6 INTEGRATION

6.1 Antiderivatives and the Rules of Integration

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1. An antiderivative of a continuous function f on an interval I is a function F such that $F'(x) = f(x)$ for every x in I. For example, an antiderivative of $f(x) = x^2$ on $(-\infty, \infty)$ is the function $F(x) = \frac{1}{3}x^3$ on $(-\infty, \infty)$.

2. If $f'(x) = g'(x)$ for all x in I, then $f(x) = g(x) + C$ for all x in I, where C is an arbitrary constant.

3. The indefinite integral of f is the family of functions $F(x) + C$, where F is an antiderivative of f and C is an arbitrary constant.

4. No, the power rule holds only for $n \neq -1$. Rather, $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C, x \neq 0$.

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1. $F(x) = \frac{1}{3}x^3 + 2x^2 - x + 2$, so $F'(x) = x^2 + 4x - 1 = f(x)$.

2. $F(x) = xe^x + \pi$, so $F'(x) = xe^x + e^x = e^x(x+1) = f(x)$.

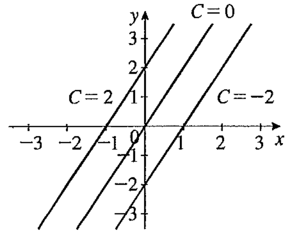
3. $F(x) = (2x^2-1)^{1/2}$, so $F'(x) = \frac{1}{2}(2x^2-1)^{-1/2}(4x) = 2x(2x^2-1)^{-1/2} = f(x)$.

4. $F(x) = x\ln x - x$, so $F'(x) = x\left(\frac{1}{x}\right) + \ln x - 1 = \ln x = f(x)$.

5. **a.** $G'(x) = \frac{d}{dx}(2x) = 2 = f(x)$

   b. $F(x) = G(x) + C = 2x + C$

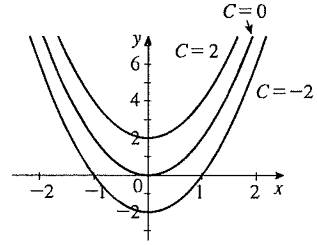
   c.



6. **a.** $G'(x) = 4x = f(x)$, and so G is an antiderivative of f.

   b. $H(x) = G(x) + C = 2x^2 + C$, where C is an arbitrary constant.

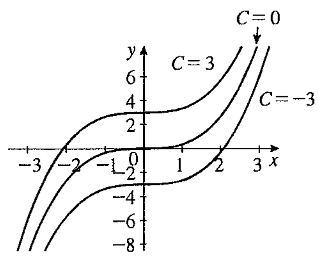
   c.



7. **a.** $G'(x) = \frac{d}{dx}\left(\frac{1}{3}x^3\right) = x^2 = f(x)$

b. $F(x) = G(x) + C = \frac{1}{3}x^3 + C$

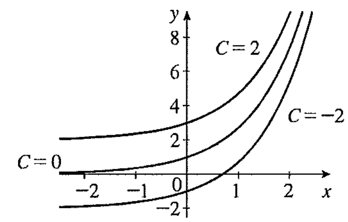
c.



8. **a.** $G(x) = e^x$, so $G'(x) = e^x = f(x)$.

b. $F(x) = e^x + C$, where C is an arbitrary constant.

c.



9. $\int 6\,dx = 6x + C$.

10. $\int \sqrt{2}\,dx = \sqrt{2}x + C$.

11. $\int x^3\,dx = \frac{1}{4}x^4 + C$.

12. $\int 2x^5\,dx = 2\left(\frac{1}{6}x^6\right) + C = \frac{1}{3}x^6 + C$.

13. $\int x^{-4}\,dx = -\frac{1}{3}x^{-3} + C$.

14. $\int 3t^{-7}\,dt = 3\left(-\frac{1}{6}t^{-6}\right) + C = -\frac{1}{2}t^{-6} + C$.

15. $\int x^{2/3}\,dx = \frac{3}{5}x^{5/3} + C$.

16. $\int 2u^{3/4}\,du = 2\left(\frac{4}{7}u^{7/4}\right) + C = \frac{8}{7}u^{7/4} + C$.

17. $\int x^{-5/4}\,dx = -4x^{-1/4} + C$.

