92. The acceleration is $\frac{dv}{dt}$, where r is the velocity of the aircraft. So suppose that $\frac{dv}{dt} = c$, a constant. Then $v = \int \frac{dv}{dt}\, dt = c\, dt = dt + k$, where k is the constant of integration. We have $v(0) = 160 \text{ mph} = \frac{160}{60} \times 88 = \frac{704}{3}$ ft/sec. This gives $v(0) = k = \frac{704}{3}$, so $v(t) = ct + \frac{704}{3}$. Because the aircraft was brought to rest in 1 second, we have $v(1) = 0$. Using this condition, we find $v(1) = c + \frac{704}{3} = 0$, and $c = -\frac{704}{3}$ ft/sec^2, and the deceleration is equivalent to $\frac{704}{3} \times \frac{1}{32} = \frac{22}{3}$, that is, $7\frac{1}{3}$ g's.

93. The time taken by runner A to cross the finish line is $t = \frac{200}{22} = \frac{100}{11}$ sec. Let a be the constant acceleration of runner B as he begins to spurt. Then $\frac{dv}{dt} = a$, so the velocity of runner B as he runs towards the finish line is $v = \int a\, dt = at + c$. At $t = 0$, $v = 20$ and so $v = at = 20$. Now $\frac{ds}{dt} = v = at + 20$, so $s = \int (at + 20)\, dt = \frac{1}{2}at^2 + 20t + k$, where k is the constant of integration. Next, $s(0) = 0$ gives $s = \frac{1}{2}at^2 + 20t = \left(\frac{1}{2}at + 20\right)t$. In order for runner B to cover 220 ft in $\frac{100}{11}$ sec, we must have $\left[\frac{1}{2}a\left(\frac{100}{11}\right) + 20\right]\frac{100}{11} = 220$, so $\frac{50}{11}a + 20 = \frac{220 \cdot 11}{100} = \frac{121}{5}$, $\frac{50}{11}a = \frac{121}{5} - 20 = \frac{21}{5}$, and $a = \frac{21}{5} \cdot \frac{11}{50} = 0.924$ ft/sec^2. Therefore, runner B must have an acceleration of at least 0.924 ft/sec^2.

94. $h(t) = \int h'(t) = \int \left(-\frac{1}{25}\right)\left(\sqrt{20} - \frac{1}{50}t\right) dt = -\frac{1}{25}\left(\sqrt{20}t - \frac{1}{100}t^2\right) + C$. Next, we use the initial condition $h(0) = 20$ to obtain $h(0) = C = 20$. Therefore, the required expression is $h(t) = -\frac{1}{25}\left(\sqrt{20}t - \frac{1}{100}t^2\right) + 20$.

95. Suppose the acceleration is k. The distance covered is $s = f(t)$ and satisfies $f''(t) = k$. Thus, $f'(t) = v(t) = \int k\, dt = kt + C_1$. Next, $v(0) = 0$ gives $v(t) = kt$, and so $s = f(t) = \int kt\, dt = \frac{1}{2}kt^2 + C_2$. Now $f(0) = 0$ gives $s = \frac{1}{2}kt^2$. If it traveled 800 ft, we have $800 = \frac{1}{2}kt^2$, so $t = \frac{40}{\sqrt{k}}$. Its speed at this time is $v(t) = kt = k\left(\frac{40}{\sqrt{k}}\right) = 40\sqrt{k}$. We want the speed to be at least 240 ft/sec, so we require $40\sqrt{k} > 240$, implying that $k > 36$. In other words, the acceleration must be at least 36 ft/sec^2.

96. True. See the proof in Section 6.1 of the text.

97. False. $\int f(x)\, dx = F(x) + C$, where C is an arbitrary constant.

98. True. Use the Sum Rule followed by the Constant Multiple Rule.

99. False. $\int \frac{d}{dx}[f(x)]\, dx = \int f'(x)\, dx = f(x) + C$, where C is a constant of integration.

100. False. Take $f(x) = 1$ and $g(x) = 1$. Then $\int f(x) g(x)\, dx = \int 1\, dx = x + C$, whereas $\left[\int f(x)\, dx\right]\left[\int g(x)\, dx\right] = \left(\int 1\, dx\right)\left(\int 1\, dx\right) = (x + C)(x + D) = x^2 + (C + D)x + CD$.

