Section 6.4 The Fundamental Theorem of Calculus:

Let f be continuous on [a, b]. Then, $\int_a^b f(x)dx = F(b) - F(a)$ where F is any antiderivative of f; that is, $F'(x) = f(x)$.

Example 1: $A = \int_1^3 x\,dx$; $F(x) = \frac{1}{2}x^2 + C$; $F(3) = (\frac{1}{2})(3)^2 + C = \frac{9}{2} + C$

$F(1) = (\frac{1}{2})(1)^2 + C = (\frac{1}{2}) + C$ so $F(3) - F(1) = \frac{9}{2} + C - ((\frac{1}{2}) + C) = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4$

Example 2: Note: In the example above that C will always drop out so we will drop the constant of integration from our calculations.

$\int_0^3 x^2\,dx$; $F(x) = (\frac{1}{3})(x)^3$ so $F(3) = 9$ and $F(0) = 0$ so $F(3) - F(0) = 9 - 0 = 9$

Section 6.5 Evaluating Definite Integrals (using substitution to make the tougher ones a cinch.)

This is much like Section 6.2 only because you can change what interval you are integrating over with respect to u (instead of x) you can complete the problem with the easier respect to u form. It is like dropping step 5 in section 6.2

Example 1: $\int_0^1 x(x^2+1)^5\,dx$; let $u = x^2+1$ then when $x=0$; $u=1$ and when $x=1$; $u=2$ so now the interval is from 1 to 2. $du = (2x)dx$ so $\frac{1}{2}du = x\,dx$ substituting gives

$\frac{1}{2}\int_1^2 u^5\,du$; $F(u) = \frac{1}{2} * \frac{1}{6}u^6 = \frac{1}{12}u^6$; $F(2) = \frac{1}{12}(2^6) = \frac{64}{12} = 5\frac{1}{3}$; $F(1) = \frac{1}{12}$;

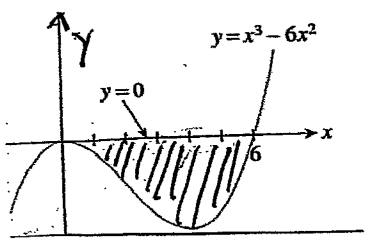
$F(2) - F(1) = 5\frac{4}{12} - \frac{1}{12} = 5\frac{3}{12} = 5\frac{1}{4}$

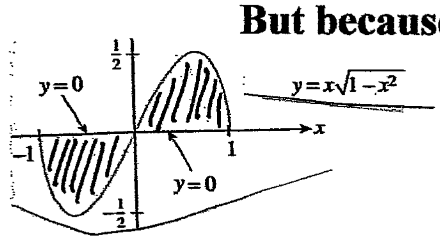
Section 6.6 Area Between Two Curves just cases using intersections of curve with line y=0 and graph given.

Example 1: $\int_0^6 (x^3 - 6x^2)\,dx$

Example 2: $\int_{-1}^0 x\sqrt{1-x^2} + \int_0^1 x\sqrt{1-x^2}$

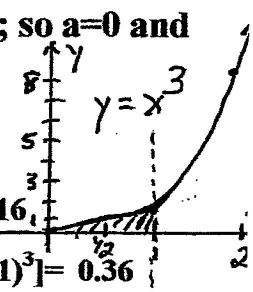
But because of symmetry $= 2\int_0^1 x\sqrt{1-x^2}$





Section 6.3 Area and the Definite Integral (computing the Riemann sum) note: $\Delta x = \frac{(b-a)}{n}$

Example: Let $f(x) = x^3$, and compute the Riemann sum of f over the interval [0,1]; so a=0 and b=1; with two (n = 2) and then five (n=5) subintervals of equal length.