18. $\int 3x^{-2/3}\,dx = 3\left(\frac{x^{1/3}}{\frac{1}{3}}\right) + C = 9x^{1/3} + C$.

19. $\int \frac{2}{x^3}\,dx = 2\int x^{-3}\,dx = -x^{-2} + C = -\frac{1}{x^2} + C$.

20. $\int \frac{1}{3x^5}\,dx = \frac{1}{3}\int x^{-5}\,dx = \frac{1}{3}\left(-\frac{1}{4}x^{-4}\right) + C$
$= -\frac{1}{12x^4} + C$.

21. $\int \pi\sqrt{t}\,dt = \pi\int t^{1/2}\,dt = \pi\left(\frac{2}{3}t^{3/2}\right) + C$
$= \frac{2\pi}{3}t^{3/2} + C$.

22. $\int \frac{3}{\sqrt{t}}\,dt = 3\int t^{-1/2}\,dt = 6t^{1/2} + C = 6\sqrt{t} + C$.

23. $\int (3 - 4x)\,dx = \int 3\,dx - 4\int x\,dx = 3x - 2x^2 + C$.

24. $\int \left(1 + u + u^2\right)du = u + \frac{1}{2}u^2 + \frac{1}{3}u^3 + C$.

25. $\int \left(x^2 + x + x^{-3}\right)dx = \int x^2\,dx + \int x\,dx + \int x^{-3}\,dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{2}x^{-2} + C$.

26. $\int \left(0.3t^2 + 0.02t + 2\right)dt = 0.3\left(\frac{1}{3}t^3\right) + 0.02\left(\frac{1}{2}t^2\right) + 2t + C = 0.1t^3 + 0.01t^2 + 2t + C$.

27. $\int 5e^x\,dx = 5e^x + C$.

28. $\int (1 + e^x)\,dx = x + e^x + C$.

29. $\int (1 + x + e^x)\,dx = x + \frac{1}{2}x^2 + e^x + C$.

30. $\int \left(2 + x + 2x^2 + e^x\right)dx = 2x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + e^x + C$.

31. $\int \left(4x^3 - \frac{2}{x^2} - 1\right) dx = \int \left(4x^3 - 2x^{-2} - 1\right) dx = x^4 + 2x^{-1} - x + C = x^4 + \frac{2}{x} - x + C.$

32. $\int \left(6x^3 + 3x^{-2} - x\right) dx = \frac{3}{2}x^4 - 3x^{-1} - \frac{1}{2}x^2 + C.$

33. $\int \left(x^{5/2} + 2x^{3/2} - x\right) dx = \frac{2}{7}x^{7/2} + \frac{4}{5}x^{5/2} - \frac{1}{2}x^2 + C.$

34. $\int \left(t^{3/2} + 2t^{1/2} - 4t^{-1/2}\right) dt = \frac{2}{5}t^{5/2} + \frac{4}{3}t^{3/2} - 8t^{1/2} + C.$

35. $\int \left(x^{1/2} + 2x^{-1/2}\right) dx = \frac{2}{3}x^{3/2} + 4x^{1/2} + C.$

36. $\int \left(x^{2/3} - x^{-2}\right) dx = \frac{3}{5}x^{5/3} + \frac{1}{x} + C.$

37. $\int \left(\frac{u^3 + 2u^2 - u}{3u}\right) du = \frac{1}{3}\int \left(u^2 + 2u - 1\right) du = \frac{1}{9}u^3 + \frac{1}{3}u^2 - \frac{1}{3}u + C.$

38. $\int \left(x^2 - x^{-2}\right) dx = \frac{1}{3}x^3 + x^{-1} = \frac{1}{3}x^3 + \frac{1}{x} + C.$

39. $\int (2t + 1)(t - 2)\, dt = \int \left(2t^2 - 3t - 2\right) dt = \frac{2}{3}t^3 - \frac{3}{2}t^2 - 2t + C.$

40. $\int u^{-2}\left(1 - u^2 + u^4\right) du = \int \left(u^{-2} - 1 + u^2\right) du = -u^{-1} - u + \frac{1}{3}u^3 + C.$

41. $\int \frac{1}{x^2}\left(x^4 - 2x^2 + 1\right) dx = \int \left(x^2 - 2 + x^{-2}\right) dx = \frac{1}{3}x^3 - 2x - x^{-1} + C = \frac{1}{3}x^3 - 2x - \frac{1}{x} + C.$