6.2 Integration by Substitution

Concept Questions

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1. To find $I = \int f(g(x))g'(x)\, dx$ by the Method of Substitution, let $u = g(x)$, so that $du = g'(x)\, dx$. Making the substitution, we obtain $I = \int f(u)\, du$, which can be integrated with respect to u. Finally, replace u by $u = g(x)$ to evaluate the integral.

2. For $I = xe^{-x^2}dx$, we let $u = -x^2$, so that $du = -2x\,dx$ and $x\,dx = -\frac{1}{2}du$. Then $I = -\frac{1}{2}\int e^u\,du$, which is easily integrated. But the substitution does not work for $J = \int e^{-x^2}\,dx$ because it does not reduce J to the form $f(u)\,du$, where f is easily integrable.

Exercises page 430

1. Put $u = 4x + 3$, so $du = 4\,dx$ and $dx = \frac{1}{4}\,du$. Then $\int 4(4x+3)^4\,dx = \int u^4\,du = \frac{1}{5}u^5 + C = \frac{1}{5}(4x+3)^5 + C$.

2. Let $u = 2x^2 + 1$, so $du = 4x\,dx$. Then $\int 4x\left(2x^2+1\right)^7 dx = \int u^7\,du = \frac{1}{8}u^8 + C = \frac{1}{8}\left(2x^2+1\right)^8 + C$.

3. Let $u = x^3 - 2x$, so $du = \left(3x^2 - 2\right)dx$. Then
$\int \left(x^3 - 2x\right)^2\left(3x^2-2\right)dx = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}\left(x^3-2x\right)^3 + C$.

4. Put $u = x^3 - x^2 + x$, so $du = \left(3x^2 - 2x + 1\right)dx$. Then
$\int \left(3x^2-2x+1\right)\left(x^3-x^2+x\right)^4 dx = \int u^4\,du = \frac{1}{5}u^5 + C = \frac{1}{5}\left(x^3-x^2+x\right)^5 + C$.

5. Let $u = 2x^2 + 3$, so $du = 4x\,dx$. Then
$$\int \frac{4x}{\left(2x^2+3\right)^3}\,dx = \int \frac{1}{u^3}\,du = \int u^{-3}\,du = -\frac{1}{2}u^{-2} + C = -\frac{1}{2\left(2x^2+3\right)^2} + C.$$

6. Let $u = x^3 + 2x$, so $du = \left(3x^2+2\right)dx$. Then
$$\int \frac{3x^2+2}{\left(x^3+2x\right)^2}\,dx = \int \frac{du}{u^2} = \int u^{-2}\,du = -u^{-1} + C = -\frac{1}{x^3+2x} + C.$$

7. Put $u = t^3 + 2$, so $du = 3t^2\,dt$ and $t^2\,dt = \frac{1}{3}\,du$. Then
$\int 3t^2\sqrt{t^3+2}\,dt = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\left(t^3+2\right)^{3/2} + C$.

8. Let $u = t^3 + 2$, so $du = 3t^2\,dt$. Then $\int 3t^2\left(t^3+2\right)^{3/2} dt = \int u^{3/2}\,du = \frac{2}{5}u^{5/2} + C = \frac{2}{5}\left(t^3+2\right)^{5/2} + C$.

9. Let $u = x^2 - 1$, so $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. Then
$\int 2\left(x^2-1\right)^9 x\,dx = 2\int \frac{1}{2}u^9\,du = \frac{1}{10}u^{10} + C = \frac{1}{10}\left(x^2-1\right)^{10} + C$.

10. Let $u = 2x^3 + 3$, so $du = 6x^2\,dx$ and $x^2\,dx = \frac{1}{6}\,du$. Then
$\int x^2\left(2x^3+3\right)^4 dx = \frac{1}{6}\int u^4\,du = \frac{1}{30}u^5 + C = \frac{1}{30}\left(2x^3+3\right)^5 + C$.

