Chap 2 Review

61. We have $f(x) = x$ if $x > 0$ and $f(x) = -x$ if $x < 0$. Therefore, when $x > 0$,
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{x+h-x}{h} = \lim_{h \to 0} \frac{h}{h} = 1$, and when $x < 0$,
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-x-h-(-x)}{h} = \lim_{h \to 0} \frac{-h}{h} = -1$. Because the right-hand limit does not equal the left-hand limit, we conclude that $\lim_{h \to 0} f(x)$ does not exist.

62. From $f(x) - f(a) = \left[\frac{f(x) - f(a)}{x - a}\right](x - a)$, we see that
$\lim_{x \to a} [f(x) - f(a)] = \lim_{x \to a} \left[\frac{f(x) - f(a)}{x - a}\right] \lim_{x \to a} (x - a) = f'(a) \cdot 0 = 0$, and so $\lim_{x \to a} f(x) = f(a)$. This shows that f is continuous at $x = a$.

Using Technology

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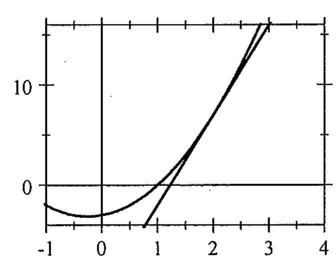
1. a. 9

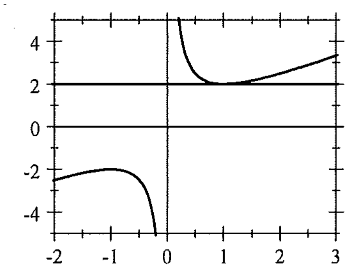
b.



c. $y = 9x - 11$

2. a. 0

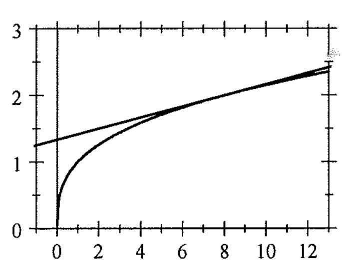
b.



c. $y = 2$

3. a. 0.083

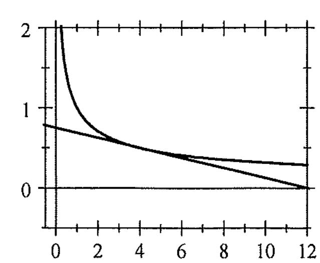
b.



c. $y = \frac{1}{12}x + \frac{4}{3}$

4. a. -0.0625

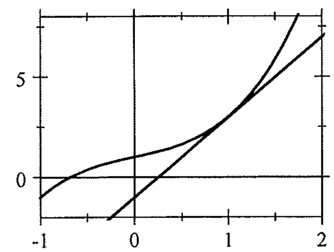
b.



c. $y = -\frac{1}{16}x + \frac{3}{4}$

5. a. 4

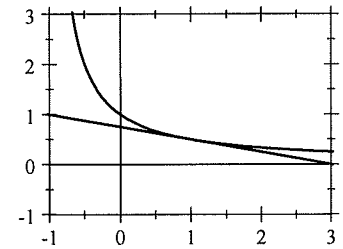
b.



c. $y = 4x - 1$

6. a. -0.25

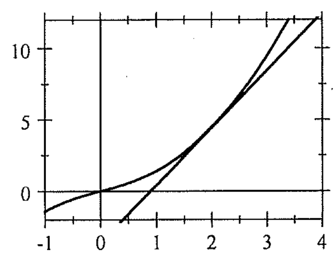
b.



c. $y = -\frac{1}{4}x + \frac{3}{4}$

7. a. 4.02

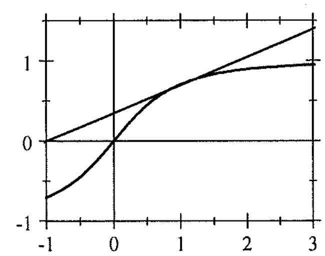
b.



c. $y = 4.02x - 3.57$

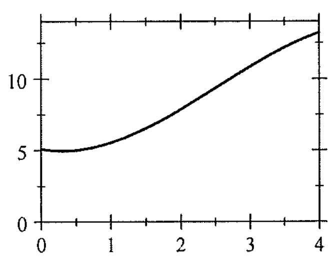
8. a. 0.35

b.



c. $y = 0.35x + 0.35$

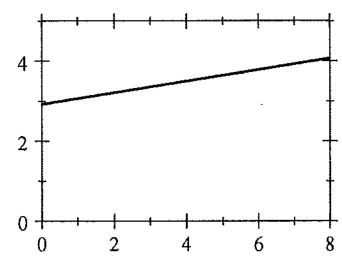
9. a.



b. $f'(3) = 2.8826$ (million per decade)

10. a. $S(t) = -0.000114719t^2 + 0.144618t + 2.92202$

b.



c. \$3.786 billion

d. \$143 million/yr

CHAPTER 2 Concept Review Questions page 156

1. domain, range, B

2. domain, $f(x)$, vertical, point

3. $f(x) \pm g(x)$, $f(x)g(x)$, $\dfrac{f(x)}{g(x)}$, $A \cap B$, $A \cap B$, 0

4. $g(f(x))$, f, $f(x)$, g

5. a. $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $a_n \neq 0$ and n is a positive integer

b. linear, quadratic, cubic **c.** quotient, polynomials **d.** x^r, where r is a real number

