62. $\lim_{i\to 0} R\left[\frac{(1+i)^n-1}{i}\right] = R\lim_{i\to 0}\left[\frac{(1+i)^n-1}{i}\right]$. Consider the function $f(x)=x^n$. Then by definition of the derivative, $f'(1)=\lim_{h\to 0}\frac{(1+h)^n-1}{h}$. With the variable h taken to be i, we see that $\lim_{i\to 0} R\left[\frac{(1+i)^n-1}{i}\right] = Rf'(1) = Rnx^{n-1}\big|_{x=1} = nR$. Thus, if the interest rate is 0, then after n payments of R dollars each, the future value of the annuity will be nR dollars as expected.

63. True. With $m=1$, the effective rate is $r_{\text{eff}} = \left(1+\frac{r}{1}\right)^1 - 1 = r$.

64. False. If Susan had gotten annual increases of 5% over 5 years, her salary would have been $A = 50{,}000\,(1+0.05)^5 \approx 63{,}814.08$, or approximately \$63,814 and not \$60,000 after 5 years.

Using Technology page 370

1. \$5872.78
2. \$475.47
3. 8.95%/yr
4. 11.158%
5. \$29,743.30
6. \$94,038.74

5.4 Differentiation of Exponential Functions

Concept Questions page 376

1. a. $f'(x)=e^x$

b. $g'(x)=e^{f(x)}\cdot f'(x)$

2. a. $f'(x)=ke^{kx}$

b. If $k>0$, then $f'(x)>0$ and f is increasing on $(-\infty,\infty)$. If $k<0$, then $f'(x)<0$ and f is decreasing on $(-\infty,\infty)$.

Exercises page 376

1. $f(x)=e^{3x}$, so $f'(x)=3e^{3x}$

2. $f(x)=3e^x$, so $f'(x)=3e^x$.

3. $g(t)=e^{-t}$, so $g'(t)=-e^{-t}$

4. $f(x)=e^{-2x}$, so $f'(x)=-2e^{-2x}$.

5. $f(x)=e^x+x^2$, so $f'(x)=e^x+2x$.

6. $f(x)=2e^x-x^2$, so $f'(x)=2e^x-2x=2(e^x-x)$.

7. $f(x)=x^3e^x$, so $f'(x)=x^3e^x+e^x(3x^2)=x^2e^x(x+3)$.

8. $f(u)=u^2e^{-u}$, so $f'(u)=2ue^{-u}+u^2e^{-u}(-1)=u(2-u)e^{-u}$.

9. $f(x)=\frac{e^x}{x}$, so $f'(x)=\frac{x(e^x)-e^x(1)}{x^2}=\frac{e^x(x-1)}{x^2}$.

10. $f(x)=\frac{x}{e^x}$, so $f'(x)=\frac{e^x(1)-xe^x}{e^{2x}}=\frac{1-x}{e^x}$.

11. $f(x) = 3\left(e^x + e^{-x}\right)$, so $f'(x) = 3\left(e^x - e^{-x}\right)$.

12. $f(x) = \dfrac{e^x + e^{-x}}{2}$, so $f'(x) = \dfrac{e^x - e^{-x}}{2}$.

13. $f(w) = \dfrac{e^w + 2}{e^w} = 1 + \dfrac{2}{e^w} = 1 + 2e^{-w}$, so $f'(w) = -2e^{-w} = -\dfrac{2}{e^w}$.

14. $f(x) = \dfrac{e^x}{e^x + 1}$, so $f'(x) = \dfrac{\left(e^x + 1\right)e^x - e^x\left(e^x\right)}{\left(e^x + 1\right)^2} = \dfrac{e^x}{\left(e^x + 1\right)^2}$.

15. $f(x) = 2e^{3x-1}$, so $f'(x) = 2e^{3x-1}(3) = 6e^{3x-1}$.

16. $f(t) = 4e^{3t+2}$, so $f'(t) = 4e^{3t+2}(3) = 12e^{3t+2}$.

17. $h(x) = e^{-x^2}$, so $h'(x) = e^{-x^2}(-2x) = -2xe^{-x^2}$.

18. $f(x) = e^{x^2-1}$, so $f'(x) = e^{x^2-1}(2x) = 2xe^{x^2-1}$.

19. $f(x) = 3e^{-1/x}$, so $f'(x) = 3e^{-1/x} \cdot \dfrac{d}{dx}\left(-\dfrac{1}{x}\right) = 3e^{-1/x}\left(\dfrac{1}{x^2}\right) = \dfrac{3e^{-1/x}}{x^2}$.

20. $f(x) = e^{1/(2x)}$, so $f'(x) = e^{1/2x} \cdot \dfrac{d}{dx}\left(\dfrac{1}{2x}\right) = \frac{1}{2}e^{1/(2x)}\left(-x^{-2}\right) = -\dfrac{e^{1/(2x)}}{2x^2}$.

21. $f(x) = \left(e^x + 1\right)^{25}$, so $f'(x) = 25\left(e^x + 1\right)^{24}e^x = 25e^x\left(e^x + 1\right)^{24}$.

22. $f(x) = \left(4 - e^{-3x}\right)^3$, so $f'(x) = 3\left(4 - e^{-3x}\right)^2\left(-e^{-3x}\right)(-3) = 9e^{-3x}\left(4 - e^{-3x}\right)^2$.

23. $f(x) = e^{\sqrt{x}}$, so $f'(x) = e^{\sqrt{x}}\dfrac{d}{dx}\left(x^{1/2}\right) = e^{\sqrt{x}}\frac{1}{2}x^{-1/2} = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$.

24. $f(t) = -e^{-\sqrt{2t}}$, so $f'(t) = -e^{-\sqrt{2t}}\dfrac{d}{dt}\left(-\sqrt{2t}\right) = e^{-\sqrt{2t}}\left(\frac{1}{2}\right)(2t)^{-1/2}(2) = \dfrac{e^{-\sqrt{2t}}}{\sqrt{2t}}$.

25. $f(x) = (x - 1)e^{3x+2}$, so $f'(x) = (x - 1)(3)e^{3x+2} + e^{3x+2} = e^{3x+2}(3x - 3 + 1) = e^{3x+2}(3x - 2)$.

