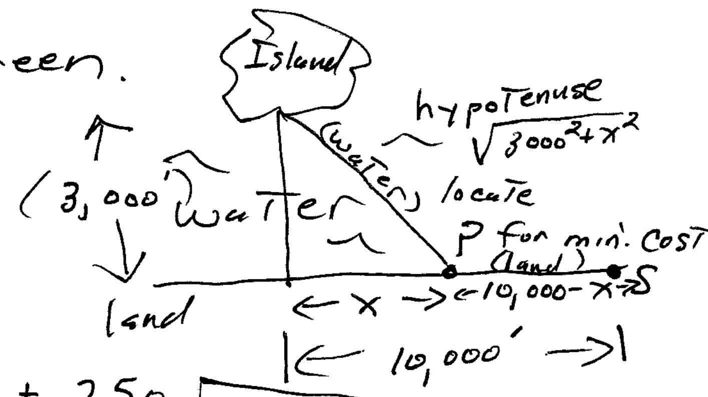
Optimization II Pg 329 # 27

Show problem on screen.



$$C(x) = 1.50(10{,}000 - x) + 2.50\sqrt{3000^2 + x^2} \qquad 0 \le x \le 10{,}000$$

$$C'(x) = -1.50 + 2.5\left(\tfrac{1}{2}\right)(9{,}000{,}000 + x^2)^{-1/2}(2x) = 0$$

$$C'(x) = -1.50 + \frac{2.5x}{(9{,}000{,}000 + x^2)^{1/2}} = 0$$

$$\frac{2.5x}{(9{,}000{,}000 + x^2)^{1/2}} = 1.5$$

$$2.5x = 1.5(9{,}000{,}000 + x^2)^{1/2}$$

$$6.25x^2 = 2.25(9{,}000{,}000 + x^2)$$

$$4x^2 = 20{,}250{,}000$$

$$x^2 = \cancel{5625}\ 5{,}062{,}500$$

$$x = 2250 \text{ ft}$$

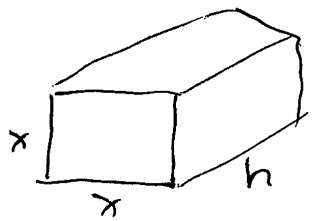
Note: $C(0) = \$22{,}500 \qquad C(2250) = 21{,}000 \qquad C(10{,}000) = 26{,}101$

so to minimize use $x = 2{,}250$ ft

gives absolute minimum

Optimization II Section 4.5

Pg. 328 #12 Show on screen.



Hint: The length (h) plus The girth ($4x$) is $\boxed{4x + h}$

Find dimentions to give Largest Volume?

regulations $4x + h = 108$

$$\boxed{h = 108 - 4x}$$

$$V = x^2 h = x^2(108 - 4x) = 108x^2 - 4x^3$$

$$V' = 216x - 12x^2 = 0$$

$$12x(18 - x) = 0$$

$$x = 0 \ \& \ x = 18$$

To find the interval we know $x \geq 0$ and $h \geq 0$

$$108 - 4x \geq 0$$

$$-4x \geq -108$$

$$x \leq 27$$

$$[0, 27]$$

So try all three and see which is largest.

$V(0) = 0 \text{ in}^3$ $\boxed{V(18) = 11{,}664 \text{ in}^3}$ $V(27) = 0$

so $x = 18''$

$$h = 108 - 4(18) = 108 - 72 = 36''$$

The dimensions $18'' \times 18'' \times 36''$ would maximize volume

$$V_{max} = 11{,}664 \text{ in}^3$$

Optimization II Pg 329 #25.

Place problem on screen. Note diagram.

given in hint: $S = kh^2w$

note $w^2 + h^2 = 24^2$

$h^2 = (576 - w^2)$

(Need a single variable before taking the derivative)

$$f(w) = S = kw(576 - w^2) = k(576w - w^3)$$

$$f'(w) = k(576 - 3w^2) = 0$$

$$-3w^2 = -576$$

$$w^2 = 192$$

$$w = \pm\sqrt{192} \doteq \pm 13.86 \text{ inches}$$

note $f''(w) = -6kw < 0$ for $w > 0$ so sad face so maximum $w = \sqrt{192}$

$f''(\sqrt{192}) = -6k\sqrt{192} < 0$

$$h^2 = 576 - w^2 = 576 - 192 = 384$$

$$h \doteq 19.60 \doteq \sqrt{384}$$

So $w \doteq 13.86$ inches and $h \doteq 19.60$ inches would give the strongest beam.