For all three when n=2; $\Delta x = \frac{(1-0)}{2} = \frac{1}{2}$ and when n=5; $\Delta x = \frac{(1-0)}{5} = \frac{1}{5}$

a). left gives: when n= 2; $\frac{1}{2}[(0)^3 + (\frac{1}{2})^3] = \frac{1}{16} = .0625$ when n=5; $\frac{1}{5}[(0)^3 + (\frac{1}{5})^3 + (\frac{2}{5})^3 + \ldots + (\frac{4}{5})^3] = \frac{100}{625} = 0.16$

b). right endpoints gives: when n= 2; $\frac{1}{2}[(\frac{1}{2})^3 + (1)^3] = \frac{1}{16} = 0.5625$ when n=5; $\frac{1}{5}[(\frac{1}{5})^3 + (\frac{2}{5})^3 + (\frac{3}{5})^3 + \ldots + (1)^3] = 0.36$

c). midpoints gives: when n= 2; $\frac{1}{2}[(\frac{1}{4})^3 + (\frac{3}{4})^3] = \frac{7}{32} = 0.21875$ when n=5; $\frac{1}{5}[(\frac{1}{10})^3 + (\frac{3}{10})^3 + (\frac{5}{10})^3 + \ldots + (\frac{9}{10})^3] = \frac{1225}{5000} = 0.245$

Note: Using the definite integral $\int_0^1 x^3\,dx$; $F(x) = \frac{1}{4}x^4$; $F(1) = \frac{1}{4}$; $F(0)=0$; $F(3) - F(0) = \frac{1}{4} = 0.25$ is exact.

Summary of Chapter 6 Integral Calculus Math 111

Section 6:1

Rule 1: $\int k\, dx = \text{k x} + \text{C}$ (where k & C are constants) example: $\int 14\, dx = 14\text{ x} + \text{C}$

Rule 2: $\int x^n \text{ dx} = \frac{1}{(n+1)} x^{n+1} + \text{C}$ (where $n \neq -1$) example: $\int x^3 \text{ dx} = \frac{1}{(3+1)} x^{3+1} + \text{C} = \frac{1}{4} x^4 + \text{C}$

Rule 3: $\int c\, f(x)\ dx = \text{c} \int f(x) dx$ (where c is a constant) example: $\int 4x\, dx = 4\int x\, dx = 4(\tfrac{1}{2})x^2 + C = 2x^2 + C$

Rule 4: $\int [\, f(x) \pm g(x)]\ dx = \int f(x)\ dx \pm \int g(x)\ dx$ example: $\int (3x^2 + 2x)dx = 3(1/3)x^3 + 2(1/2)x^2 + C = x^3 + x^2 + C$

Rule 5: $\int e^x\ dx = e^x + \text{C}$ example: $\int 9e^x\ dx = 9 \int e^x\ dx = 9e^x + \text{C}$

Rule 6: $\int x^{-1}\, dx = \int \frac{1}{x} \text{dx} = \ln |x| + \text{C}$ (where $x \neq 0$) example: $\int (\frac{5}{x})\ dx = 5\int \frac{1}{x} \text{dx} = 5 \ln|x| + \text{C}$

Section 6.2

Example 1: $\int 4x\ (2x^2 + 1)^7 \text{ dx}$ simplify by letting $u = (2x^2 + 1)$ then $du/dx = 4x$ so $du = 4x\ dx$

Substituting above gives a much easier problem: $\int u^7 \text{ du} = (1/8)\ u^8 + C = (1/8)\ (2x^2 + 1)^8 + C$

Example 2: $\int \frac{2}{(x-2)} \text{dx} = \int 2\ (x-2)^{-1} \text{ dx}$ simplify by letting $u = (x - 2)$ then $du/dx = 1$ so $du = dx$

Substituting above gives $2\int u^{-1} \text{ du} = 2\int (\frac{1}{u}) \text{ du} = 2 \ln|u| + C = \ln u^2 + C = \ln (x-2)^2 + C$

Example 3: $\int 3\, x^2 \sqrt{(x^3 + 2)} \text{ dx} = \int 3\, x^2\, (x^3 + 2)^{(1/2)} \text{ dx}$ let $u = (x^3 + 2)$ then $du/dx = 3\ x^2$

$du = (3\ x^2)\ dx$

gives $\int u^{1/2} \text{ du} = (\frac{1}{(\frac{1}{2}+1)})\ u^{3/2} + C = \frac{1}{\frac{3}{2}} u^{3/2} + C = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (x^3 + 2)^{3/2} + C$

Example 4: $\int x^2\ (2x^3 + 3)^4 \text{ dx}$ let $u = (2x^3 + 3)$ so $du = 6\ x^2$ du so $(\frac{1}{6})$ du $= x^2$ dx

Substituting simplifies to: $\frac{1}{6} \int u^4 \text{ du} = \frac{1}{6}\ (\frac{1}{5})\ u^5 + C = \frac{1}{30} (2x^3 + 3)^5 + C$