26. $f(s) = \left(s^2 + 1\right)e^{-s^2}$, so $f'(s) = 2se^{-s^2} + \left(s^2 + 1\right)e^{-s^2}(-2s) = -2s^3e^{-s^2}$.

27. $f(x) = \dfrac{e^x - 1}{e^x + 1}$, so $f'(x) = \dfrac{\left(e^x + 1\right)\left(e^x\right) - \left(e^x - 1\right)\left(e^x\right)}{\left(e^x + 1\right)^2} = \dfrac{e^x\left(e^x + 1 - e^x + 1\right)}{\left(e^x + 1\right)^2} = \dfrac{2e^x}{\left(e^x + 1\right)^2}$.

28. $g(t) = \dfrac{e^{-t}}{1 + t^2}$, so

$$g'(t) = \frac{\left(1 + t^2\right)\left(-e^{-t}\right) - \left(e^{-t}\right)(2t)}{\left(1 + t^2\right)^2} = \frac{e^{-t}\left(-1 - t^2 - 2t\right)}{\left(1 + t^2\right)^2} = \frac{-e^{-t}\left(t^2 + 2t + 1\right)}{\left(1 + t^2\right)^2} = \frac{-e^{-t}(t + 1)^2}{\left(1 + t^2\right)^2}.$$

29. $f(x) = e^{-4x} + e^{3x}$, so $f'(x) = -4e^{-4x} + 3e^{3x}$ and $f''(x) = 16e^{-4x} + 9e^{3x}$.

30. $f(t) = 3e^{-2t} - 5e^{-t}$, so $f'(t) = -6e^{-2t} + 5e^{-t}$ and $f''(t) = 12e^{-2t} - 5e^{-t}$.

31. $f(x) = 2xe^{3x}$, so $f'(x) = 2e^{3x} + 2xe^{3x}(3) = 2(3x+1)e^{3x}$ and $f''(x) = 6e^{3x} + 2(3x+1)e^{3x}(3) = 6(3x+2)e^{3x}$.

32. $f(t) = t^2e^{-2t}$, so $f'(t) = 2te^{-2t} + t^2e^{-2t}(-2) = 2t(1-t)e^{-2t}$ and $f''(t) = (2-4t)e^{-2t} + 2t(1-t)e^{-2t}(-2) = 2\left(2t^2-4t+1\right)e^{-2t}$.

33. $y = f(x) = e^{2x-3}$, so $f'(x) = 2e^{2x-3}$. To find the slope of the tangent line to the graph of f at $x = \frac{3}{2}$, we compute $f'\left(\frac{3}{2}\right) = 2e^{3-3} = 2$. Next, using the point-slope form of the equation of a line, we find that $y - 1 = 2\left(x - \frac{3}{2}\right) = 2x - 3$, or $y = 2x - 2$.

34. $y = e^{-x^2}$. The slope of the tangent line at any point is $y' = e^{-x^2}(-2x) = -2xe^{-x^2}$. The slope of the tangent line when $x = 1$ is $m = -2e^{-1}$. Therefore, an equation of the tangent line is $y - \frac{1}{e} = -\frac{2}{e}(x-1)$, or $y = -\frac{2}{e}x + \frac{3}{e}$.

35. $f(x) = e^{-x^2/2}$, so $f'(x) = e^{-x^2/2}(-x) = -xe^{-x^2/2}$. Setting $f'(x) = 0$, gives $x = 0$ as the only critical point of f. From the sign diagram, we conclude that f is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.

+ + + + 0 − − − − sign of f'

x: 0

36. $f(x) = x^2e^{-x}$, so $f'(x) = 2xe^{-x} + x^2e^{-x}(-1) = x(2-x)e^{-x}$. Observe that $f'(x) = 0$ if $x = 0$ or 2. The sign diagram of f' shows that f is increasing on $(0, 2)$ and decreasing on $(-\infty, 0)$ and $(2, \infty)$.

− − − 0 + + + 0 − − − sign of f'

x: 0, 2

37. $f(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$, so $f'(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$ and $f''(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$. Setting $f''(x) = 0$ gives $e^x = e^{-x}$ or $e^{2x} = 1$, and so $x = 0$. From the sign diagram for f'',we conclude that f is concave upward on $(0, \infty)$ and concave downward on $(-\infty, 0)$.

− − − − 0 + + + + sign of f''

x: 0

38. $f(x) = xe^x$, so $f'(x) = e^x + xe^x = (x+1)e^x$ and $f''(x) = (x+1)e^x + e^x = (x+2)e^x$. Setting $f''(x) = 0$ gives $x = -2$. The sign diagram of f''shows that f is concave downward on $(-\infty, -2)$ and concave upward on $(-2, \infty)$.

− − − − 0 + + + + + + sign of f''

x: −2, 0

39. $f(x) = xe^{-2x}$, so $f'(x) = e^{-2x} + xe^{-2x}(-2) = (1-2x)e^{-2x}$ and $f''(x) = -2e^{-2x} + (1-2x)e^{-2x}(-2) = 4(x-1)e^{-2x}$. Observe that $f''(x) = 0$ if $x = 1$. The sign diagram of f'' shows that $\left(1, e^{-2}\right)$ is an inflection point.

− − − − 0 + + + + sign of f''

x: 1

40. $f(x) = 2e^{-x^2} = 2\left(e^{-x^2}\right)$, so $f'(x) = 2(-2x)e^{-x^2} = -4xe^{-x^2}$ and $f''(x) = -4x(-2x)e^{-x^2} - 4e^{-x^2} = -4e^{-x^2}(-2x^2+1) = 4e^{-x^2}(2x^2-1)$. Setting $f''(x) = 0$ gives $2x^2 = 1$, $x^2 = \frac{1}{2}$, and so $x = \pm\frac{\sqrt{2}}{2}$. The sign diagram for f'' shows that $\left(-\frac{\sqrt{2}}{2}, 2e^{-1/2}\right)$ and $\left(\frac{\sqrt{2}}{2}, 2e^{-1/2}\right)$ are inflection points.