6. $f(x)$, L, a

7. a. L^r **b.** $L \pm M$ **c.** LM **d.** $\dfrac{L}{M}$, $M \neq 0$

8. a. L, x **b.** M, negative, absolute

9. a. right **b.** left **c.** L, L

10. a. continuous **b.** discontinuous **c.** every

11. a. $a, a, g(a)$ **b.** everywhere **c.** Q

12. a. $[a, b]$, $f(c) = M$ **b.** $f(x) = 0$, (a, b)

13. a. $f'(a)$ **b.** $y - f(a) = m(x - a)$

14. a. $\dfrac{f(a+h) - f(a)}{h}$ **b.** $\displaystyle\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

CHAPTER 2 Review Exercises page 157

1. a. $9 - x \geq 0$ gives $x \leq 9$, and the domain is $(-\infty, 9]$.

b. $2x^2 - x - 3 = (2x-3)(x+1)$, and $x = \frac{3}{2}$ or -1. Because the denominator of the given expression is zero at these points, we see that the domain of f cannot include these points and so the domain of f is $(-\infty, -1) \cup \left(-1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$.

2. a. We must have $2 - x \geq 0$ and $x + 3 \neq 0$. This implies $x \leq 2$ and $x \neq -3$, so the domain of f is $(-\infty, -3) \cup (-3, 2]$.

b. The domain is $(-\infty, \infty)$.

3. a. $f(-2) = 3(-2)^2 + 5(-2) - 2 = 0$.

b. $f(a+2) = 3(a+2)^2 + 5(a+2) - 2 = 3a^2 + 12a + 12 + 5a + 10 - 2 = 3a^2 + 17a + 20$.

c. $f(2a) = 3(2a)^2 + 5(2a) - 2 = 12a^2 + 10a - 2$.

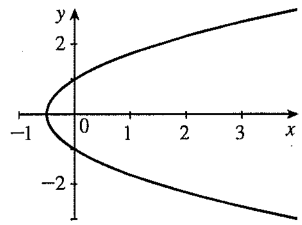
d. $f(a+h) = 3(a+h)^2 + 5(a+h) - 2 = 3a^2 + 6ah + 3h^2 + 5a + 5h - 2$.

4. a. $f(x-1) + f(x+1) = \left[2(x-1)^2 - (x-1) + 1\right] + \left[2(x+1)^2 - (x+1) + 1\right]$

$$= \left(2x^2 - 4x + 2 - x + 1 + 1\right) + \left(2x^2 + 4x + 2 - x - 1 + 1\right) = 4x^2 - 2x + 6.$$

b. $f(x+2h) = 2(x+2h)^2 - (x+2h) + 1 = 2x^2 + 8xh + 8h^2 - x - 2h + 1$.

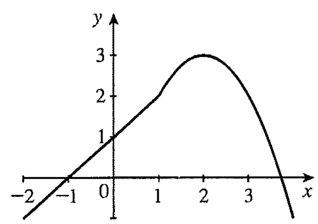
5. a.



b. For each value of $x > 0$, there are two values of y. We conclude that y is not a function of x. (We could also note that the function fails the vertical line test.)

c. Yes. For each value of y, there is only one value of x.

6.



7. a. $f(x)g(x) = \frac{2x+3}{x}$.

b. $\frac{f(x)}{g(x)} = \frac{1}{x(2x+3)}$.

c. $f(g(x)) = \frac{1}{2x+3}$.

d. $g(f(x)) = 2\left(\frac{1}{x}\right) + 3 = \frac{2}{x} + 3$.

8. a. $(f \circ g)(x) = f(g(x)) = 2g(x) - 1 = 2(x^2+4) - 1 = 2x^2 + 7$ and
$(g \circ f)(x) = g(f(x)) = [f(x)]^2 + 4 = (2x-1)^2 + 4 = 4x^2 - 4x + 5$.

b. $(f \circ g)(x) = f(g(x)) = 1 - g(x) = 1 - \frac{1}{3x+4} = \frac{3x+3}{3x+4} = \frac{3(x+1)}{3x+4}$ and
$(g \circ f)(x) = g(f(x)) = \frac{1}{3f(x)+4} = \frac{1}{3(1-x)+4} = \frac{1}{7-3x}$.

c. $(f \circ g)(x) = f(g(x)) = g(x) - 3 = \frac{1}{\sqrt{x+1}} - 3$ and
$(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{f(x)+1}} = \frac{1}{\sqrt{(x-3)+1}} = \frac{1}{\sqrt{x-2}}$.

9. a. Take $f(x) = 2x^2 + x + 1$ and $g(x) = \frac{1}{x^3}$.

b. Take $f(x) = x^2 + x + 4$ and $g(x) = \sqrt{x}$.

10. We have $c(4)^2 + 3(4) - 4 = 2$, so $16c + 12 - 4 = 2$, or $c = -\frac{6}{16} = -\frac{3}{8}$.

11. $\lim_{x\to 0}(5x-3) = 5(0) - 3 = -3$.

12. $\lim_{x\to 1}(x^2+1) = (1)^2 + 1 = 1 + 1 = 2$.

13. $\lim_{x\to -1}(3x^2+4)(2x-1) = [3(-1)^2+4][2(-1)-1] = -21$.

14. $\lim_{x\to 3}\frac{x-3}{x+4} = \frac{3-3}{3+4} = 0$

15. $\lim_{x\to 2}\frac{x+3}{x^2-9} = \frac{2+3}{4-9} = -1$.

16. $\lim_{x\to -2}\frac{x^2-2x-3}{x^2+5x+6}$ does not exist. (The denominator is 0 at $x = -2$.)