11. Let $u = 1 - x^5$, so $du = -5x^4\,dx$ and $x^4\,dx = -\frac{1}{5}\,du$. Then
$$\int \frac{x^4}{1-x^5}\,dx = -\frac{1}{5}\int \frac{du}{u} = -\frac{1}{5}\ln|u| + C = -\frac{1}{5}\ln\left|1-x^5\right| + C.$$

12. Let $u = x^3 - 1$, so $du = 3x^2\,dx$ and $x^2\,dx = \frac{1}{3}\,du$. Then
$$\int \frac{x^2}{\sqrt{x^3-1}}\,dx = \frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{1}{3}\int u^{-1/2}\,du = \frac{2}{3}u^{1/2} + C = \frac{2}{3}\sqrt{x^3-1} + C.$$

13. Let $u = x - 2$, so $du = dx$. Then $\displaystyle\int \frac{2}{x-2}\,dx = 2\int \frac{du}{u} = 2\ln|u| + C = \ln u^2 + C = \ln(x-2)^2 + C$.

14. Let $u = x^3 - 3$, so $du = 3x^2\,dx$ and $\frac{1}{3}du = x^2\,dx$. Then $\int \frac{x^2}{x^3-3}\,dx = \int \frac{du}{3u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3-3| + C$.

15. Let $u = 0.3x^2 - 0.4x + 2$. Then $du = (0.6x - 0.4)\,dx = 2(0.3x - 0.2)\,dx$. Thus, $\int \frac{0.3x - 0.2}{0.3x^2 - 0.4x + 2}\,dx = \int \frac{1}{2u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(0.3x^2 - 0.4x + 2) + C$.

16. Let $u = 0.2x^3 + 0.3x$. Then $du = (0.6x^2 + 0.3)\,dx = 0.3(2x^2+1)\,dx$. Thus, $\int \frac{2x^2+1}{0.2x^3+0.3x}\,dx = \int \frac{1}{0.3u}\,du = \frac{1}{0.3}\ln|u| + C = \frac{10}{3}\ln|0.2x^3 + 0.3x| + C$.

17. Let $u = 3x^2 - 1$, so $du = 6x\,dx$ and $x\,dx = \frac{1}{6}\,du$. Then $\int \frac{2x}{3x^2-1}\,dx = 2\int \frac{x}{3x^2-1}\,dx = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|3x^2-1| + C$.

18. $I = \int \frac{x^2-1}{x^3-3x+1}\,dx$. Let $u = x^3 - 3x + 1$. Then $du = (3x^2-3)\,dx = 3(x^2-1)\,dx$ and $(x^2-1)\,dx = \frac{1}{3}\,du$. Therefore, $I = \int \frac{1}{3}u^{-1}\,du = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3-3x+1| + C$.

19. Let $u = -2x$, so $du = -2\,dx$ and $dx = -\frac{1}{2}\,du$. Then $\int e^{-2x}\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-2x} + C$.

20. Let $u = -0.02x$, so $du = -0.02\,dx$ and $dx = -\frac{1}{0.02}\,du = -50\,du$. Then $\int e^{-0.02x}\,dx = -50\int e^u\,du = -50e^{-0.02x} + C$.

21. Let $u = 2 - x$, so $du = -dx$ and $dx = -du$. Then $\int e^{2-x}\,dx = -\int e^u\,du = -e^u + C = -e^{2-x} + C$.

22. Let $u = 2t + 3$, so $du = 2\,dt$ and $dt = \frac{1}{2}\,du$. Then $\int e^{2t+3}\,dt = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{2t+3} + C$.

23. Let $u = -x^2$, so $du = -2x\,dx$ and $x\,dx = -\frac{1}{2}\,du$. Then $\int xe^{-x^2}\,dx = \int -\frac{1}{2}e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-x^2} + C$.

24. Let $u = x^3 - 1$, so $du = 3x^2\,dx$ and $x^2\,dx = \frac{1}{3}\,du$. Then $\int x^2e^{x^3-1}\,dx = \int \frac{1}{3}e^u\,du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3-1} + C$.

