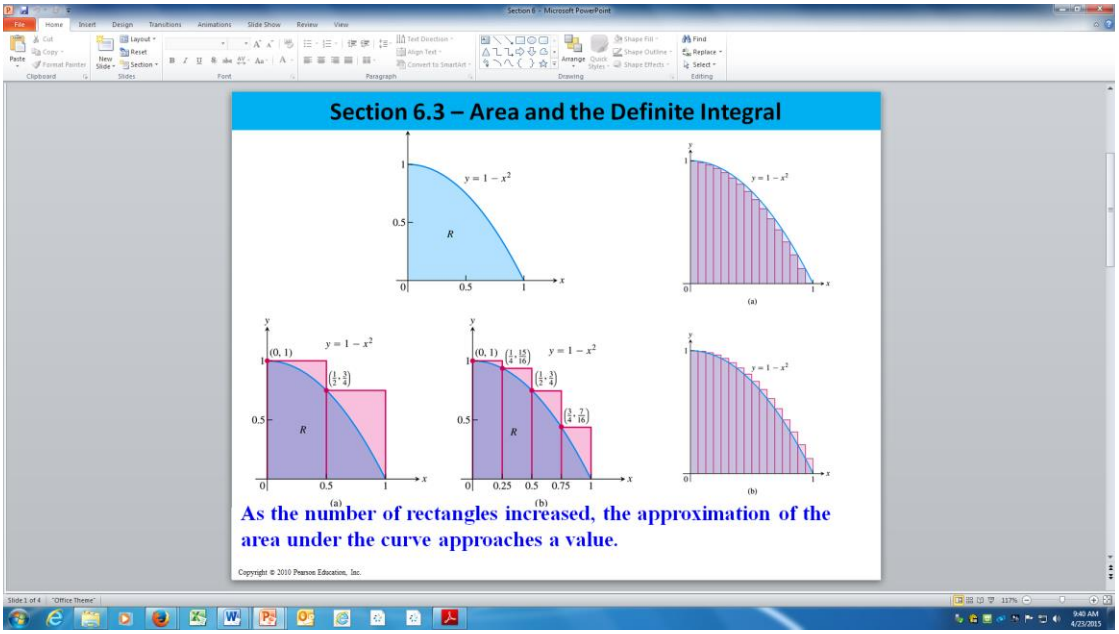
Section 6.3 – Area and the Definite Integral
y = 1 − x²
R
(a)
(b)
(0, 1)
(1/2, 3/4)
(1/4, 15/16)
(3/4, 7/16)
As the number of rectangles increased, the approximation of the area under the curve approaches a value.
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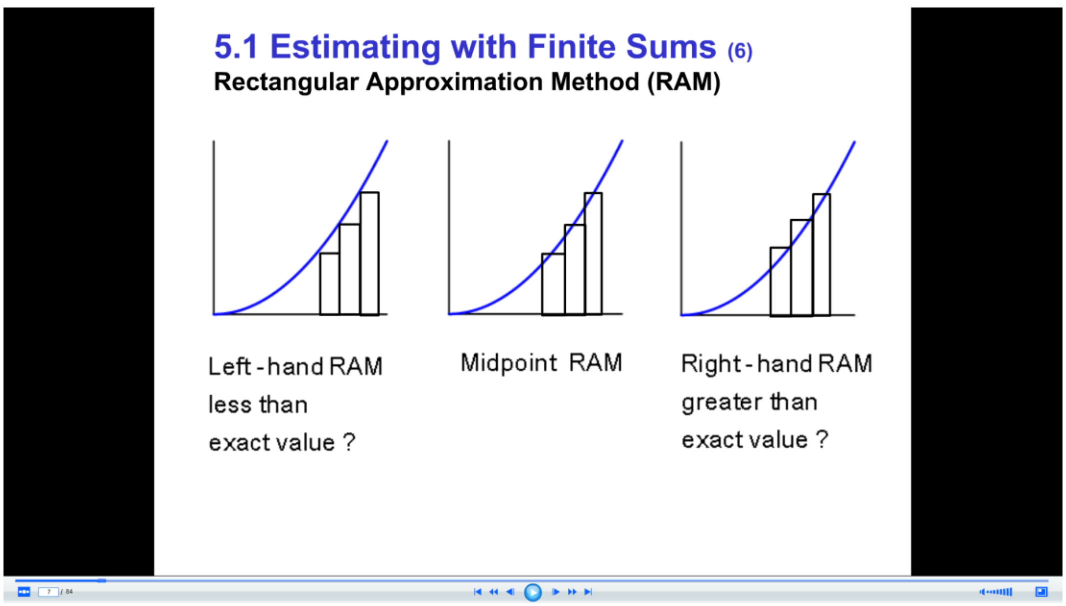
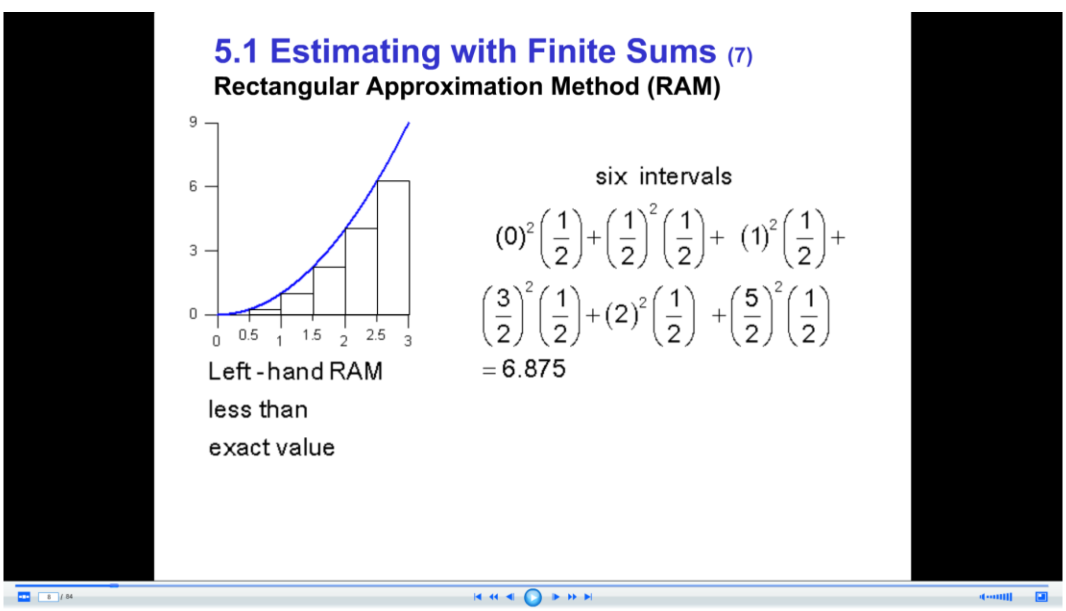
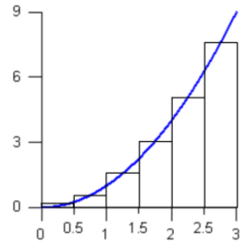