17. $\lim_{x\to 3} \sqrt{2x^3 - 5} = \sqrt{2(27) - 5} = 7.$

18. $\lim_{x\to 3} \frac{4x-3}{\sqrt{x+1}} = \frac{12-3}{\sqrt{4}} = \frac{9}{2}.$

19. $\lim_{x\to 1^+} \frac{x-1}{x(x-1)} = \lim_{x\to 1^+} \frac{1}{x} = 1.$

20. $\lim_{x\to 1^-} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1^-} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1^-} \frac{x-1}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1^-} \frac{1}{\sqrt{x}+1} = \frac{1}{2}.$

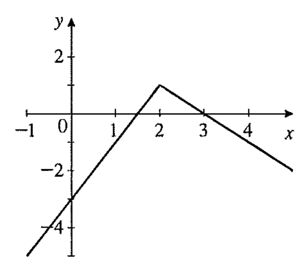
21. $\lim_{x\to \infty} \frac{x^2}{x^2-1} = \lim_{x\to \infty} \frac{1}{1-\frac{1}{x^2}} = 1.$

22. $\lim_{x\to -\infty} \frac{x+1}{x} = \lim_{x\to -\infty} \left(1 + \frac{1}{x}\right) = 1.$

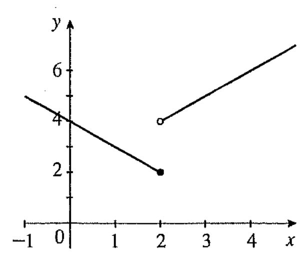
23. $\lim_{x\to \infty} \frac{3x^2+2x+4}{2x^2-3x+1} = \lim_{x\to \infty} \frac{3+\frac{2}{x}+\frac{4}{x^2}}{2-\frac{3}{x}+\frac{1}{x^2}} = \frac{3}{2}.$

24. $\lim_{x\to -\infty} \frac{x^2}{x+1} = \lim_{x\to -\infty} \left(x \cdot \frac{1}{1+\frac{1}{x}}\right) = -\infty$, so the limit does not exist.

25. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (-x+3) = -2+3 = 1$ and
$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x-3) = 2(2) - 3 = 4 - 3 = 1.$
Therefore, $\lim_{x\to 2} f(x) = 1.$



26. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x+2) = 4$ and
$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (4-x) = 2$. Therefore, $\lim_{x\to 2} f(x)$ does not exist.



27. The function is discontinuous at $x = 2$.

28. Because the denominator $4x^2 - 2x - 2 = 2(2x^2 - x - 1) = 2(2x+1)(x-1) = 0$ if $x = -\frac{1}{2}$ or 1, we see that f is discontinuous at these points.

29. Because $\lim_{x\to-1} f(x) = \lim_{x\to-1} \frac{1}{(x+1)^2} = \infty$ (does not exist), we see that f is discontinuous at $x = -1$.

30. The function is discontinuous at $x = 0$.

31. a. Let $f(x) = x^2 + 2$. Then the average rate of change of y over $[1, 2]$ is $\frac{f(2) - f(1)}{2-1} = \frac{(4+2)-(1+2)}{1} = 3$.

Over $[1, 1.5]$, it is $\frac{f(1.5) - f(1)}{1.5-1} = \frac{(2.25+2)-(1+2)}{0.5} = 2.5$. Over $[1, 1.1]$, it is $\frac{f(1.1) - f(1)}{1.1-1} = \frac{(1.21+2)-(1+2)}{0.1} = 2.1$.

b. Computing $f'(x)$ using the four-step process., we obtain

$f'(x) = \lim_{h\to0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to0} \frac{h(2x+h)}{h} = \lim_{h\to0} (2x+h) = 2$. Therefore, the instantaneous rate of change of f at $x = 1$ is $f'(1) = 2$, or 2 units per unit change in x.

32. $f(x) = 4x + 5$. We use the four-step process:

Step 1 $f(x+h) = 4(x+h) + 5 = 4x + 4h + 5$.

Step 2 $f(x+h) - f(x) = 4x + 4h + 5 - 4x - 5 = 4h$.

Step 3 $\frac{f(x+h) - f(x)}{h} = \frac{4h}{h} = 4$.

Step 4 $f'(x) = \lim_{h\to0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to0} (4) = 4$.

33. $f(x) = \frac{3}{2}x + 5$. We use the four-step process:

Step 1 $f(x+h) = \frac{3}{2}(x+h) + 5 = \frac{3}{2}x + \frac{3}{2}h + 5$.

Step 2 $f(x+h) - f(x) = \frac{3}{2}x + \frac{3}{2}h + 5 - \frac{3}{2}x - 5 = \frac{3}{2}h$.

Step 3 $\frac{f(x+h) - f(x)}{h} = \frac{3}{2}$.

Step 4 $f'(x) = \lim_{h\to0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to0} \frac{3}{2} = \frac{3}{2}$.

Therefore, the slope of the tangent line to the graph of the function f at the point $(-2, 2)$ is $\frac{3}{2}$. To find the equation of the tangent line to the curve at the point $(-2, 2)$, we use the point-slope form of the equation of a line, obtaining $y - 2 = \frac{3}{2}[x - (-2)]$ or $y = \frac{3}{2}x + 5$.

34. $f(x) = -x^2$. We use the four-step process:

Step 1 $f(x+h) = -(x+h)^2 = -x^2 - 2xh - h^2$.

Step 2 $f(x+h) - f(x) = \left(-x^2 - 2xh - h^2\right) - \left(-x^2\right) = -2xh - h^2 = h(-2x - h)$.

Step 3 $\frac{f(x+h) - f(x)}{h} = -2x - h$.

Step 4 $f'(x) = \lim_{h\to0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to0} (-2x - h) = -2x$.