25. $\int (e^x - e^{-x})\,dx = \int e^x\,dx - \int e^{-x}\,dx = e^x - \int e^{-x}\,dx$. To evaluate the second integral on the right, let $u = -x$ so $du = -dx$ and $dx = -du$. Then $\int (e^x - e^{-x})\,dx = e^x + \int e^u\,du = e^x + e^u + C = e^x + e^{-x} + C$.

26. $\int (e^{2x} + e^{-3x})\,dx = \int e^{2x}\,dx + \int e^{-3x}\,dx$. To evaluate the first integral, let $u = 2x$, and to evaluate the second, let $u = -3x$. We find $\int (e^{2x} + e^{-3x})\,dx = \frac{1}{2}e^{2x} - \frac{1}{3}e^{-3x} + C$.

27. Let $u = 1 + e^x$, so $du = e^x\,dx$. Then $\int \frac{2e^x}{1+e^x}\,dx = 2\int \frac{e^x}{1+e^x}\,dx = 2\int \frac{du}{u} = 2\ln|u| + C = 2\ln(1+e^x) + C$.

28. Let $u = 1 + e^{2x}$, so $du = 2e^{2x}\,dx$. Then $e^{2x}\,dx = \frac{1}{2}\,du$, so $\int \frac{e^{2x}}{1+e^{2x}}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(1+e^{2x}) + C$.

29. Let $u = \sqrt{x} = x^{1/2}$. Then $du = \frac{1}{2}x^{-1/2}\,dx$ and $2\,du = x^{-1/2}\,dx$, so $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int 2e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C$.

30. Let $u = e^{-1/x}$. Then $du = -\frac{1}{x^2}e^{-1/2}\,dx$, so $\int \frac{e^{-1/x}}{x^2}\,dx = \int -u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}e^{-2/x} + C$.

31. Let $u = e^{3x} + x^3$, so $du = \left(3e^{3x} + 3x^2\right)dx = 3\left(e^{3x} + x^2\right)dx$ and $\left(e^{3x} + x^2\right)dx = \frac{1}{3}du$. Then $\int \frac{e^{3x} + x^2}{\left(e^{3x} + x^3\right)^3}\,dx = \frac{1}{3}\int \frac{du}{u^3} = \frac{1}{3}\int u^{-3}\,du = -\frac{u^{-2}}{6} + C = -\frac{1}{6\left(e^{3x} + x^3\right)^2} + C$.

32. Let $u = e^x + e^{-x}$, so $du = e^x - e^{-x}\,dx$. Then $\int \frac{e^x - e^{-x}}{\left(e^x + e^{-x}\right)^{3/2}}\,dx = \int \frac{du}{u^{3/2}} = \int u^{-3/2}\,du = -2u^{-1/2} + C = -2\left(e^x + e^{-x}\right)^{-1/2} + C$.

33. Let $u = e^{2x} + 1$, so $du = 2e^{2x}\,dx$ and $\frac{1}{2}\,du = e^{2x}\,dx$. Then $\int e^{2x}\left(e^{2x} + 1\right)^3 dx = \int \frac{1}{2}u^3\,du = \frac{1}{8}u^4 + C = \frac{1}{8}\left(e^{2x} + 1\right)^4 + C$.

34. Let $u = 1 + e^{-x}\,dx$, so $du = -e^{-x}\,dx$. Then $\int e^{-x}\left(1 + e^{-x}\right)dx = \int -u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}\left(1 + e^{-x}\right)^2 + C$.

35. Let $u = \ln 5x$, so $du = \frac{1}{x}\,dx$. Then $\int \frac{\ln 5x}{x}\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln 5x)^2 + C$.

36. Let $v = \ln u$, so $dv = \frac{1}{u}\,du$. Then $\int \frac{(\ln u)^3}{u}\,du = \int v^3\,dv = \frac{1}{4}v^4 + C = \frac{1}{4}(\ln u)^4 + C$.