+ + + 0 − − − − − − 0 + + + sign of f''

$-\frac{\sqrt{2}}{2}$ 0 $\frac{\sqrt{2}}{2}$ x

41. $f(x) = e^{-x^2}$, so $f'(x) = -2xe^{-x^2}$ and $f''(x) = -2e^{-x^2} - 2xe^{-x^2}(-2x) = -2e^{-x^2}(1-2x^2) = 0$ implies $x = \pm\frac{\sqrt{2}}{2}$. The sign diagram of f'' shows that the graph of f has inflection points at $\left(-\frac{\sqrt{2}}{2}, e^{-1/2}\right)$ and $\left(\frac{\sqrt{2}}{2}, e^{-1/2}\right)$. The slope of the tangent line at $\left(-\frac{\sqrt{2}}{2}, e^{-1/2}\right)$ is $f'\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}e^{-1/2}$, and the tangent line has equation $y - e^{-1/2} = \sqrt{2}e^{-1/2}\left(x + \frac{\sqrt{2}}{2}\right)$, which can be simplified to $y = e^{-1/2}\left(\sqrt{2}x + 2\right)$. The slope of the tangent line at $\left(\frac{\sqrt{2}}{2}, e^{-1/2}\right)$ is $f'\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}e^{-1/2}$, and this tangent line has equation $y - e^{-1/2} = -\sqrt{2}e^{-1/2}\left(x - \frac{\sqrt{2}}{2}\right)$ or $y = e^{-1/2}\left(-\sqrt{2}x + 2\right)$.

+ + + 0 − − − − − − 0 + + + sign of f''

$-\frac{\sqrt{2}}{2}$ 0 $\frac{\sqrt{2}}{2}$ x

42. $f(x) = xe^{-x}$, so $f'(x) = e^{-x} + xe^{-x}(-1) = (1-x)e^{-x}$ and $f''(x) = -e^{-x} + (1-x)e^{-x}(-1) = (x-2)e^{-x} = 0$ implies $x - 2 = 0$ or $x = 2$. The sign diagram of f'' shows that the graph of f has an inflection point at $(2, 2e^{-2})$. The slope of the tangent line at that point is $f'(2) = -e^{-2}$. The tangent line has equation $y - 2e^{-2} = -e^{-2}(x-2) = -e^{-2}x + 4e^{-2}$ or $y = -\frac{1}{e^2}x + \frac{4}{e^2}$.

− − − − 0 + + + + sign of f''

2 x

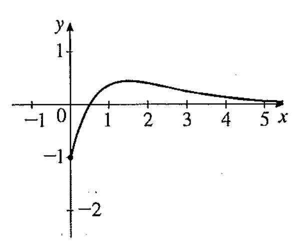
43. $f(x) = e^{-x^2}$, so $f'(x) = -2xe^{-x^2} = 0$ if $x = 0$, the only critical point of f. From the table, we see that f has an absolute minimum value of e^{-1} attained at $x = -1$ and $x = 1$. It has an absolute maximum at $(0, 1)$.

x	-1	0	1
$f(x)$	e^{-1}	1	e^{-1}

44. $h(x) = e^{x^2-4}$, so $h'(x) = 2xe^{x^2-4}$. Setting $h'(x) = 0$ gives $x = 0$ as the only critical point of h. We see that $h(0) = e^{-4}$ is the absolute minimum and $h(-2) = h(2) = 1$ are the absolute maxima of h.

x	-2	0	2
$h(x)$	1	e^{-4}	1

45. $g(x) = (2x-1)e^{-x}$, so $g'(x) = 2e^{-x} + (2x-1)e^{-x}(-1) = (3-2x)e^{-x} = 0$ if $x = \frac{3}{2}$. The graph of g shows that $\left(\frac{3}{2}, 2e^{-3/2}\right)$ is an absolute maximum, and $(0, -1)$ is an absolute minimum.



46. $f(x) = xe^{-x^2}$, so

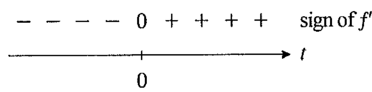
$f'(x) = e^{-x^2} + xe^{-x^2}(-2x) = (1 - 2x^2)e^{-x^2} = 0$ if $x = \pm\frac{\sqrt{2}}{2}$.

From the table, we see that f has an absolute minimum at $(0, 0)$ and an absolute maximum at $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}e^{-1/2}\right)$.

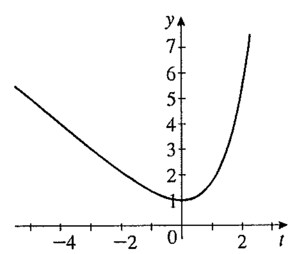
x	0	$\frac{\sqrt{2}}{2}$	2
$f(x)$	0	$\frac{\sqrt{2}}{2}e^{-1/2}$	$2e^{-4}$

47. $f(t) = e^t - t$. We first gather the following information on f.

1. The domain of f is $(-\infty, \infty)$.
2. Setting $t = 0$ gives 1 as the y-intercept.
3. $\lim_{t \to -\infty} (e^t - t) = \infty$ and $\lim_{t \to \infty} (e^t - t) = \infty$.
4. There is no asymptote.
5. $f'(t) = e^t - 1$ if $t = 0$, a critical point of f. From the sign diagram for f', we see that f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

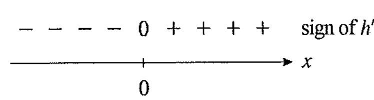


6. From the results of part 5, we see that $(0, 1)$ is a relative minimum of f.
7. $f''(t) = e^t > 0$ for all t, so the graph of f is concave upward on $(-\infty, \infty)$.
8. There is no inflection point.

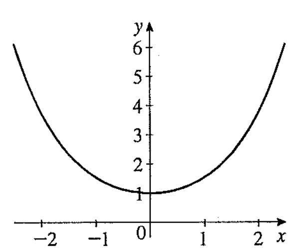


48. $h(x) = \dfrac{e^x + e^{-x}}{2}$. We first gather the following information on h.