The slope of the tangent line is $f'(2) = -2(2) = -4$. An equation of the tangent line is $y - (-4) = -4(x - 2)$, or $y = -4x + 4$.

35. $f(x) = -\frac{1}{x}$. We use the four-step process:

Step 1 $f(x+h) = -\frac{1}{x+h}$.

Step 2 $f(x+h) - f(x) = -\frac{1}{x+h} - \left(-\frac{1}{x}\right) = -\frac{1}{x+h} + \frac{1}{x} = \frac{h}{x(x+h)}$.

Step 3 $\frac{f(x+h) - f(x)}{h} = \frac{1}{x(x+h)}$.

Step 4 $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{x(x+h)} = \frac{1}{x^2}$.

36. a. f is continuous at $x = a$ because the three conditions for continuity are satisfied at $x = a$; that is, 1. $f(x)$ is defined. 2. $\lim_{x \to a} f(x)$ exists. 3. $\lim_{x \to a} f(x) = f(a)$.

b. f is not differentiable at $x = a$ because the graph of f has a kink at $x = a$.

37. $S(4) = 6000(4) + 30{,}000 = 54{,}000$.

38. a. The line passes through $(0, 2.4)$ and $(5, 7.4)$ and has slope $m = \frac{7.4 - 2.4}{5 - 0} = 1$. Letting y denote the sales, we see that an equation of the line is $y - 2.4 = 1(t - 0)$, or $y = t + 2.4$. We can also write this in the form $S(t) = t + 2.4$.

b. The sales in 2011 are $S(3) = 3 + 2.4 = 5.4$, or \$5.4 million.

39. a. $C(x) = 6x + 30{,}000$.

b. $R(x) = 10x$.

c. $P(x) = R(x) - C(x) = 10x - (6x + 30{,}000) = 4x - 30{,}000$.

d. $P(6000) = 4(6000) - 30{,}000 = -6000$, or a loss of \$6000. $P(8000) = 4(8000) - 30{,}000 = 2000$, or a profit of \$2000. $P(12{,}000) = 4(12{,}000) - 30{,}000 = 18{,}000$, or a profit of \$18,000.

40. Substituting the first equation into the second yields $3x - 2\left(\frac{3}{4}x + 6\right) + 3 = 0$, so $\frac{3}{2}x - 12 + 3 = 0$ and $x = 6$. Substituting this value of x into the first equation then gives $y = \frac{21}{2}$, so the point of intersection is $\left(6, \frac{21}{2}\right)$.

41. The profit function is given by $P(x) = R(x) - C(x) = 20x - (12x + 20{,}000) = 8x - 20{,}000$.

42. We solve the system $\begin{cases} 3x + p - 40 = 0 \\ 2x - p + 10 = 0 \end{cases}$ Adding these two equations, we obtain $5x - 30 = 0$, or $x = 6$. Thus, $p = 2x + 10 = 12 + 10 = 22$. Therefore, the equilibrium quantity is 6000 and the equilibrium price is \$22.

43. The child should receive $D(35) = \frac{500(35)}{150} \approx 117$, or approximately 117 mg.

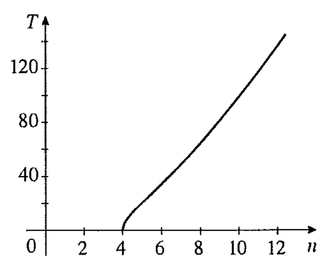
44. When 1000 units are produced, $R(1000) = -0.1(1000)^2 + 500(1000) = 400{,}000$, or \$400,000.

45. $R(30) = -\frac{1}{2}(30)^2 + 30(30) = 450$, or \$45,000.

46. $N(0) = 200(4+0)^{1/2} = 400$, and so there are 400 members initially. $N(12) = 200(4+12)^{1/2} = 800$, and so there are 800 members after one year.

47. The population will increase by $P(9) - P(0) = [50{,}000 + 30(9)^{3/2} + 20(9)] - 50{,}000$, or 990, during the next 9 months. The population will increase by $P(16) - P(0) = [50{,}000 + 30(16)^{3/2} + 20(16)] - 50{,}000$, or 2240 during the next 16 months.

48. $T = f(n) = 4n\sqrt{n-4}$. $f(4) = 0$, $f(5) = 20\sqrt{1} = 20$, $f(6) = 24\sqrt{2} \approx 33.9$, $f(7) = 28\sqrt{3} \approx 48.5$, $f(8) = 32\sqrt{4} = 64$, $f(9) = 36\sqrt{5} \approx 80.5$, $f(10) = 40\sqrt{6} \approx 98$, $f(11) = 44\sqrt{7} \approx 116$, and $f(12) = 48\sqrt{8} \approx 135.8$.



49. We need to find the point of intersection of the two straight lines representing the given linear functions. We solve the equation $2.3 + 0.4t = 1.2 + 0.6t$, obtaining $1.1 = 0.2t$ and thus $t = 5.5$. This tells us that the annual sales of the Cambridge Drug Store first surpasses that of the Crimson Drug store $5\frac{1}{2}$ years from now.

50. We solve $-1.1x^2 + 1.5x + 40 = 0.1x^2 + 0.5x + 15$, obtaining $1.2x^2 - x - 25 = 0$, $12x^2 - 10x - 250 = 0$, $6x^2 - 5x - 125 = 0$, and $(x-5)(6x+25) = 0$. Therefore, $x = 5$. Substituting this value of x into the second supply equation, we have $p = 0.1(5)^2 + 0.5(5) + 15 = 20$. So the equilibrium quantity is 5000 and the equilibrium price is \$20.