42. $\int t^{1/2}\left(t^2 + t - 1\right) dt = \int \left(t^{5/2} + t^{3/2} - t^{1/2}\right) dt = \frac{2}{7}t^{7/2} + \frac{2}{5}t^{5/2} - \frac{2}{3}t^{3/2} + C.$

43. $\int \frac{ds}{(s + 1)^{-2}} = \int (s + 1)^2\, ds = \int \left(s^2 + 2s + 1\right) ds = \frac{1}{3}s^3 + s^2 + s + C.$

44. $\int \left(x^{1/2} + 3x^{-1} - 2e^x\right) dx = \frac{2}{3}x^{3/2} + 3\ln|x| - 2e^x + C.$

45. $\int \left(e^t + t^e\right) dt = e^t + \frac{1}{e + 1}t^{e+1} + C.$

46. $$\int \left(\frac{1}{x^2} - \frac{1}{\sqrt[3]{x^2}} + \frac{1}{\sqrt{x}}\right) dx = \int \left(x^{-2} - x^{-2/3} + x^{-1/2}\right) dx = -x^{-1} - 3x^{1/3} + 2x^{1/2} + C$$
$$= -\frac{1}{x} - 3x^{1/3} + 2\sqrt{x} + C.$$

47. $\int \frac{x^3 + x^2 - x + 1}{x^2}\, dx = \int \left(x + 1 - \frac{1}{x} + \frac{1}{x^2}\right) dx = \frac{1}{2}x^2 + x - \ln|x| - x^{-1} + C.$

48. $\int \frac{t^3 + \sqrt[3]{t}}{t^2}\, dt = \int \left(t + t^{-5/3}\right) dt = \frac{1}{2}t^2 - \frac{3}{2}t^{-2/3} + C.$

49. $$\int \frac{\left(x^{1/2} - 1\right)^2}{x^2}\, dx = \int \frac{x - 2x^{1/2} + 1}{x^2}\, dx = \int \left(x^{-1} - 2x^{-3/2} + x^{-2}\right) dx$$
$$= \ln|x| + 4x^{-1/2} - x^{-1} + C = \ln|x| + \frac{4}{\sqrt{x}} - \frac{1}{x} + C.$$

50. $\displaystyle\int (x+1)^2\left(1-\frac{1}{x}\right)dx = \int \left(x^2+2x+1\right)\left(1-\frac{1}{x}\right)dx = \int \left(x^2+x-1-\frac{1}{x}\right)dx$
$= \frac{1}{3}x^3 + \frac{1}{2}x^2 - x - \ln|x| + C.$

51. $\int f'(x)\,dx = \int (3x+1)\,dx = \frac{3}{2}x^2 + x + C$. The condition $f(1) = 3$ gives $f(1) = \frac{3}{2} + 1 + C = 3$, so $C = \frac{1}{2}$. Therefore, $f(x) = \frac{3}{2}x^2 + x + \frac{1}{2}$.

52. $f(x) = \int f'(x)\,dx = \int \left(3x^2 - 6x\right)dx = x^3 - 3x^2 + C$. Using the given initial condition, we have $f(2) = 8 - 12 + C = 4$, so $C = 8$. Therefore, $f(x) = x^3 - 3x^2 + 8$.

53. $f'(x) = 3x^2 + 4x - 1$, so $f(x) = x^3 + 2x^2 - x + C$. Using the given initial condition, we have $f(2) = 8 + 2(4) - 2 + C = 9$, so $16 - 2 + C = 9$, or $C = -5$. Therefore, $f(x) = x^3 + 2x^2 - x - 5$.

54. $\displaystyle f(x) = \int f'(x)\,dx = \int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = 2x^{1/2} + C$. Using the given condition, we obtain $f(4) = 2\sqrt{4} + C = 4 + C = 2$, so $C = -2$. Therefore, $f(x) = 2\sqrt{x} - 2$.

55. $\displaystyle f(x) = \int f'(x)\,dx = \int \left(1 + \frac{1}{x^2}\right)dx = \int \left(1 + x^{-2}\right)dx = x - \frac{1}{x} + C$. Using the given initial condition, we have $f(1) = 1 - 1 + C = 3$, so $C = 3$. Therefore, $f(x) = x - \dfrac{1}{x} + 3$.

