4/21/2015

Concept #3 & Concept #1 What is the antiderivative?

Get out yesterdays handout

Section 6.1

Six Rules of indefinite integrals

Rules 1-4 example

$$\int (x^3 + 5x + 13)\,dx$$

(Rule 4) gives

$$\int x^3\,dx + \int 5x\,dx + \int 13\,dx$$

Rule 3 gives

$$\int x^3\,dx + 5\int x\,dx + \int 13\,dx$$

Rule 1 gives

$$\int x^3\,dx + 5\int x\,dx + 13x + C$$

OR

$$\int x^3\,dx + 5\int x^1\,dx + \int 13x^0\,dx$$

use Rule 2 on each

$$\frac{1}{(1+3)}x^{(1+3)} + 5\left(\frac{1}{(1+1)}\right)x^{(1+1)} + 13\left(\frac{1}{(0+1)}\right)x^{(0+1)} + C$$

$$\int (x^3 + 5x + 13)\,dx = \frac{1}{4}x^4 + \frac{5}{2}x^2 + 13x + C$$

The antiderivative

Check quick

$$\frac{d}{dx}\left(\frac{1}{4}x^4 + \frac{5}{2}x^2 + 13x + C\right)$$

$$= x^3 + 5x + 13$$

check by $\frac{d}{dx}$(antiderivative)

Note: (back to concept #2)

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4/21/2015 — six absent yesterday
→ New Calendar
→ collecting Chap 6 hmwk plus 1
→ No More gnats
→ quick look at powerpoint
1:1 correspondence
N = Counting #'s

~~Concept #1~~

What is the antiderivative?

$$f(x) = \int f'(x)\,dx$$

$F(x) = x^2$ is an antiderivative of $f(x) = 2x$

because $F'(x) = \frac{d}{dx}(x^2) = 2x = f(x)$

on just skip (Pg.2) To concept #3 To save Time

$$\int (2x)\,dx = 2\left(\frac{1}{1+1}\right)x^{(1+1)} + C = x^2 + C$$

antiderivative

$$\frac{d}{dx}(x^2+C) = 2x$$

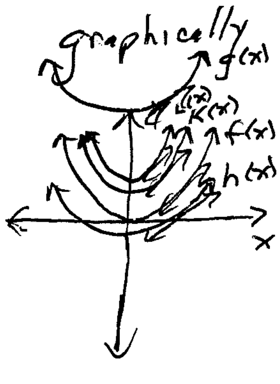
when done Taking the integral ($\int$) you can Take the derivative of your answer ~~and get the~~ (the antiderivative and get the orginal back.

So the derivative of the antiderivative is the orginal function.

Concept #2 Why is the $+C$ on each antiderivative

example: Take the derivative of each of the following:

graphically



$f(x) = x^2$

$g(x) = x^2 + 13$

$h(x) = x^2 - \frac{2}{3}$

$K(x) = x^2 + \sqrt{7}$ ≈ 2.6

$L(x) = x^2 + e$ $\approx 2.718281828459045\ldots$

$f'(x) = g'(x) = h'(x) = K'(x) = L'(x) = 2x$

So

$\int (2x)\,dx$ can be any one of These, so we just say $\int (2x)\,dx = \underline{x^2 + C}$

Note The $+C$ just moves graph up & down y axis
all derivatives equal (all slopes =)

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Concept #4 Can one ever calculate C?
They call the process an initial - value problem.

example: Find the function f if it is Known that $f'(x) = 3x^2 - 4x + 8$ and $f(1) = 9$
find $f(x)$???

$$f(x) = \int (3x^2 - 4x + 8)\,dx$$
$$= \int 3x^2 dx - \int 4x\,dx + \int 8\,dx$$
$$= 3\int x^2 dx - 4\int x\,dx + 8\int dx$$
$$= 3\left(\tfrac{1}{2+1}\right)x^{(2+1)} - 4\left(\tfrac{1}{(1+1)}\right)x^{(1+1)} + 8\left(\tfrac{1}{0+1}\right)x^{0+1} + C$$
$$= 3\left(\tfrac{1}{3}\right)x^3 - 4\left(\tfrac{1}{2}\right)x^2 + 8(1)x^1 + C$$
$$= x^3 - 2x^2 + 8x + C$$