37. Let $u = 3\ln x$, so $du = \frac{3}{x}\,dx$. Then $3\int \frac{1}{x\ln x}\,dx = 3\int \frac{du}{u} = 3\ln|u| + C = 3\ln|\ln x| + C$.

38. Let $u = \ln x$, so $du = \frac{1}{x}\,dx$. Then $\int \frac{1}{x(\ln x)^2}\,dx = \int \frac{1}{u^2}\,du = \int u^{-2}\,du = -u^{-1} + C = -\frac{1}{\ln x} + C$.

39. Let $u = \ln x$, so $du = \frac{1}{x}\,dx$. Then $\int \frac{\sqrt{\ln x}}{x}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(\ln x)^{3/2} + C$.

40. Let $u = \ln x$, so $du = \frac{1}{x}\,dx$. Then $\int \frac{(\ln x)^{7/2}}{x}\,dx = \int u^{7/2}\,du = \frac{2}{9}u^{9/2} + C = \frac{2}{9}(\ln x)^{9/2} + C$.

41. $\int \left(xe^{x^2} - \frac{x}{x^2+2}\right)dx = \int xe^{x^2}\,dx - \int \frac{x}{x^2+2}\,dx$. To evaluate the first integral, let $u = x^2$, so $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. Then $\int xe^{x^2}\,dx = \frac{1}{2}\int e^u\,du + C_1 = \frac{1}{2}e^u + C_1 = \frac{1}{2}e^{x^2} + C_1$. To evaluate the second integral, let $u = x^2 + 2$, so $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. Then $\int \frac{x}{x^2+2}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C_2 = \frac{1}{2}\ln\left(x^2+2\right) + C_2$.
Therefore, $\int \left(xe^{x^2} - \frac{x}{x^2+2}\right)dx = \frac{1}{2}e^{x^2} - \frac{1}{2}\ln\left(x^2+2\right) + C$.

42. $\int \left(xe^{-x^2} + \frac{e^x}{e^x+3}\right)dx = \int xe^{-x^2}\,dx + \int \frac{e^x}{e^x+3}\,dx$. To evaluate the first integral, let $u = -x^2$, so $du = -2x\,dx$ and $x\,dx = -\frac{1}{2}\,du$. Then $\int xe^{-x^2}\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C_1 = -\frac{1}{2}e^{-x^2} + C_1$. To evaluate the second integral, let $u = e^x + 3$, so $du = e^x\,dx$. Then $\int \frac{e^x}{e^x+3}\,dx = \int \frac{du}{u} = \ln|u| + C_2 = \ln\left(e^x + 3\right) + C_2$.
Therefore, $\int \left(xe^{-x^2} - \frac{e^x}{e^x+3}\right)dx = -\frac{1}{2}e^{-x^2} + \ln\left(e^x + 3\right) + C$.

43. Let $u = \sqrt{x} - 1$, so $du = \frac{1}{2}x^{-1/2}\,dx = \dfrac{1}{2\sqrt{x}}\,dx$ and $dx = 2\sqrt{x}\,du$. Also, we have $\sqrt{x} = u + 1$, so $x = (u+1)^2 = u^2 + 2u + 1$ and $dx = 2(u+1)\,du$. Thus,

$$\int \frac{x+1}{\sqrt{x}-1}\,dx = \int \frac{u^2+2u+2}{u} \cdot 2(u+1)\,du = 2\int \frac{(u^3+3u^2+4u+2)}{u}\,du$$
$$= 2\int \left(u^2 + 3u + 4 + \tfrac{2}{u}\right) du = 2\left(\tfrac{1}{3}u^3 + \tfrac{3}{2}u^2 + 4u + 2\ln|u|\right) + C$$
$$= 2\left[\tfrac{1}{3}\left(\sqrt{x}-1\right)^3 + \tfrac{3}{2}\left(\sqrt{x}-1\right)^2 + 4\left(\sqrt{x}-1\right) + 2\ln\left|\sqrt{x}-1\right|\right] + C.$$