56. $f(x) = \int \left(e^x - 2x\right)dx = e^x - x^2 + C$. Using the initial condition, we have $f(0) = e^0 - 0 + C = 1 + C = 2$, so $C = 1$. Thus, $f(x) = e^x - x^2 + 1$.

57. $\displaystyle f(x) = \int \frac{x+1}{x}\,dx = \int \left(1 + \frac{1}{x}\right)dx = x + \ln|x| + C$. Using the initial condition, we have $f(1) = 1 + \ln 1 + C = 1 + C = 1$, so $C = 0$. Thus, $f(x) = x + \ln|x|$.

58. $f'(x) = 1 + e^x + \dfrac{1}{x}$, so $f(x) = xe^x + \ln|x| + C$. Using the initial condition, we have $f(1) = 1 + e + \ln 1 + C$, and so $3 + e = 1 + e + C$ and $C = 2$. Therefore, $f(x) = xe^x + \ln|x| + 2$.

59. $f(x) = \int f'(x)\,dx = \int \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2}\left(2x^{1/2}\right) + C = x^{1/2} + C$, and $f(2) = \sqrt{2} + C = \sqrt{2}$ implies $C = 0$. Thus, $f(x) = \sqrt{x}$.

60. $f(t) = \int f'(t)\,dt = \int \left(t^2 - 2t + 3\right)dt = \frac{1}{3}t^3 - t^2 + 3t + C$. $f(1) = \frac{1}{3} - 1 + 3 + C = 2$ implies $C = -\frac{1}{3}$, so $f(t) = \frac{1}{3}t^3 - t^2 + 3t - \frac{1}{3}$.

61. $f'(x) = e^x + x$, so $f(x) = e^x + \frac{1}{2}x^2 + C$ and $f(0) = e^0 + \frac{1}{2}(0) + C = 1 + C$. Thus, $3 = 1 + C$, and so $2 = C$. Therefore, $f(x) = e^x + \frac{1}{2}x^2 + 2$.

62. $\displaystyle f(x) = \int \left(\frac{2}{x} + 1\right)dx = 2\ln|x| + x + C$. $f(1) = 2\ln 1 + 1 + C = 2$, so $f(x) = 2\ln|x| + x + 1$.

63. The net amount on deposit in branch A is given by the area under the graph of f from $t = 0$ to $t = 180$. On the other hand, the net amount on deposit in branch B is given by the area under the graph of g over the same interval. Branch A has a larger amount on deposit because the rate at which money was deposited into branch A was always greater than the rate at which money was deposited into branch B over the period in question.

64. Because $f(t) \geq g(t)$ for all t in $[0, T]$, we see that the velocity of car A is always greater than or equal to that of car B. We conclude accordingly that after T seconds, car A will be ahead of car B.

65. The number in year t is $N(t) = \int R(t)\,dt = \int 14.3\,dt = 14.3t + C$. To determine C, we use the condition $N(0) = 90.1$, giving $C = 90.1$. Therefore, $N(t) = 14.3t + 90.1$. The estimated number of users in 2015 is $N(4) = 14.3(4) + 90.1 = 147.3$, or 147.3 million.

66. The percentage in year t is $P(t) = \int R(t)\,dt = \int 0.7dt = 0.7t + C$. To determine C, we use the condition $P(0) = 75$, giving $C = 75$. Therefore, $P(t) = 0.7t + 75$. The percentage of households that owned multiple sets in 2010 was $P(10) = 0.7(10) + 75 = 82$.

67. Let f be the position function of the maglev. Then $f'(t) = v(t)$. Therefore, $f(t) = \int f'(t)\,dt = \int v(t)\,dt = \int (0.2t + 3)\,dt = 0.1t^2 + 3t + C$. If we measure the position of the maglev from the station, then the required function is $f(t) = 0.1t^2 + 3t$.

68. **a.** $R(x) = \int R'(x)\,dx = \int (-0.009x + 12)\,dx = -0.0045x^2 + 12x + C$. But $R(0) = C = 0$, and so $R(x) = -0.0045x^2 + 12x$.

b. $R(x) = px$, and so $-0.0045x^2 + 12x = px$. Thus, $p = -0.0045x + 12$.

