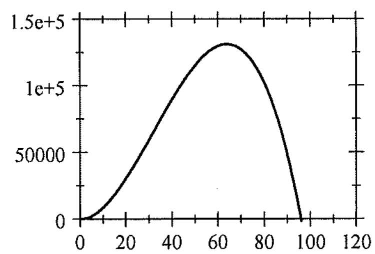
1. 

2. 

3. We see that $f(t) = 0$ when $t = 98$. We conclude that the rocket returns to Earth 98 seconds later.

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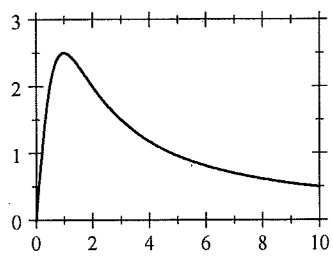
1. 

2. The highest point on the graph of S is $(1, 2.5)$, and this tells us that the sales of the laser disc reach a maximum of \$2.5 million one year after its release.

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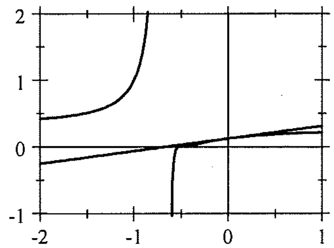
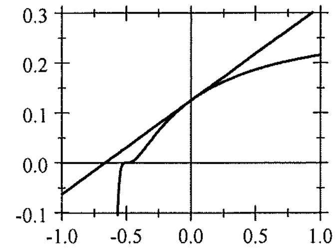
1.



2.



3. The slope of the tangent line is 0.1875, as expected.

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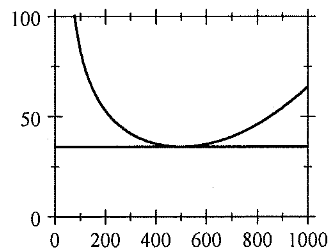
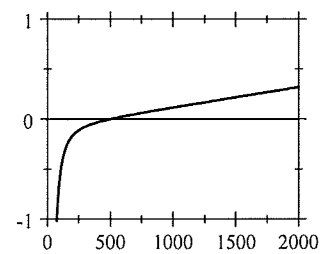
1, 2.



The slope is zero. Yes, the point of tangency is the lowest point on the graph of $\overline{C}$.

3.



Yes. At the lowest point on the graph of $\overline{C}$, the derivative of $\overline{C}$ must be zero.