44. Let $v = e^{-u} + u$. Then $dv = \left(-e^{-u} + 1\right) du$ and $-dv = \left(e^{-u} - 1\right) du$. Therefore,

$$\int \frac{e^{-u}-1}{e^{-u}+u}\,du = \int \left(-\frac{dv}{v}\right) = -\ln|v| = -\ln\left|e^{-u}+u\right| + C.$$

45. Let $u = x - 1$, so $du = dx$. Also, $x = u + 1$, and so

$$\int x(x-1)^5\,dx = \int (u+1)u^5\,du = \int \left(u^6 + u^5\right) du = \tfrac{1}{7}u^7 + \tfrac{1}{6}u^6 + C = \tfrac{1}{7}(x-1)^7 + \tfrac{1}{6}(x-1)^6 + C$$
$$= \frac{(6x+1)(x-1)^6}{42} + C.$$

46. $\displaystyle\int \frac{t}{t+1}\,dt = \int \left(1 - \frac{1}{t+1}\right) dt = \int dt - \int \frac{1}{t+1}\,dt = t - \ln|t+1| + C.$

47. Let $u = 1 + \sqrt{x}$, so $du = \frac{1}{2}x^{-1/2}\,dx$ and $dx = 2\sqrt{x} = 2(u-1)\,du$. Then

$$\int \frac{1-\sqrt{x}}{1+\sqrt{x}}dx = \int \left(\frac{1-(u-1)}{u}\right) \cdot 2(u-1)\,du = 2\int \frac{(2-u)(u-1)}{u}\,du = 2\int \frac{-u^2+3u-2}{u}\,du$$
$$= 2\int \left(-u + 3 - \frac{2}{u}\right) du = -u^2 + 6u - 4\ln|u| + C$$
$$= -\left(1+\sqrt{x}\right)^2 + 6\left(1+\sqrt{x}\right) - 4\ln\left(1+\sqrt{x}\right) + C$$
$$= -1 - 2\sqrt{x} - x + 6 + 6\sqrt{x} - 4\ln\left(1+\sqrt{x}\right) + C = -x + 4\sqrt{x} + 5 - 4\ln\left(1+\sqrt{x}\right) + C.$$

48. Let $u = 1 - \sqrt{x}$, so $du = -\dfrac{1}{2\sqrt{x}}\,dx$ and $dx = -2\sqrt{x}\,du$. Then $\sqrt{x} = 1 - u$ and $dx = -2(1-u)\,du$, so

$$\int \frac{1+\sqrt{x}}{1-\sqrt{x}}\,dx = \int \frac{2-u}{u}(-2)(1-u)\,du = -2\int \frac{(u-2)(u-1)}{u}\,du = -2\int \frac{u^2-3u+2}{u}\,du$$
$$= -2\int \left(u - 3 + \frac{2}{u}\right)du = -2\left(\tfrac{1}{2}u^2 - 3u + 2\ln|u|\right) + C = 6u - u^2 - 4\ln|u| + C$$
$$= 6\left(1-\sqrt{x}\right) - \left(1-\sqrt{x}\right)^2 - 4\ln\left(1-\sqrt{x}\right) + C.$$

49. $I = \int v^2(1-v)^6\,dv$. Let $u = 1 - v$, so $du = -dv$. Also, $1 - u = v$, and so $(1-u)^2 = v^2$. Therefore,

$$I = \int -\left(1 - 2u + u^2\right)u^6 du = \int -\left(u^6 - 2u^7 + u^8\right) du = -\left(\tfrac{1}{7}u^7 - \tfrac{1}{4}u^8 + \tfrac{1}{9}u^9\right) + C$$
$$= -u^7\left(\tfrac{1}{7} - \tfrac{1}{4}u + \tfrac{1}{9}u^2\right) + C = -\tfrac{1}{252}(1-v)^7\left[36 - 63(1-v) + 28\left(1 - 2v + v^2\right)\right]$$
$$= -\tfrac{1}{252}(1-v)^7\left[36 - 63 + 63v + 28 - 56v + 28v^2\right] = -\tfrac{1}{252}(1-v)^7\left(28v^2 + 7v + 1\right) + C.$$