Chapter 5.2 – Logarithmic Functions

The following relationship is true:

$$y = \log_2 x \;\; if \;\; and \;\; only \; if \;\; 2^y = x$$

Consider the logarithm,

$$\log_{10} 100 = 2$$

What relationship must be true according to this logarithm? ________________

Simply, $If \;\; \log_a b = c, \, then \; a^c = b$. (Both “a” and “b” MUST be POSITIVE!!!) This is known as the equivalent exponential form. In fact, logarithmic and exponential functions are inverses of each other.

<u>CONVERSIONS:</u>

Convert to either exponential or logarithmic form:

1. $81 = 3^4$

2. $\log_9 3 = \dfrac{1}{2}$

3. $7 = \sqrt{49}$

PROPERTIES OF LOGARITHMIC FUNCTIONS (Numerous, but not difficult to remember and apply)

(1) $\log_b 1 = 0$

(2) $\log_b b = 1$

(3) $\log_b b^x = x$

(4) $b^{\log_b x} = x$

(5) $\log_b M \cdot N = \log_b M + \log_b N$

(6) $\log_b \dfrac{M}{N} = \log_b M - \log_b N$

(7) $\log_b M^p = p \log_b M$

(8) $\log_b M = \log_b N, \quad iff \quad M = N$

Rewrite the following using as many laws of logarithms as possible:

4. $\log_5 x^3 y^4$

5. $\log_3 \dfrac{x^2+1}{3^x}$

6. $\log \dfrac{x^2\sqrt{x^2-1}}{10^x}$

<u>NATURAL LOGARITHMS (ln) and Base "e".</u>

The "natural logarithm", ln, is the inverse of base "e". Reviewing properties (3) and (4) above, we see that $\log_b b^x = x$. That is, taking the logarithm of an exponent with the same base, b, causes them to cancel each other out and we are left with just "x", the input value.

Also, $b^{\log_b x} = x$. That is, raising base b to the power of a logarithm with that same base, results in a cancellation, and again we are left with just "x".

Similarly, "ln" and "e" cancel each other out. Namely,

$$\ln(e^x) = x \quad \text{and}$$

$$e^{\ln x} = x$$

Solve for x to 4 decimal places. [Hint: Use ln]

7. $10^x = 3$

8. $e^x = 2$

9. $4^x = 3$

10. $10^x = 7$

11. $4^x = 5$

12. $e^x = 6$

13. $\log_4 x = 2$

14. $\log_x 36 = 2$

15. $\log_2 81 = x$

Chapter 5.4 – Differentiation of Exponential Functions

There are two rules to keep in mind when finding the derivative of a function that contains e^x.

Rule 1: The Derivative of the Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

Note that this derivative only works when the power of e is JUST x.

Rule 1a: The Derivative of the Constant Exponential Function

$$\frac{d}{dx}(ce^x) = ce^x$$

Note that the coefficient remains. Again, this only works when the power of e is JUST x.

EXAMPLE 1

Find the derivative of $y = 9e^x$.

Find the derivative of $y = 7e^x$

Rule 2: The Chain Rule for Exponential Functions

$$\frac{d}{dx}(e^{f(x)}) = f'(x) \cdot e^{f(x)}$$

The Chain Rule looks a bit confusing but with an example, it simply states that the part with the "e" will appear again in your answer as well at the derivative of the function that "e" is raised to.

EXAMPLE 2

Find the derivative of $f(x) = e^{x^2}$

$f'(x) = 2xe^{x^2}$

Find the derivative of $y = 6e^{x^3}$

Find the derivative of the following:

1. $f(x) = 3e^x$
2. $g(t) = e^{-t}$
3. $f(x) = 4e^x - x^4$
4. $f(x) = x^3 e^x$
5. $f(x) = \dfrac{x}{e^x}$
6. $f(x) = \left(e^x + 2\right)^{18}$
7. $f(x) = -7x^e + 2e^x + 4x^3$ ***
8. $g(t) = t^7 - t^5 + e^3 - t + e^t - e^{t^9}$

$h(x) = e^{-3x}, \quad find \;\; h^{(8)}(x)$