51. The life expectancy of a female whose current age is 65 is $C(65) \approx 16.80$ (years). The life expectancy of a female whose current age is 75 is $C(75) \approx 10.18$ (years).

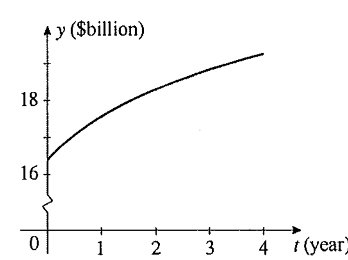
52. **a.** The amount of Medicare benefits paid out in 2010 is $B(0) = 0.25$, or \$250 billion.

b. The amount of Medicare benefits projected to be paid out in 2040 is $B(3) = 0.09(3)^2 + (0.102)(3) + 0.25 = 1.366$, or \$1.366 trillion.

53. $N(0) = 648$, or 648,000, $N(1) = -35.8 + 202 + 87.7 + 648 \approx 902$ or 902,000, $N(2) = -35.8(2)^3 + 202(2)^2 + 87.8(2) + 648 = 1345.2$ or 1,345,200, and $N(3) = -35.8(3)^3 + 202(3)^2 + 87.8(3) + 648 = 1762.8$ or 1,762,800.

54. **a.** $A(0) = 16.4$, or \$16.4 billion; $A(1) = 16.4(1+1)^{0.1} \approx 17.58$, or \$17.58 billion; $A(2) = 16.4(2+1)^{0.1} \approx 18.30$, or \$18.3 billion; $A(3) = 16.4(3+1)^{0.1} \approx 18.84$, or \$18.84 billion; and $A(4) = 16.4(4+1)^{0.1} \approx 19.26$, or \$19.26 billion. The nutritional market grew over the years 1999 to 2003.

b.



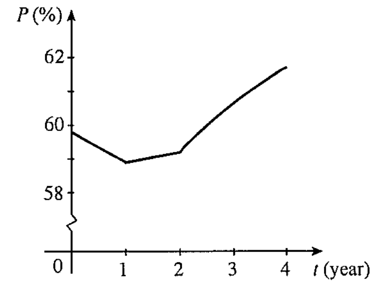
55. **a.** $f(t) = 267$; $g(t) = 2t^2 + 46t + 733$.

b. $h(t) = (f+g)(t) = f(t) + g(t) = 267 + (2t^2 + 46t + 733) = 2t^2 + 46t + 1000$.

c. $h(13) = 2(13)^2 + 46(13) + 1000 = 1936$, or 1936 tons.

56. a. $P(0) = 59.8$, $P(1) = 0.3(1) + 58.6 = 58.9$,
$P(2) = 56.79(2)^{0.06} \approx 59.2$, $P(3) = 56.79(3)^{0.06} \approx 60.7$, and
$P(4) = 56.79(4)^{0.06} \approx 61.7$.

b.



c. $P(3) \approx 60.7$, or 60.7%.

57. a. $f(r) = \pi r^2$.

b. $g(t) = 2t$.

c. $h(t) = (f \circ g)(t) = f(g(t)) = \pi [g(t)]^2 = 4\pi t^2$.

d. $h(30) = 4\pi (30^2) = 3600\pi$, or 3600π ft^2.

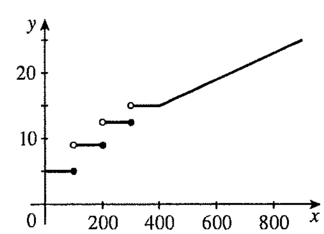
58. Measured in inches, the sides of the resulting box have length $20 - 2x$ and the height is x, so its volume is $V = x(20 - 2x)^2$ in^3.

59. Let h denote the height of the box. Then its volume is $V = (x)(2x)h = 30$, so that $h = \dfrac{15}{x^2}$. Thus, the cost is

$$\begin{aligned} C(x) &= 30(x)(2x) + 15[2xh + 2(2x)h] + 20(x)(2x) \\ &= 60x^2 + 15(6xh) + 40x^2 = 100x^2 + (15)(6)x\left(\frac{15}{x^2}\right) \\ &= 100x^2 + \frac{1350}{x}. \end{aligned}$$

60.

$$C(x) = \begin{cases} 5 & \text{if } 1 \le x \le 100 \\ 9 & \text{if } 100 < x \le 200 \\ 12.50 & \text{if } 200 < x \le 300 \\ 15.00 & \text{if } 300 < x \le 400 \\ 7 + 0.02x & \text{if } x > 400 \end{cases}$$



61. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty} \left(20 + \dfrac{400}{x}\right) = 20$. As the level of production increases without bound, the average cost of producing the commodity steadily decreases and approaches \$20 per unit.

62. a. $C'(x)$ gives the instantaneous rate of change of the total manufacturing cost c in dollars when x units of a certain product are produced.

b. Positive

c. Approximately \$20.

63. True. If $x < 0$, then $\sqrt{x}$ is not defined, and if $x > 0$, then $\sqrt{-x}$ is not defined. Therefore $f(x)$ is defined nowhere, and is not a function.

64. False. Let $f(x) = x^{1/3} + 1$. Then $f'(x) = \frac{1}{3}x^{-2/3}$, so $f'(1) = \frac{1}{3}$ and an equation of the tangent line to the graph of f at the point $(1, 2)$ is $y - 2 = \frac{1}{3}(x - 1)$ or $y = \frac{1}{3}x + \frac{5}{3}$. This tangent line intersects the graph of f at the point $(-8, -1)$, as can be easily verified.

CHAPTER 2 Before Moving On... page 160

1. a. $f(-1) = -2(-1) + 1 = 3$.

b. $f(0) = 2$.

c. $f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 + 2 = \frac{17}{4}$.

