Chapter 5.5 – Differentiation of Logarithmic Functions

Just like exponential functions, logarithmic functions have specific rules for finding the derivative. The logarithmic function that we will be differentiating will be the natural logarithm.

Rule 1: The Derivative of ln x

$$\frac{d}{dx}\ln|x| = \frac{1}{x}, \quad x \neq 0$$

EXAMPLE 3

Find the derivative of each function:

$$f(x) = x^2 \cdot \ln x$$

$$g(x) = \frac{\ln x}{x^2}$$

Rule 2: The Chain Rule for Logarithmic Functions

$$\frac{d}{dx}[\ln f(x)] = \frac{f'(x)}{f(x)}, \quad f(x) > 0$$

Simply put, the derivative goes in the numerator and the original function in the denominator.

<u>EXAMPLE 4</u>

Find the derivative of $y = \ln x^4$.

Find the derivative of $y = 4\ln x$

What do you notice about the above solutions?______________________________________

Explain why this happened. ____

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Find the derivative of the function:

1. $f(x) = \ln 5x$
2. $f(x) = \ln(3x+1)$
3. $g(x) = 2\ln x^5$ (2 ways)
4. $h(t) = \ln\sqrt{t}$ (2 ways)

*** $y = x^2(x-1)(x^2+4)^3$

Chapter 5.6 – Exponential Functions as Mathematical Models

There are many practical situations in the life and social sciences in which mathematical modeling can be applied. The application that we will focus on deals with exponential growth and exponential decay (also known as negative growth). The exponential function that we first examined had the form:

$$f(x) = b^x, \quad b > 1$$

In particular, we will focus on the formula $f(x) = e^x$ from which we derive the following:

$$\begin{aligned}
&Q(t) = Q_0 e^{kt}, \quad where \\
&Q(t) = quantity\ after\ \ time\ \ t \\
&Q_0 \quad = initial\ quantity\,(quantity\ at\ time = 0) \\
&k \qquad = growth\,/\,decay\ cons\tan t \\
&t \qquad = time
\end{aligned}$$

Note: “k” is positive for growth and negative for decay.

Next, we will look at the rate of change of the function $Q(t) = Q_0 e^{kt}$. Remember, the rate of change is synonymous with the derivative. Therefore, finding the derivative results in:

$$\begin{aligned}
Q'(t) &= \frac{d}{dt} \cdot Q_0 e^{kt} \\
&= Q_0 \cdot \left(\frac{d}{dt} e^{kt} \right) \\
&= Q_0 \cdot k \cdot e^{kt} \\
&= k \cdot Q_0 e^{kt} \\
&= k \cdot Q(t)
\end{aligned}$$

In other words, the derivative (rate of change) of the Exponential Growth Function is equal to the growth constant multiplied by the function.

EXAMPLE 1:

An ant colony grows exponentially. Suppose there are 100 ants initially and 600 ants after 2 months.

a. How many ants will there be after 4 months?
b. What is the rate of growth after 4 months?

EXAMPLE 2:

Carbon-9b7 is a radioactive isotope of carbon that decays exponentially, having a half-life of 16 days. That is, after 16 days, only half of the original quantity is left. Suppose there were 2000kg of pure Carbon 9b7 initially.

a. How much is left after t days?
b. How much is left after 8 days?
c. How long does it take for only 500kg to be left?