69. $P'(x) = -0.004x + 20$, so $P(x) = -0.002x^2 + 20x + C$. Because $C = -16{,}000$, we find that $P(x) = -0.002x^2 + 20x - 16{,}000$. The company realizes a maximum profit when $P'(x) = 0$, that is, when $x = 5000$ units. Next, $P(5000) = -0.002(5000)^2 + 20(5000) - 16{,}000 = 34{,}000$. Thus, the maximum profit of \$34,000 is realized at a production level of 5000 units.

70. $C(x) = \int C'(x)\,dx = \int (0.002x + 100)\,dx = 0.001x^2 + 100x + k$, but $C(0) = k = 4000$, and so $C(x) = 0.001x^2 + 100x + 4000$.

71. **a.** The amount of wind energy generated in year t is $A(t) = \int r(t)\,dt = \int (5.018t - 3.204)\,dt = 2.509t^2 - 3.204t + C$. To determine C, we use the condition $A(0) = 1.8$, giving $C = 1.8$. Therefore, $A(t) = 2.509t^2 - 3.204t + 1.8$.

b. The amount of wind energy generated in 2012 was $A(7) = 2.509(7)^2 - 3.204(7) + 1.8 = 102.313$, or approximately 102.3 terawatt-hours.

c. The amount generated in 2013 was $A(8) = 2.509(8)^2 - 3.204(8) + 1.8 = 136.744$, or approximately 136.7 terawatt-hours.

72. **a.** $f(t) = \int r(t)\,dt = \int (0.0058t + 0.159)\,dt = 0.0029t^2 + 0.159t + C$. $f(0) = 1.6$, and so $0 + 0 + C = 1.6$, or $C = 1.6$. Therefore, $f(t) = 0.0029t^2 + 0.159t + 1.6$.

b. The national health expenditure in 2015 will be $f(13) = 0.0029\left(13^2\right) + 0.159(13) + 1.6 = 4.1571$, or approximately \$4.16 trillion.

73. The total number of acres grown in year t is $N(t) = \int R(t)\,dt = \int (150t + 14.82)\,dt = 75t^2 + 14.82t + C$. Using the condition $N(0) = 27.2$, we find $N(0) = C = 27.2$. Therefore, $N(t) = 75t^2 + 14.82t + 27.2$. The number of acres grown in 2012 is given by $N(6) = 75(6)^2 + 14.82(6) + 27.2 \approx 2816.12$, or approximately 2816.1 acres.

74. The position of the car is $s(t) = \int f(t)\,dt = \int 2\sqrt{t}\,dt = \int 2t^{1/2}\,dt = 2\left(\frac{2}{3}t^{3/2}\right) + C = \frac{4}{3}t^{3/2} + C$. $s(0) = 0$ implies that $s(0) = C = 0$, so $s(t) = \frac{4}{3}t^{3/2}$.

75. **a.** $h(t) = \int h'(t)\,dt = \int (-32t + 4)\,dt = -16t^2 + 4t + C$. But $h(0) = C = 400$, so $h(t) = -16t^2 + 4t + 400$.

b. It strikes the ground when $h(t) = 0$; that is, when $-16t^2 + 4t + 400 = 0$. Using the quadratic formula, we find that $t = \dfrac{-4 \pm \sqrt{16 - 4(-16)(400)}}{2(-16)} \approx 5.13$ or -4.88. We disregard the negative root since t must be nonnegative, and conclude that $t \approx 5.13$.

c. Its velocity is $-32(5.13) + 4 = 160.16$, or approximately 160.16 ft/sec downward.

76. The rate of change of the population at any time t is $P'(t) = 4500t^{1/2} + 1000$. Therefore, $P(t) = 3000t^{3/2} + 1000t + C$. But $P(0) = 30{,}000$, and this implies that $P(t) = 3000t^{3/2} + 1000t + 30{,}000$. Finally, the projected population 9 years after the construction has begun is $P(9) = 3000(9)^{3/2} + 1000(9) + 30{,}000 = 120{,}000$.