1. The domain of h is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 1 as the y-intercept.
3. $\lim_{x \to -\infty} h(x) = \lim_{x \to \infty} h(x) = \infty$.
4. There is no asymptote.
5. $h'(x) = \frac{1}{2}(e^x - e^{-x}) = 0$ implies $e^x = e^{-x}$, $e^{2x} = 1$, and $x = 0$, a critical point of h. The sign diagram of h' shows that h is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

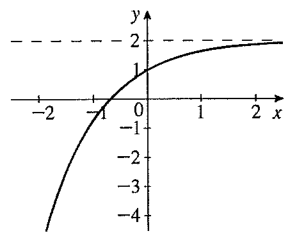


6. The results of part 5 show that $(0, 1)$ is a relative minimum of h.
7. $h''(x) = \frac{1}{2}(e^x + e^{-x})$ is always positive, so the graph of h is concave upward everywhere.
8. The results of part 7 show that h has no inflection point.



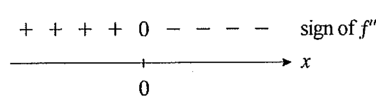
49. $f(x) = 2 - e^{-x}$. We first gather the following information on f.

1. The domain of f is $(-\infty, \infty)$.
2. Setting $x = 0$ gives 1 as the y-intercept.
3. $\lim_{x\to-\infty}(2 - e^{-x}) = -\infty$ and $\lim_{x\to\infty}(2 - e^{-x}) = 2$.
4. From the results of part 3, we see that $y = 2$ is a horizontal asymptote of f.
5. $f'(x) = e^{-x} > 0$ for all x in $(-\infty, \infty)$, so f is increasing on $(-\infty, \infty)$.
6. Because there is no critical point, f has no relative extremum.
7. $f''(x) = -e^{-x} < 0$ for all x in $(-\infty, \infty)$ and so the graph of f is concave downward on $(-\infty, \infty)$.
8. There is no inflection point.

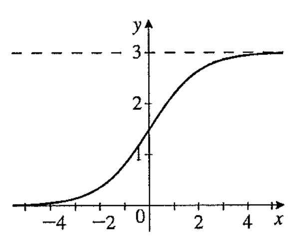


50. $f(x) = \dfrac{3}{1 + e^{-x}}$. We first gather the following information on f.

1. The domain of f is $(-\infty, \infty)$.
2. Letting $x = 0$ gives $\frac{3}{2}$ as the y-intercept.
3. $\lim_{x\to-\infty}\dfrac{3}{1 + e^{-x}} = 0$ and $\lim_{x\to\infty}\dfrac{3}{1 + e^{-x}} = 3$.
4. From the results of part 3, we see that $y = 0$ and $y = 3$ are horizontal asymptotes of f.
5. $f'(x) = 3\dfrac{d}{dx}\left(1 + e^{-x}\right)^{-1} = -3\left(1 + e^{-x}\right)^{-2}\left(e^{-x}\right)(-1) = \dfrac{3e^{-x}}{\left(1 + e^{-x}\right)^2}$. Observe that $f'(x) > 0$ for all x in $(-\infty, \infty)$, so f is increasing on $(-\infty, \infty)$.
6. f has no relative extremum since there is no critical point.
7. $$f''(x) = \frac{\left(1 + e^{-x}\right)^2\left(-3e^{-x}\right) - 3e^{-x}(2)\left(1 + e^{-x}\right)\left(-e^{-x}\right)}{\left(1 + e^{-x}\right)^4} = \frac{3e^{-x}\left(1 + e^{-x}\right)\left[2e^{-x} - \left(1 + e^{-x}\right)\right]}{\left(1 + e^{-x}\right)^4}$$
$$= \frac{3e^{-x}\left(e^{-x} - 1\right)}{\left(1 + e^{-x}\right)^3}.$$
Observe that $f''(x) = 0$ if $e^{-x} = 1$, or $x = 0$. The sign diagram of f'' shows that f is concave upward on $(-\infty, 0)$ and concave downward on $(0, \infty)$.



8. The results of part 7 show that $\left(0, \frac{3}{2}\right)$ is an inflection point of f.



51. $x^2 + y^3 = 2e^{2y}$, so $2x + 3y^2y' = 4e^{2y}y'$ and $y'\left(4e^{2y} - 3y^2\right) = 2x$. Thus, $\dfrac{dy}{dx} = \dfrac{2x}{4e^{2y} - 3y^2}$.

52. $xy^2 + \sqrt{x}e^y = 10$, so $y^2 + 2xy\frac{dy}{dx} + \frac{1}{2}x^{-1/2}e^y + x^{1/2}e^y\frac{dy}{dx} = 0$. Multiplying by $x^{1/2}$, we have $x^{1/2}y^2 + 2x^{3/2}y\frac{dy}{dx} + \frac{1}{2}e^y + xe^y\frac{dy}{dx} = 0$, $\left(2x^{3/2}y + xe^y\right)\frac{dy}{dx} = -\left(\frac{1}{2}e^y + x^{1/2}y^2\right)$, and finally $\frac{dy}{dx} = -\frac{e^y + 2\sqrt{x}y^2}{2x\left(\sqrt{x}y + e^y\right)}$.

53. $x = y + e^y$, so $1 = y' + e^y y' = (1 + e^y)\,y'$. Differentiating again, $0 = \left(e^y y'\right)y' + (1 + e^y)\,y''$, and so $\frac{d^2y}{dx^2} = -\frac{e^y\,(dy/dx)^2}{1 + e^y}$. From our first differentiation, we have $\frac{dy}{dx} = \frac{1}{1 + e^y}$, so we can write $\frac{d^2y}{dx^2} = -\frac{e^y}{(1 + e^y)^3}$.

54. $e^x - e^y = y - x$, so $e^x - e^y y' = y' - 1$ and $(1 + e^y)\,y' = e^x + 1$. Differentiating again, $\left(e^y y'\right)y' + (1 + e^y)\,y'' = e^x$, so $y'' = \frac{e^x - e^y\left(y'\right)^2}{1 + e^y}$; that is, $\frac{d^2y}{dx^2} = \frac{e^x - e^y\,(dy/dx)^2}{1 + e^y}$.

55. $xy + e^y = e$, so $y + x\frac{dy}{dx} + e^y\frac{dy}{dx} = 0$. When $x = 0$ and $y = 1$, we have $1 + 0\cdot\frac{dy}{dx} + e^1\frac{dy}{dx} = 0$, so $e\frac{dy}{dx} = -1$ and $\frac{dy}{dx} = -\frac{1}{e}$.