2. a. $(f + g)(x) = f(x) + g(x) = \dfrac{1}{x+1} + x^2 + 1$.

b. $(fg)(x) = f(x)g(x) = \dfrac{x^2+1}{x+1}$.

c. $(f \circ g)(x) = f(g(x)) = \dfrac{1}{g(x)+1} = \dfrac{1}{x^2+2}$.

d. $(g \circ f)(x) = g(f(x)) = [f(x)]^2 + 1 = \dfrac{1}{(x+1)^2} + 1$.

3. $4x + h = 108$, so $h = 108 - 4x$. The volume is $V = x^2h = x^2(108 - 4x) = 108x^2 - 4x^3$.

4. $\displaystyle\lim_{x \to -1} \frac{x^2 + 4x + 3}{x^2 + 3x + 2} = \lim_{x \to -1} \frac{(x+3)(x+1)}{(x+2)(x+1)} = 2$.

5. a. $\displaystyle\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left(x^2 - 1\right) = 0$.

b. $\displaystyle\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^3 = 1$.

Because $\displaystyle\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$, f is not continuous at 1.

6. The slope of the tangent line at any point is

$$\begin{aligned}
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) + 1 - \left(x^2 - 3x + 1\right)}{h} \\
&= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x - 1}{h} \\
&= \lim_{h \to 0} \frac{h(2x + h - 3)}{h} = \lim_{h \to 0} (2x + h - 3) = 2x - 3.
\end{aligned}$$

Therefore, the slope at 1 is $2(1) - 3 = -1$. An equation of the tangent line is $y - (-1) = -1(x - 1)$, or $y + 1 = -x + 1$, or $y = -x$.

CHAPTER 2 Explore & Discuss

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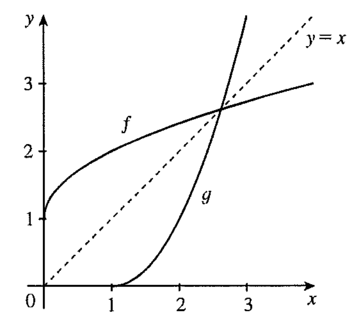
1. $(g \circ f)(x) = g(f(x)) = [f(x) - 1]^2 = [(\sqrt{x} + 1) - 1]^2 = (\sqrt{x})^2 = x$ and $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} + 1 = \sqrt{(x-1)^2} + 1 = (x - 1) + 1 = x$.

2. From the figure, we see that the graph of one is the mirror reflection of the other if we place a mirror along the line $y = x$.



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1. As x approaches 0 from either direction, $h(x)$ oscillates more and more rapidly between -1 and 1 and therefore cannot approach a specific number. But this says $\lim_{x \to 0} h(x)$ does not exist.

2. The function f fails to have a limit at $x = 0$ because $f(x)$ approaches 1 from the right but -1 from the left. The function g fails to have a limit at $x = 0$ because $g(x)$ is unbounded on either side of $x = 0$. The function h here does not approach any number from either the right or the left and has no limit at 0, as explained earlier.

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1. $\lim_{x \to \infty} f(x)$ does not exist because no matter how large x is, $f(x)$ takes on values between -1 and 1. In other words, $f(x)$ does not approach a definite number as x approaches infinity. Similarly, $\lim_{x \to -\infty} f(x)$ fails to exist.

2. The function of Example 10 fails to have a limit at infinity (negative infinity) because $f(x)$ increases (decreases) without bound as x approaches infinity (negative infinity). On the other hand, the function whose graph is depicted here, though bounded (its values lie between -1 and 1), does not approach any specific number as x increases (decreases) without bound and this is the reason it fails to have a limit at infinity or negative infinity.

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The average rate of change of a function f is measured over an interval. Thus, the average rate of change of f over the interval $[a, b]$ is the number $\dfrac{f(b) - f(a)}{b - a}$. On the other hand, the instantaneous rate of change of a function measures the rate of change of the function at a point. As we have seen, this quantity can be found by taking the limit of an appropriate difference quotient. Specifically, the instantaneous rate of change of f at $x = a$ is $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$.

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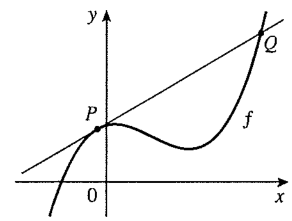
Yes. Here the line tangent to the graph of f at P also intersects the graph at the point Q lying on the graph of f.



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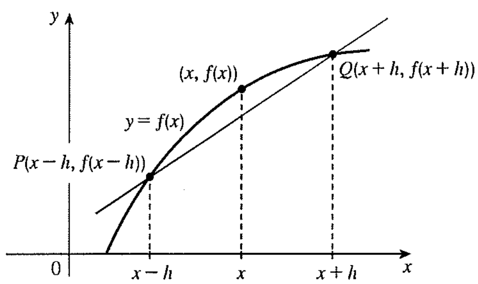
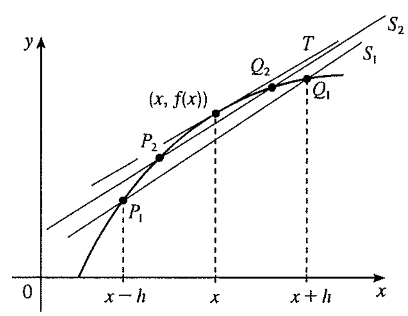
1. The quotient gives the slope of the secant line passing through $P\,(x-h, f\,(x-h))$ and $Q\,(x+h, f\,(x+h))$. It also gives the average rate of change of f over the interval $[x-h, x+h]$.



