Calculus Differentiation Basic Rules

Slope of a Tangent Line definition $f'(x) = slope = \left(\frac{rise}{run}\right) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$

Rule 1: Derivative of a constant

$\frac{d}{dx}(c) = 0$ example: $f(x) = 13$; $f'(x) = 0$

Rule 2: The Power Rule

$\frac{d}{dx}(x^n) = n(x^{n-1})$ example: $f(x) = x^{13}$; $f'(x) = 13x^{12}$

Rule 3: Derivation of a Constant Multiple of a Function

$\frac{d}{dx}[c\, f(x)] = c\frac{d}{dx}[f(x)]$ example: $f(x) = 5x^3$; $f'(x) = 15x^2$

Rule 4: The Sum Rule

$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ example: $f(x) = 4x^3 - 3x^2 + 5x - 13$

$f'(x) = 12x^2 - 6x + 5$

Rule 5: The Product Rule

$\frac{d}{dx}[f(x)\ g(x)] = f(x)\ g'(x) + g(x)\ f'(x)$ example: $f(x) = (2x^2 - 1)(x^3 + 13)$

$f'(x) = (2x^2 - 1)(3x^2) + (x^3 + 13)(4x) = 6x^4 - 3x^2 + 4x^4 + 52x = 10x^4 - 3x^2 + 52x$

Rule 6: The Quotient Rule **example:** $f(x) = \frac{(x^2+1)}{(x^2-1)}$

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ $f'(x) = \frac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2} = -\frac{4x}{(x^2-1)^2}$

Rule 7: The Chain Rule **example:** $g(x) = x^{1/2}$ $f(x) = (3x^2 - x)$

If $h(x) = g[f(x)]$, then **$h(x) = (3x^2 - x)^{1/2}$**

$h'(x) = \frac{d}{dx} g[f(x)] = g'[f(x)]\ f'(x)$ **$h'(x) = \frac{1}{2}(3x^2 - x)^{-1/2}(6x - 1)$**

Rule 8: The General Power Rule **example:** $f(x) = (3x^2 - x)^{1/2}$

$f'(x) = \frac{d}{dx}[f(x)]^n = n[f(x)]^{(n-1)} f'(x)$ **$f'(x) = \frac{1}{2}(3x^2 - x)^{-1/2}(6x - 1)$**

Summary of Principal Formulas and Terms

Logarithms allow you to solve for the unknown when the unknown is a power.

Example: $2^x = 131{,}072$ becomes $x = \log_2(131{,}072) = \frac{\log(131{,}072)}{\log(2)}$ or $\frac{\ln(131{,}072)}{\ln(2)} = 17$

Memorize: $10^2 = 100$ iff $\log_{10}(100) = 2$

Logarithmic Rules

(1) $\log_b 1 = 0$ example: $\log_{13} 1 = 0$ since $13^0 = 1$ Note: $\log = \log_{10}$; $\ln = \log_e$

(2) $\log_b b = 1$ example: $\log_{13} 13 = 1$ since $13^1 = 13$

(3) $\log_b b^x = x$ example: $\log_{13} 13^x = x$ since $13^x = 13^x$

(4) $b^{\log_b x} = x$ example: $13^{\log_{13} x} = x$ since in log form $\log_{13} x = \log_{13} x$

(5) $\log_b M * N = \log_b M + \log_b N$ example: LOG 12 = LOG (3 * 4) = LOG 3 + LOG 4

(6) $\log_b M/N = \log_b M - \log_b N$ example: LOG 2 = LOG (8/4) = LOG 8 - LOG 4

(7) my favorite $\log_b M^p = p \log_b M$ example: $\log_{10} 100 = \log_{10} 10^2 = 2 \log_{10} 10 = 2$

(8) logical $\log_b M = \log_b N$ iff $M = N$ example: $\log_{13}(3x-1) = \log_{13} 8$; $3x-1 = 8$; $x = 3$

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = 2.71828182845904523536\ldots$$

Note: $\log = \log_{10}$; $\ln = \log_e$ thus $\ln e^x = x \ln e = x$ as $\ln e = 1$ and $e^{\ln X} = X$ (for x > 0) as in log form $\ln x = \ln x$

The Four Rules of Derivatives of exponential & logarithmic functions:

1. $\frac{d}{dx}(e^x) = e^x$ **example:** $f(x) = e^x$; $f'(x) = e^x$

2. $\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} f'(x)$ **example:** $f(x) = e^{(13x)}$; $f'(x) = e^{13x}(13) = 13\, e^{13x}$

3. $\frac{d}{dx}\ln|x| = \frac{1}{X}$ **examples:** $f(x) = x^2 \ln x$; $f'(x) = x^2\left(\frac{1}{X}\right) + \ln x\,(2x) = x + 2x \ln x = x(1 + 2\ln x)$

   $f(x) = \frac{\ln x}{X}$; $f'(x) = [\,x\left(\frac{1}{X}\right) - \ln x\,(1)] / x^2 = [1 - \ln x] / x^2$

4. $\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$ **example:** $f(x) = \ln(13x^2 + 13x + 13)$; $f'(x) = \frac{(26x+13)}{(13x2+13x+13)}$