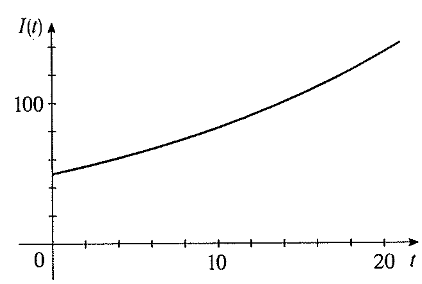
56. $x + y - e^{x-y} = 1$, so $1 + y' - e^{x-y}\left(1 - y'\right) = 0$. Substituting $x = y = 1$ into this equation yields $1 + y' - e^0\left(1 - y'\right) = 0$, so $1 + y' - \left(1 - y'\right) = 0$ and $y' = 0$. An equation of the tangent line is thus $y - 1 = 0\,(x - 1)$ or $y = 1$.

57. a. The number of video viewers in 2012 was $N(5) = 135e^{0.067(5)}$ or approximately 188.7 million.

b. $N'(t) = 135\,(0.067)\,e^{0.067t} = 9.045e^{0.067t}$, so the number of viewers was changing at the rate of $N'(5) = 9.045e^{0.067(5)}$ or approximately 12.6 million viewers per year in 2012.

58. a. In 1990, the index was $I(0) = 50e^{0.05(0)} = 50$. In 2011, it was $I(21) = 50e^{0.05(21)} \approx 143$.

b.



c. $I'(t) = 50\,(0.05)\,e^{0.05t}$, so the index was changing at the rate of $I'(10) \approx 4$, or 4 per year.

59. We find $f'(t) = \frac{d}{dt}\left(20.5e^{0.74t}\right) = 20.5\left(0.74e^{0.74t}\right) = 15.17e^{0.74t}$, so $f'(2) = 15.17e^{0.74(2)} \approx 66.64$. Thus, the value of stolen drugs is increasing at the rate of \$66.64 million per year at the beginning of 2008.

60. a. $G(t) = 1.58e^{-0.213t}$. The projected annual average population growth rate in 2020 will be $G(3) \approx 0.834$, or approximately 0.83%/decade.

b. $G'(t) = -0.33654e^{-0.213t}$, so the projected annual average population growth rate will be changing at the rate of $G'(3) = -0.1776$, that is, it is decreasing at approximately 0.18%/decade/decade.

61. a. $S(t) = 20{,}000\left(1 + e^{-0.5t}\right)$, so $S'(t) = 20{,}000\left(-0.5e^{-0.5t}\right) = -10{,}000e^{-0.5t}$. Thus, $S'(1) = -10{,}000e^{-0.5} \approx -6065$, or $-\$6065$/day/day; $S'(2) = -10{,}000e^{-1} \approx -3679$, or $-\$3679$/day/day; $S'(3) = -10{,}000\left(e^{-1.5}\right) \approx -2231$, or $-\$2231$/day/day; and $S'(4) = -10{,}000e^{-2} \approx -1353$, or $-\$1353$/day/day.

b. $S(t) = 20{,}000\left(1 + e^{-0.5t}\right) = 27{,}400$, so $1 + e^{-0.5t} = \frac{27{,}400}{20{,}000}$, $e^{-0.5t} = \frac{274}{200} - 1$, $-0.5t = \ln\left(\frac{274}{200} - 1\right)$, and so

$$t = \frac{\ln\left(\frac{274}{200} - 1\right)}{-0.5} \approx 2, \text{ or 2 days.}$$

62. a. $A(t) = 0.23te^{-0.4t}$, so $A\left(\frac{1}{2}\right) = 0.23\left(\frac{1}{2}\right)e^{-0.2} \approx 0.094$ and $A(8) = 0.23(8)e^{-3.2} \approx 0.075$.

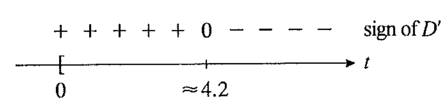
b. $A'(t) = 0.23\left[t(-0.4)e^{-0.4t} + e^{-0.4t}\right] = 0.23e^{-0.4t}(-0.4t + 1)$. Thus, $A'\left(\frac{1}{2}\right) = 0.23e^{-0.2}(0.8) = 0.151$ and $A'(8) = 0.23e^{-3.2}(-2.2) \approx -0.021$.

63. $N(t) = 5.3e^{0.095t^2 - 0.85t}$.

a. $N'(t) = 5.3e^{0.095t^2 - 0.85t}(0.19t - 0.85)$. Because $N'(t)$ is negative for $0 \le t \le 4$, we see that $N(t)$ is decreasing over that interval.

b. To find the rate at which the number of polio cases was decreasing at the beginning of 1959, we compute $N'(0) = 5.3e^{0.095(0^2) - 0.85(0)}(0.85) \approx 5.3(-0.85) = -4.505$, or 4505 cases per year per year (t is measured in thousands). To find the rate at which the number of polio cases was decreasing at the beginning of 1962, we compute $N'(3) = 5.3e^{0.095(9) - 0.85(3)}(0.57 - 0.85) \approx (-0.28)(0.9731) \approx -0.273$, or 273 cases per year per year.

64. a. $D'(t) = \frac{d}{dt}\left(6.9te^{-0.24t}\right) = 6.9(1 - 0.24t)e^{-0.24t}$. Setting $D'(t) = 0$ gives $6.9(1 - 0.24t) = 0$, $-0.24t = -1$, or $t \approx 4.2$. The sign diagram for $D'(t)$ shows that D is increasing on $(0, 4.2)$ and decreasing on $(4.2, 40)$. Thus, the difference in size between the autistic brain and the normal brain increases between the time of birth and 4.2 years of age and decreases thereafter.