but given $f(1) = 9$

so $f(1) = 1^3 - 2(1)^2 + 8(1) + C = 9$
$$= 1 - 2 + 8 + C = 9$$
$$= 7 + C = 9$$
$$-7 \qquad -7$$
$$C = 2$$

thus $\boxed{f(x) = x^3 - 2x^2 + 8x + 2}$

$$\int (3x^2 - 4x + 8)\,dx = x^3 - 2x^2 + 8x + C = f(x)$$
$$\int f'(x) = f(x)$$

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Concept #5

Rule 5 : $\int e^x dx = e^x + C$

check: $\frac{d}{dx}(e^x + C) = e^x$

Rule 6: $\int x^{-1} dx = \int \left(\frac{1}{x}\right) dx = \ln|x| + C$

(where $x \neq 0$)

check: $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$

Note in Rule 2 (where $n \neq -1$)

note if you Tried rule 2 $\int x^{-1} dx = \frac{1}{(-1+1)} x^{(-1+1)} + C$

Problem $= \frac{1}{0} x^0 + C$

Cannot divide by zero

helps you remember

Concept #6

Note Section 6.2 Notes

examples 1, 2, and 3

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Board Work

Bring Up Pg 418 6.1 hmwk page

(Middle group To board)

① Pg 418 #12. $\int 2x^5 dx = 2\left(\frac{1}{6}\right)x^6 + C$

$= \frac{1}{3}x^6 + C$ check with $\frac{d}{dx}$

② #14 $\int 3t^{-7} dt = 3\left(\frac{1}{(-7+1)}\right)t^{-6} + C$

-6

$= -\frac{1}{2}t^{-6} + C$ check with $\frac{d}{dx}$

③ #30 $\int (2 + x + 2x^2 + e^x)\, dx$

$= 2x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + e^x + C$

check with $\frac{d}{dx}$

④ Pg 430 Note look at 6.2 notes

#1 $\int 4(4x+3)^4 dx$

let $u = (4x+3)$ Then $du = 4dx$

check with $\frac{d}{dx}$

gives

$\int u^4 du = \left(\frac{1}{5}\right)u^5 + C = \frac{1}{5}(4x+3)^5 + C$

let
$u = (x^3 + 2x)$
$du = (3x^2 + 2)dx$

#6 $\int \frac{(3x^2+2)}{(x^3+2x)^2} dx = \int u^{-2} du = \frac{1}{-1}u^{-1} + C$

check with $\frac{d}{dx}$ $= -\frac{1}{(x^3+2x)} + C$

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Boardwork

(Last Group To Board)

Pg 430 #4 $\int (3x^2-2x+1)(x^3-x^2+x)^4\,dx$

let $u = (x^3-x^2+x)$

Then $du = (3x^2-2x+1)\,dx$ check with $\frac{d}{dx}$

becomes

$\int u^4\,du = \left(\frac{1}{5}\right)u^5 + C = \boxed{\left(\frac{1}{5}\right)(x^3-x^2+x)^5 + C}$

#14. $\int \frac{x^2}{(x^3-3)}\,dx$ Note: start with

let $u = \cancel{x}^3 - 3$

$du = 3x^2\,dx$ gives $\int \frac{1}{3}u^{-1}\,du$

$\frac{1}{3}du = x^2\,dx$

$= \frac{1}{-1+1}$ opps $\ln(x)$ Rule 6

$= \frac{1}{3}\ln(\cancel{u}) + C$

(1st Group To Board) Brady Bunch

$= \frac{1}{3}\ln(x^3-3) + C$

check with $\left(\frac{d}{dx}\right)$

#10 $\int x^2(2x^3+3)^4\,dx$

let $u = (2x^3+3)$ Then $du = 6x^2\,dx$

so $\frac{1}{6}du = x^2\,dx$

gives $\int \frac{1}{6}u^4\,du = \frac{1}{6}\left(\frac{1}{5}\right)u^5 + C$

$= \left(\frac{1}{30}\right)(2x^3+3)^5 + C$

check with $\frac{d}{dx}$

$(5)\left(\frac{1}{30}\right)(2x^3+3)^4(6x^2) = x^2(2x^3+3)^4$

The end for Today