2. The limit gives the slope of the tangent line to the graph of f at the point $(x, f\,(x))$. It also gives the instantaneous rate of change of f at the point $(x, f\,(x))$. As h gets smaller and smaller, the secant lines approach the tangent line T.



3. The observation in part (b) suggests that this definition makes sense. We can also justify this observation as follows: From the definition of $f'\,(x)$, we have $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x+h) - f\,(x)}{h}$.
Replacing h by $-h$ gives $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x-h) - f\,(x)}{-h} = \lim_{h\to 0} \dfrac{f\,(x) - f\,(x-h)}{h}$. Thus,
$2f'\,(x) = \lim_{h\to 0} \left[\dfrac{f\,(x+h) - f\,(x)}{h} + \dfrac{f\,(x) - f\,(x-h)}{h}\right]$, and so $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x+h) - f\,(x-h)}{2h}$, in agreement with the result of Example 3.

4. **Step 1** Compute $f\,(x+h)$ and $f\,(x-h)$.
Step 2 Form the difference $f\,(x+h) - f\,(x-h)$.
Step 3 Form the quotient $\dfrac{f\,(x+h) - f\,(x-h)}{2h}$.
Step 4 Compute $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x+h) - f\,(x-h)}{2h}$.
For the function $f\,(x) = x^2$, we have the following:
Step 1 $f\,(x+h) = (x+h)^2 = x^2 + 2xh + h^2$ and $f\,(x-h) = (x-h)^2 = x^2 - 2xh + h^2$.
Step 2 $f\,(x+h) - f\,(x-h) = \left(x^2 + 2xh + h^2\right) - \left(x^2 - 2xh + h^2\right) = 4xh$.
Step 3 $\dfrac{f\,(x+h) - f\,(x)}{2h} = \dfrac{4xh}{2h} = 2x$.
Step 4 $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x+h) - f\,(x)}{2h} = \lim_{h\to 0} 2x = 2x$, in agreement with the result of Example 3.

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No. The slope of the tangent line to the graph of f at $(a, f(a))$ is defined by $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, and because the limit must be unique (see the definition of a limit), there is only one number $f'(a)$ giving the slope of the tangent line. Furthermore, since there can only be one straight line with a given slope, $f'(a)$, passing through a given point, $(a, f(a))$, our conclusion follows.

CHAPTER 2 Exploring with Technology

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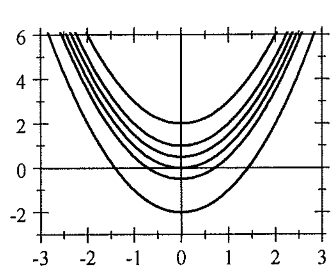
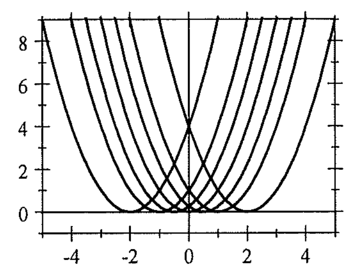
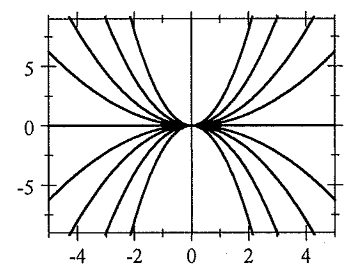
1.



2.



3.



4. The graph of $f(x) + c$ is obtained by translating the graph of f along the y-axis by c units. The graph of $f(x + c)$ is obtained by translating the graph of f along the x-axis by c units. Finally, the graph of cf is obtained from that of f by "expanding"(if $c > 1$) or "contracting"(if $0 < c < 1$) that of f. If $c < 0$, the graph of cf is obtained from that of f by reflecting it with respect to the x-axis as well as expanding or contracting it.

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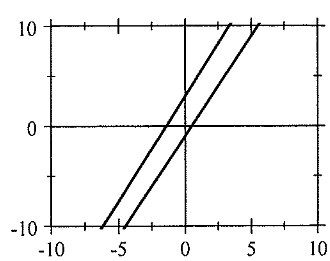
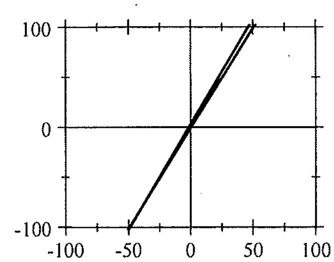
1. a.



The lines seem to be parallel to each other and do not appear to intersect.

b.

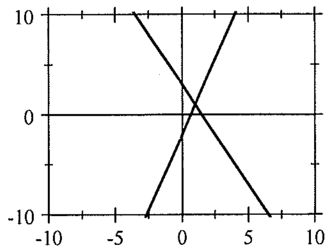


They appear to intersect. But finding the point of intersection using TRACE and ZOOM with any degree of accuracy seems to be an impossible task. Using the intersection feature of the graphing utility yields the point of intersection $(-40, -81)$ immediately.

c. Substituting the first equation into the second gives $2x - 1 = 2.1x + 3$, $-4 = 0.1x$, and thus $x = -40$. The corresponding y-value is -81.

d. Using TRACE and ZOOM is not effective. The intersection feature gives the desired result immediately. The algebraic method also yields the answer with little effort and without the use of a graphing utility.

2. a.



Plotting the straight lines L_1 and L_2 and using TRACE and ZOOM repeatedly, you will see that the iterations approach the answer $(1, 1)$. Using the intersection feature of the graphing utility gives the result $x = 1$ and $y = 1$, immediately.

b. Substituting the first equation into the second yields $3x - 2 = -2x + 3$, so $5x = 5$ and $x = 1$. Substituting this value of x into either equation gives $y = 1$.

c. The iterations obtained using TRACE and ZOOM converge to the solution $(1, 1)$. The use of the intersection feature is clearly superior to the first method. The algebraic method also yields the desired result easily.