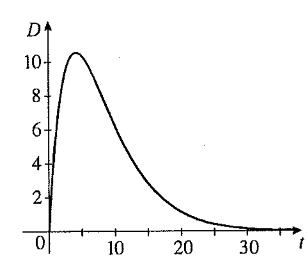


b. It is greatest at age 4.2. The difference is $D(4.2) \approx 10.6$, or 10.6%.

c.
$$D''(t) = 6.9\left[-0.24e^{-0.24t} + (1 - 0.24t)(-0.24)e^{-0.24t}\right]$$
$$= 6.9(-0.48 + 0.0576t)e^{-0.24t} = 0.$$

Setting $D''(t) = 0$ gives $-0.48 + 0.0576t = 0$, or $t \approx 8.33$. Because $D''(t) < 0$ if $0 \le t < 8.33$ and $D''(t) > 0$ if $t > 8.33$, we see that $(8.33, 7.78)$ is an inflection point of D. So the difference between the size of the autistic brain and the normal brain is decreasing at the fastest rate at age 8.3.

d.



65. a. The frequency for a 70 year old is given by $f(1) = 0.71e^{0.71(1)} \approx 1.43$ (%), and for a 90 year old it is $f(5) = 0.71e^{0.71(5)} \approx 23.51$ (%).

b. $f'(t) = \frac{d}{dt}\left(0.71e^{0.7t}\right) = 0.7\left(0.71e^{0.7t}\right) = 0.497e^{0.7t}$, which is positive for $0 < t < 5$, and so f is increasing on $(0, 5)$. This says that the frequency of Alzheimer's disease increases with age in the age range under consideration.

c. $f''(t) = \frac{d}{dt}\left(0.497e^{0.7t}\right) = 0.3479e^{0.7t}$, which is positive for $0 < t < 5$, and so f is concave upward on $(0, 5)$. This says that the frequency of Alzheimer's disease is increasing at an increasing rate in the age range under consideration.

66. a. The population aged 75 and over in 2010 was $f(0) = \dfrac{72.15}{1 + 2.7975e^{0}} \approx 19.00$, or approximately 19 million.

b.
$$f'(t) = 72.15\frac{d}{dt}\left(1 + 2.7975e^{-0.02145t}\right)^{-1}$$
$$= 72.15(-1)\left(1 + 2.7975e^{-0.02145t}\right)^{-2}(2.7975)(-0.02145)e^{-0.02145t}$$
$$= \frac{4.32946e^{-0.02145t}}{\left(1 + 2.7975e^{-0.02145t}\right)^{2}}$$

Thus, the population aged 75 and over is expected to be growing at the rate of $f'(20) \approx 0.35$, or 350,000 per year in 2030.

c. The population aged 75 and over in 2030 is expected to be $f(20) = \dfrac{72.15}{1 + 2.7975e^{-0.02145(20)}} \approx 25.57$, or 25.57 million.

67. a. $R(x) = px = 100xe^{-0.0001x}$.

b. $R'(x) = 100e^{-0.0001x} + 100xe^{-0.0001x}(-0.0001) = 100(1 - 0.0001x)e^{-0.0001x}$.

c. $R'(10{,}000) = 100[1 - 0.0001(10{,}000)]e^{-0.001} = 0$, or \$0/pair.

68. a. $N(t) = 130.7e^{-0.1155t^2} + 50$. The number of deaths in 1950 was $N(0) = 130.7 + 50 = 180.7$, or approximately 181 per 100,000 people.

b.
$$N'(t) = (130.7)(-0.1155)(2t)e^{-0.1155t^2}$$
$$= -30.1917te^{-0.1155t^2}.$$

The rates of change of the number of deaths per 100,000 people per decade are given in the table.

Year	1950	1960	1970	1980
Rate	0	−27	−38	−32

c. $N''(t) = -30.1917\left[e^{-0.1155t^2} + t(-0.1155)(2t)e^{-0.1155t^2}\right] = -30.1917\left(1 - 0.231t^2\right)e^{-0.1155t^2}$. Setting $N''(t) = 0$ gives $t \approx \pm 2.08$. So $t \approx 2$ gives an inflection point, and we conclude that the decline was greatest around 1970.

d. The number is given by $N(6) \approx 52.04$, or approximately 52.

69. The demand equation is $p(x) = 100e^{-0.0002x} + 150$. Next, $p'(x) = 100(-0.0002)e^{-0.0002x} = -0.02e^{-0.0002x}$.

a. To find the rate of change of the price per bottle when $x = 1000$, we compute $p'(1000) = -0.02e^{-0.0002(1000)} = -0.02e^{-0.2} \approx -0.0164$, or -1.64 cents per bottle. To find the rate of change of the price per bottle when $x = 2000$, we compute $p'(2000) = -0.02e^{-0.0002(2000)} = -0.02e^{-0.4} \approx -0.0134$, or -1.34 cents per bottle.

b. The price per bottle when $x = 1000$ is given by $p(1000) = 100e^{-0.0002(1000)} + 150 \approx 231.87$, or \$231.87/bottle. The price per bottle when $x = 2000$ is given by $p(2000) = 100e^{-0.0002(2000)} + 150 \approx 217.03$, or \$217.03/bottle.

70. a. $p = 240\left(1 - \dfrac{3}{3 + e^{-0.0005x}}\right) = 240\left[1 - 3\left(3 + e^{-0.0005x}\right)^{-1}\right]$,

so $p' = 720\left(3 + e^{-0.0005x}\right)^{-2}\left(-0.0005e^{-0.0005x}\right)$. Thus,

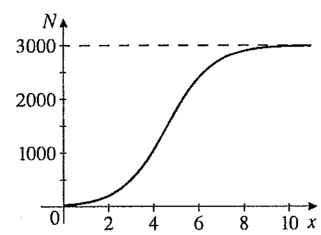
$p'(1000) = 720\left(3 + e^{-0.0005\cdot 1000}\right)^{-2}\left(-0.0005e^{-0.0005\cdot 1000}\right) = -\dfrac{0.36\,(0.606531)}{(3 + 0.606531)^2} \approx -0.0168$, or -1.68 cents per case per case.

b. $p(1000) = 240\left(1 - \dfrac{3}{3.606531}\right) \approx 40.36$, or \$40.36/case.

71. a. $N(0) = \dfrac{3000}{1 + 99} = 30$.

b. $N'(x) = 3000\dfrac{d}{dx}\left(1 + 99e^{-x}\right)^{-1} = -3000\left(1 + 99e^{-x}\right)^{-2}\left(-99e^{-x}\right) = \dfrac{297{,}000e^{-x}}{\left(1 + 99e^{-x}\right)^2}$. Because $N'(x) > 0$ for all x in $(0, \infty)$, we see that N is increasing on $(0, \infty)$.

c. From the graph of N, we see that the total number of students who contracted influenza during that particular epidemic is approximately $\lim_{x\to\infty} \dfrac{3000}{1 + 99e^{-x}} = 3000$.