5.1 Estimating with Finite Sums (6)
Rectangular Approximation Method (RAM)
Left-hand RAM
less than
exact value ?
Midpoint RAM
Right-hand RAM
greater than
exact value ?


5.1 Estimating with Finite Sums (7)
Rectangular Approximation Method (RAM)
Left-hand RAM
less than
exact value
six intervals
$(0)^2\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)+(1)^2\left(\frac{1}{2}\right)+\left(\frac{3}{2}\right)^2\left(\frac{1}{2}\right)+(2)^2\left(\frac{1}{2}\right)+\left(\frac{5}{2}\right)^2\left(\frac{1}{2}\right)$
$=6.875$

5.1 Estimating with Finite Sums (8)

Rectangular Approximation Method (RAM)



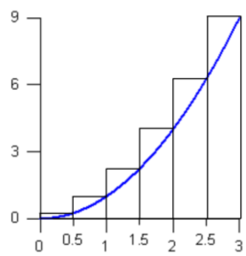
Midpoint RAM

six intervals

$$\left(\frac{1}{4}\right)^2\left(\frac{1}{2}\right)+\left(\frac{3}{4}\right)^2\left(\frac{1}{2}\right)+\left(\frac{5}{4}\right)^2\left(\frac{1}{2}\right)+$$
$$\left(\frac{7}{4}\right)^2\left(\frac{1}{2}\right)+\left(\frac{9}{4}\right)^2\left(\frac{1}{2}\right)+\left(\frac{11}{4}\right)^2\left(\frac{1}{2}\right)$$
$$=8.9375$$

5.1 Estimating with Finite Sums (9)

Rectangular Approximation Method (RAM)



Right-hand RAM
less than
exact value

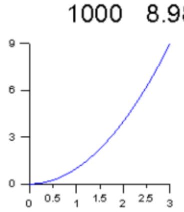
six intervals

$$\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)+\ (1)^2\left(\frac{1}{2}\right)+\left(\frac{3}{2}\right)^2\left(\frac{1}{2}\right)+$$
$$(2)^2\left(\frac{1}{2}\right)+\left(\frac{5}{2}\right)^2\left(\frac{1}{2}\right)+(3)^2\left(\frac{1}{2}\right)$$
$$=11.375$$

5.1 Estimating with Finite Sums (10)

Rectangular Approximation Method (RAM)

n	$LRAM_n$	$MRAM_n$	$RRAM_n$
6	6.875	8.9375	11.375
12	7.980625	8.984375	10.15625
24	8.4453125	8.99609375	9.5703125
48	8.720703125	8.999023438	9.283203125
100	8.86545	8.999775	9.13545
1000	8.9865045	8.99999775	9.0135045



All three sums approach the same number

Area Under A Curve

Definition: A sum such as the one below is called a **Riemann sum:**

$$\left[f(x_1)+f(x_2)+f(x_3)+\ldots+f(x_n)\right]\Delta x \text{ where } \Delta x = \frac{b-a}{n}$$

Area Under A Curve

The area under a curve can be approached by taking an infinite Riemann sum.

Definition: The Definite Integral

Let f(x) be defined on [a,b]. If

$$\lim_{n\to\infty}\left[f(x_1)+f(x_2)+f(x_3)+\ldots+f(x_n)\right]\Delta x$$

exists for all choices of representative points $x_1, x_2, x_3, \ldots x_n$ in the n subintervals of [a,b] of equal width $\Delta x = \frac{b-a}{n}$, then this limit is called the **definite integral of f(x) from a to b** and is denoted by $\int_a^b f(x)dx$.

The number **a** is the **lower limit of integration**, and the number **b** is the **upper limit of integration.**

Definition: The Definite Integral

Thus

$$\int_a^b f(x)dx = \lim_{n \to \infty}\left[f(x_1) + f(x_2) + f(x_3) + \ldots + f(x_n)\right]\Delta x$$

As long as f(x) is a **continuous** function on a closed interval, **it has a definite integral** on that interval. f(x) is said to be **integrable** when its integral exists.

The Definite Integral

Please make this important distinction between the indefinite integral of a function and the definite integral of a function:

The indefinite integral of a function is **another function.** example: $\int x\,dx = \frac{x^2}{2} + C$

The definite integral of a function is **a number.**

example: $\int_1^3 x\,dx = 4$ You will learn how to calculate this number in the next section of your textbook.