77. The number of new subscribers at any time is $N(t) = \int \left(100 + 210t^{3/4}\right) dt = 100t + 120t^{7/4} + C$. The given condition implies that $N(0) = 5000$. Using this condition, we find $C = 5000$. Therefore, $N(t) = 100t + 120t^{7/4} + 5000$. The number of subscribers 16 months from now is $N(16) = 100(16) + 120(16)^{7/4} + 5000$, or 21,960.

78. $v(r) = \int v'(r)\,dr = \int -kr\,dr = -\frac{1}{2}kr^2 + C$. But $v(R) = -\frac{1}{2}kR^2 + C = 0$, so $C = \frac{1}{2}kR^2$. Therefore, $v(R) = -\frac{1}{2}kr^2 + \frac{1}{2}kR^2 = \frac{1}{2}k\left(R^2 - r^2\right)$.

79. $h(t) = \int h'(t)\,dt = \int \left(-3t^2 + 192t\right) dt = -t^3 + 96t^2 + C = -t^3 + 96t^2 + C$. $h(0) = C = 0$ implies $h(t) = -t^3 + 96t^2$. The altitude 30 seconds after liftoff is $h(30) = -30^3 + 96(30)^2 = 59{,}400$ ft.

80. **a.** $N(t) = \int N'(t)\,dt = \int \left(-3t^2 + 12t + 45\right) dt = -t^3 + 6t^2 + 45t + C$. But $N(0) = C = 0$, and so $N(t) = -t^3 + 6t^2 + 45t$.

b. The number is $N(4) = -4^3 + 6(4)^2 + 45(4) = 212$.

81. $C(x) = \int C'(x)\,dx = \int \left(0.000009x^2 - 0.009x + 8\right) dx = 0.000003x^3 - 0.0045x^2 + 8x + k$. $C(0) = k = 120$, and so $C(x) = 0.000003x^3 - 0.0045x^2 + 8x + 120$. Thus, $C(500) = 0.000003(500)^3 - 0.0045(500)^2 + 8(500) + 120 = \$3370$.

82. **a.** $f(x) = \int r(x)\,dx = \int \left(0.004641x^2 - 0.3012x + 4.9\right) dx = 0.001547x^3 - 0.1506x^2 + 4.9x + C$. $f(30) = 0.14$ yields $0.001547\left(30^3\right) - 0.1506\left(30^2\right) + 4.9(30) + C = 0.14$, so $C = -53.09$. Thus, $f(x) = 0.001547x^3 - 0.1506x^2 + 4.9x - 53.09$.

b. The risk of Down syndrome when the maternal age is 40 at delivery is given by $f(40) = 0.001547\left(40^3\right) - 0.1506\left(40^2\right) + 4.9(40) - 53.09 \approx 0.959$, or approximately 0.96%. When the maternal age is 45, the risk is $f(45) \approx 3.416$, or approximately 3.42%.

83. **a.** We have the initial-value problem $R'(t) = 8\sqrt{2}t^{1/2} - 32t^3$ with $R(0) = 0$. Integrating, we find $R(t) = \int \left(8\sqrt{2}t^{1/2} - 32t^3\right) dt = \frac{16\sqrt{2}}{3}t^{3/2} - 8t^4 + C$. $R(0) = 0$ implies that $C = 0$, so $R(t) = \frac{16\sqrt{2}}{3}t^{3/2} - 8t^4$.

b. $R\left(\frac{1}{2}\right) = \frac{16\sqrt{2}}{3}\left(\frac{1}{2}\right)^{3/2} - 8\left(\frac{1}{2}\right)^4 \approx 2.166$, so after $\frac{1}{2}$ hr, approximately 2.2 inches of rain had fallen.

84. a. The approximate percentage in year t is
$P(t) = \int r(t)\,dt = \int \left(-0.00025142t^2 + 0.02116t + 0.0328\right) dt = -0.000083807t^3 + 0.01058t^2 + 0.0328t + C$.
To find C, we use the condition $P(0) = 6.2$, obtaining $C = 6.2$. Therefore,
$P(t) = -0.000083807t^3 + 0.01058t^2 + 0.0328t + 6.2$.

b. The percentage is $P(53) \approx 25.1807$, or approximately 25%.