72. a. The number of units sold 24 months after introduction was $N(24) = 20{,}000\left[1 - e^{-0.05(24)}\right]^2 \approx 9766.59$, or approximately 9767.

b. $N'(t) = 20{,}000(2)\left(1 - e^{-0.05t}\right)\left[-(-0.05)e^{-0.05t}\right] = 2000e^{-0.05t}\left(1 - e^{-0.05t}\right)$. Thus, 24 months after its introduction, the product was selling at the rate of $N'(24) \approx 420.95$, or approximately 421 units per month.

73. a. Here $f(p) = 50e^{-0.02p}$, so $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{p\left(50e^{-0.02p}\right)(-0.02)}{50e^{-0.02p}} = 0.02p$.

b. Using the result from part (a), we see that $E(p) = 1$ if $0.02p = 1$, or $p = 50$; $E(p) < 1$ if $p < 50$; and $E(p) > 1$ if $p > 50$. Demand is inelastic if $0 < p < 50$, unitary if $p = 50$, and elastic if $p > 50$.

c. Because demand is inelastic when $p = 40$, decreasing the unit price slightly will cause revenue to decrease.

d. Because demand is elastic when $p = 60$, increasing the unit price slightly will cause revenue to decrease.

74. a. Using Equation 7 from Section 3.4 with $f(p) = ae^{-bp}$, we have $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{pae^{-bp}(-b)}{ae^{-bp}} = bp$.

b. Using the result from part (a), we see that $bp = 1$ if $p = 1/b$, showing that demand is unitary if $p = 1/b$. Because $bp > 1$ if $p \geq 1/b$ and $bp < 1$ if $p \leq 1/b$, we see that demand is elastic if $p > 1/b$ and inelastic if $0 < p < 1/b$.

75. **a.** $W = 2.4e^{1.84h}$, so if $h = 1.6$, $W = 2.4e^{1.84(1.6)} \approx 45.58$, or approximately 45.6 kg.

b. $\Delta W \approx dW = (2.4)(1.84)e^{1.84h}\,dh$. With $h = 1.6$ and $dh = \Delta h = 1.65 - 1.6 = 0.05$, we find $\Delta W \approx (2.4)(1.84)e^{1.84(1.6)} \cdot (0.05) \approx 4.19$, or approximately 4.2 kg.

76. The number of people is

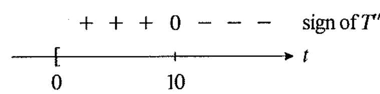
$$\Delta P \approx f'(10)\,\Delta x = 50{,}000\frac{d}{dx}\left(1 - e^{-0.01x^2}\right)(0.1) = 50{,}000\left(0.02xe^{-0.01x^2}\right)\Big|_{x=10}(0.1)$$
$$= (50{,}000)(0.02)(10)(0.3679)(0.1) \approx 367.9, \text{ or 368 people.}$$

77. $P(t) = 80{,}000e^{\sqrt{t}/2-0.09t} = 80{,}000e^{(1/2)t^{1/2}-0.09t}$, so $P'(t) = 80{,}000\left(\frac{1}{4}t^{-1/2} - 0.09\right)e^{(1/2)t^{1/2}-0.09t}$. Setting $P'(t) = 0$, we have $\frac{1}{4}t^{-1/2} = 0.09$, so $t^{-1/2} = 0.36$, $\frac{1}{\sqrt{t}} = 0.36$, and $t = \left(\frac{1}{0.36}\right)^2 \approx 7.72$. Evaluating $P(t)$ at each of its endpoints and at the point $t = 7.72$, we find $P(0) = 80{,}000$, $P(7.72) \approx 160{,}207.69$, and $P(8) \approx 160{,}170.71$. We conclude that P is optimized at $t = 7.72$. The optimal price is approximately \$160,208.

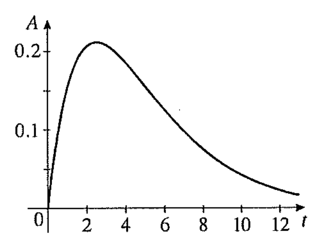
78. We want to find the maximum of dT/dt:

$T'(t) = -1000\frac{d}{dt}\left[(t+10)e^{-0.1t} + 10{,}000\right] = -1000\left[e^{-0.1t} + (t+10)e^{-0.1t}(-0.1)\right] = 100te^{-0.1t}$, so

$T''(t) = 100\frac{d}{dt}\left(te^{-0.1t}\right) = 100\left[e^{-0.1t} + te^{-0.1t}(-0.1)\right] = 100e^{-0.1t}(1 - 0.1t)$. Observe that $T''(t) = 0$ if $t = 10$, a critical number of T'. From the sign diagram of T'', we see that $t = 10$ gives a relative maximum of T'. This is in fact an absolute maximum. Thus, the maximum production will be reached in the tenth year of operation.



79. $A(t) = 0.23te^{-0.4t}$, so $A'(t) = 0.23(1 - 0.4t)e^{-0.4t}$. Setting $A'(t) = 0$ gives $t = \frac{1}{0.4} = \frac{5}{2}$. From the graph of A, we see that the proportion of alcohol is highest $2\frac{1}{2}$ hours after drinking. The level is given by $A\left(\frac{5}{2}\right) \approx 0.2115$, or approximately 0.21%.



80. **a.** $p = 8 + 4e^{-2t} + te^{-2t}$, so the price at $t = 0$ is $8 + 4$, or \$12 per unit.

b. $\frac{dp}{dt} = -8e^{-2t} + e^{-2t} - 2te^{-2t}$, so $\left.\frac{dp}{dt}\right|_{t=0} = -8e^{-2t} + e^{-2t} - 2te^{-2t}\big|_{t=0} = -8 + 1 = -7$. Thus, the price is decreasing at the rate of \$7/week.

c. The equilibrium price is $\lim_{t\to\infty}\left(8 + 4e^{-2t} + te^{-2t}\right) = 8 + 0 + 0$, or \$8 per unit.