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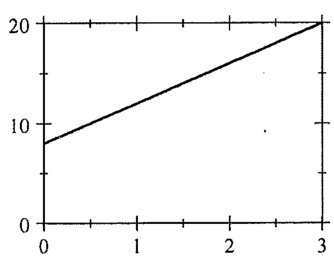
1.



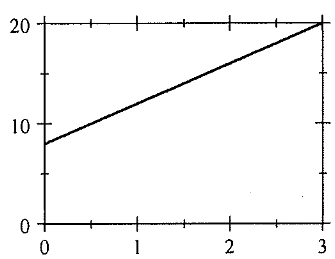
2. Using TRACE and ZOOM repeatedly, we find that $g(x)$ approaches 16 as x approaches 2.

3. If we try to use the evaluation function of the graphing utility to find $g(2)$ it will fail. This is because $x = 2$ is not in the domain of g.

4. The results obtained here confirm those obtained in the preceding example.

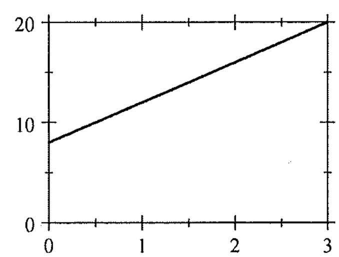
Page 109 (First Box)

1.



Using TRACE, we find $\lim_{x \to 2} \dfrac{4(x^2 - 4)}{x - 2} = 16$.

2.



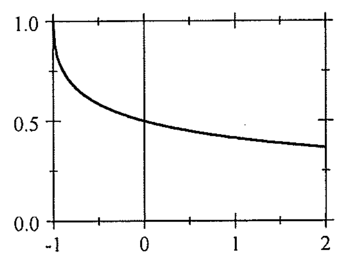
Using TRACE, we find $\lim_{x \to 2} 4(x + 2) = 16$. When $x = 2$, $y = 16$. The function $f(x) = 4(x + 2)$ is defined at $x = 2$ and so $f(2) = 16$ is defined.

3. No.

4. As we saw in Example 5, the function f is not defined at $x = 2$, but g is defined there.

Page 109 (Second Box)

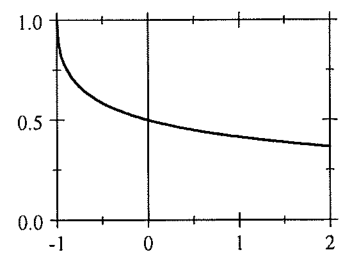
1.



Using TRACE and ZOOM, we see that

$$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x} = 0.5.$$

2.



The graph of f is the same as that of g except that the domain of f includes $x = 0$. (This is not evident from simply looking at the graphs!) Using the evaluation function to find the value of y, we obtain $y = 0.5$ when $x = 0$. This is to be expected since $x = 0$ lies in the domain of g.

3. As mentioned in part 2, the graphs are indistinguishable even though $x = 0$ is in the domain of g but not in the domain of f.

4. The functions f and g are the same everywhere except at $x = 0$ and so $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x} = \lim_{x \to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{2}$, as seen in Example 6.

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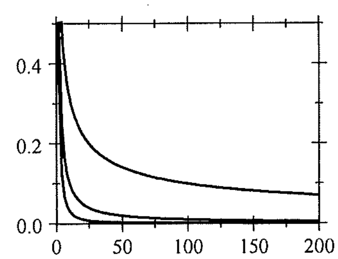
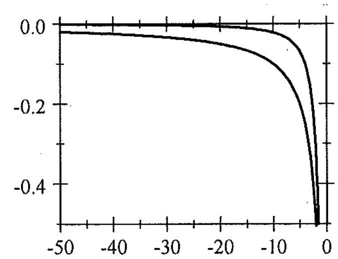
1.



The results suggest that $\frac{1}{x^n}$ goes to zero (as x increases) with increasing rapidity as n gets larger, as predicted by Theorem 2.

2.



The results suggest that $\frac{1}{x^n}$ goes to zero (as negative x increases in absolute value) with increasing rapidity as n gets larger, as predicted by Theorem 2.

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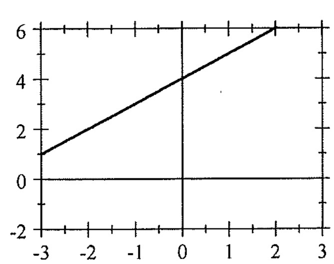
1.



2. Using ZOOM repeatedly, we find $\lim_{x \to 0} g(x) = 4$.

3. The fact that the limit found in part 2 is $f'(2)$ is an illustration of the definition of a derivative.

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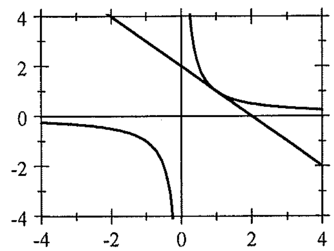

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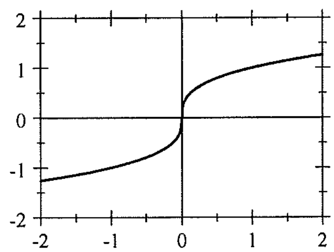
1.



2. The graphing utility will indicate an error when you try to draw the tangent line to the graph of f at $(0, 0)$. This happens because the slope of the tangent line to the graph of $f(x)$ is not defined at $x = 0$.