85. a. The percentage of people 12 and older using social networking sites or services in year t is
$P(t) = \int R(t)\,dt = \int 5.92t^{-0.158}\,dt \approx 7.031t^{0.842} + C$. To find C, we use the condition $P(1) = 7$, obtaining $7.031 + C = 7$, so $C = -0.031$. Therefore, $P(t) = 7.031t^{0.842} - 0.031$.

b. The percentage in 2013 was $P(5) = 7.031(5)^{0.842} - 0.031 \approx 27.23$, or approximately 27.2%.

86. a. $E(t) = \int 31.863t^{-0.61}\,dt = 81.7t^{0.39} + C$. Using the condition $E(0) = 81.7$ gives $C = 81.7$, so
$E(t) = 81.7\left(t^{0.39} + 1\right)$.

b. U.S. coal exports in 2013 were $E(3) = 81.7\left(3^{0.39} + 1\right) \approx 207.1$, or approximately 207.1 million short tons.

87. $S'(W) = 0.131773W^{-0.575}$, so $S = \int 0.131773W^{-0.575}\,dW = 0.310054W^{0.425} + C$. Now
$S(70) = 0.310054(70)^{0.425} + C = 1.886277 + C = 1.886277$, so $C = -0.000007 \approx 0$. Thus,
$S(75) = 0.310054(75)^{0.425} \approx 1.9424$.

88. $A(t) = \int A'(t)\,dt = \int \left(3.2922t^2 - 0.366t^3\right) dt = 1.0974t^3 - 0.0915t^4 + C$. Now $A(0) = C = 0$, so
$A(t) = 1.0974t^3 - 0.0915t^4$.

89. a. Let y denote the height of a typical preschool child. Then $R(t) = 25.8931e^{-0.993t} + 6.39$
and $y = \int R(t)\,dt = -\frac{25.8931}{0.993}e^{-0.993t} + 6.39t + C = -26.0756e^{-0.993t} + 6.39t + C$.
$y\left(\frac{1}{4}\right) = -26.0756e^{-(0.993)(1/4)} + 6.39\left(\frac{1}{4}\right) + C = 60.30$. Therefore, $C = 79.045$, and so
$y(t) = -26.0756e^{-0.993t} + 6.39t + 79.045$.

b. $y(1) = -26.0756e^{-0.993} + 6.39 + 79.045 \approx 75.7749$, or approximately 75.77 cm.

90. Denote the constant acceleration by k. Then if $s = f(t)$ is the position function of the car, we have $f''(t) = k$, so $f'(t) = v(t) = \int k\,dt = kt + C_1$. We have $v(0) = 66$, and this gives $C_1 = 66$. Therefore, $f'(t) = v(t) = kt + 66$, and so $s = f(t) = \int f'(t)\,dt = \int v(t)\,dt = (kt + 66)\,dt = \frac{1}{2}kt^2 + 66 + C_2$. Next we use the condition that $s = 0$ when $t = 0$ to obtain $s = f(t) = \frac{1}{2}kt^2 + 66t$. To find the time it takes for the car to go from 66 ft/sec to 88 ft/sec, we use the expression for $v(t)$ to write $88 = kt + 66$, giving $t = \frac{22}{k}$. Finally using the expression for s and the condition that the car covered 440 ft during this period, we have $440 = \frac{1}{2}k\left(\frac{22}{k}\right)^2 + 66\left(\frac{22}{k}\right) = \frac{242}{k} + \frac{1452}{k} = \frac{1694}{k}$, so $k = 3.85$. Therefore, the car was accelerating at the rate of 3.85 ft/sec^2.

91. Denote the constant deceleration by k. Then $f''(t) = -k$, so $f'(t) = v(t) = -kt + C_1$. Next, the given condition implies that $v(0) = 88$. This gives $C_1 = 88$, so $f'(t) = -kt + 88$. Now $s = f(t) = \int f'(t)\,dt = \int(-kt + 88)\,dt = -\frac{1}{2}kt^2 + 88t + C_2$, and $f(0) = 0$ gives $s = f(t) = -\frac{1}{2}kt^2 + 88t$. Because the car is brought to rest in 9 seconds, we have $v(9) = -9k + 88 = 0$, or $k = \frac{88}{9} \approx 9.78$, so the deceleration is 9.78 ft/sec^2. The distance covered is $s = f(9) = -\frac{1}{2}\left(\frac{88}{9}\right)(81) + 88(9) = 396$, so the stopping distance is 396 ft.