81. **a.** The temperature inside the house is given by $T(0) = 30 + 40e^0 = 70$, or 70°F.

b. The reading is changing at the rate of $T'(1) = 40(-0.98)e^{-0.98t}\big|_{t=1} \approx -14.7$. Thus, it is dropping at the rate of approximately 14.7°F/min.

c. The temperature outdoors is given by $\lim_{t\to\infty} T(t) = \lim_{t\to\infty}\left(30 + 40e^{-0.98t}\right) = 30 + 0 = 30$, or 30°F.

82. $\frac{dy}{dt} = \frac{d}{dt}\left[(y_0 - C)e^{-kt/V} + C\right] = (y_0 - C)\left(-\frac{k}{V}\right)e^{-kt/V} = \frac{k(C - y_0)}{V}e^{-kt/V}$ (g/cc per unit time).

83. $A(t) = \dfrac{150\left(1 - e^{0.022662t}\right)}{1 - 2.5e^{0.022662t}}$.

a. Let $k = 0.022662$. Then $A'(t) = 150\dfrac{d}{dt}\left[\dfrac{1 - e^{kt}}{1 - 2.5e^{kt}}\right] = 150\dfrac{\left(1 - 2.5e^{kt}\right)\left(-ke^{kt}\right) - \left(1 - e^{kt}\right)\left(-2.5ke^{kt}\right)}{\left(1 - 2.5e^{kt}\right)^2}$.

Thus, the rate of formation of chemical C one minute after the interaction begins is

$A'(1) = \dfrac{5.09895e^{0.022662}}{\left(1 - 2.5e^{0.022662}\right)^2} \approx 2.15$, or 2.15 g/min.

b. The amount of chemical C that is eventually formed is

$\lim_{t\to\infty} A(t) = 150 \lim_{t\to\infty} \dfrac{1 - e^{0.022662t}}{1 - 2.5e^{0.022662t}} = 150 \lim_{t\to\infty} \dfrac{e^{-0.022662t} - 1}{e^{-0.022662t} - 2.5} = \dfrac{150}{2.5} = 60$ g.

84. a. $y = c\left(e^{-bt} - e^{-at}\right)$, so $y' = c\left(-be^{-bt} + ae^{-at}\right) = ca\left(-\frac{b}{a}e^{-bt} + e^{-at}\right) = cae^{-at}\left[-\frac{b}{a}e^{(a-b)t} + 1\right]$. Setting $y' = 0$ gives $-\dfrac{b}{a}e^{(a-b)t} + 1 = 0$, $e^{(a-b)t} = \dfrac{a}{b}$, $\ln e^{(a-b)t} = \ln\dfrac{a}{b}$, and so $t = \dfrac{\ln\frac{a}{b}}{a-b}$. Because $y(0) = 0$ and $\lim_{t\to\infty} y = 0$, $t = \dfrac{\ln\frac{a}{b}}{a-b}$ gives the time at which the concentration is maximal.

b. $y'' = c\left(b^2e^{-bt} - a^2e^{-at}\right) = ca^2e^{-at}\left[\dfrac{b^2}{a^2}e^{(a-b)t} - 1\right]$. Setting $y'' = 0$ gives $e^{(a-b)t} = \dfrac{a^2}{b^2}$, so $t = \dfrac{2\ln\frac{a}{b}}{a-b}$.

From the sign diagram of y'', we see that the concentration of the drug is decreasing most rapidly when $t = \dfrac{2\ln\frac{a}{b}}{a-b}$ seconds.

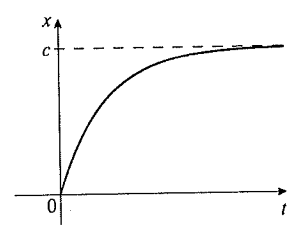
$- - - -\ 0\ + + + +$ sign of y''

$0 \qquad \dfrac{2\ln\frac{a}{b}}{a-b} \qquad t$

85. a. $x(t) = c\left(1 - e^{-at/V}\right)$, so

$x'(t) = \dfrac{d}{dt}\left(c - ce^{-at/V}\right) = \dfrac{ac}{V}e^{-at/V}$. Because $a > 0$, $c > 0$, and $V > 0$, we see that $x'(t)$ is always positive and we conclude that $x(t)$ is always increasing.

b.

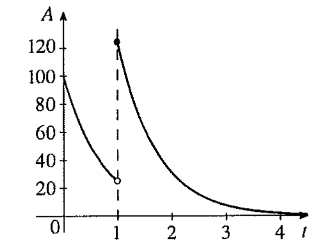


86. We are given that $c\left(1 - e^{-at/V}\right) < m$, so $1 - e^{-at/V} < \dfrac{m}{c}$, $-e^{-at/V} < \dfrac{m}{c} - 1$, and $e^{-at/V} > 1 - \dfrac{m}{c}$. Taking logarithms of both sides of the inequality, we have $-\dfrac{at}{V}\ln e > \ln\dfrac{c-m}{c}$, $-\dfrac{at}{V} > \ln\dfrac{c-m}{c}$, $-t > \dfrac{V}{a}\ln\dfrac{c-m}{c}$, and so $t < \dfrac{V}{a}\left(-\ln\dfrac{c-m}{c}\right) = \dfrac{V}{a}\ln\dfrac{c}{c-m}$. Therefore, the liquid must not be allowed to enter the organ for longer than $t = \dfrac{V}{a}\ln\dfrac{c}{c-m}$ minutes.

87. a. $A(t) = \begin{cases} 100e^{-1.4t} & \text{if } 0 \le t < 1 \\ 100\left(1 + e^{1.4}\right)e^{-1.4t} & \text{if } t \ge 1 \end{cases}$ so $A'(t) = \begin{cases} -140e^{-1.4t} & \text{if } 0 < t < 1 \\ -140\left(1 + e^{1.4}\right)e^{-1.4t} & \text{if } t > 1 \end{cases}$ Thus, after 12 hours the amount of drug is changing at the rate of $A'\left(\frac{1}{2}\right) = -140e^{-0.7} \approx -69.52$, or decreasing at the rate of 70 mg/day. After 2 days, it is changing at the rate of $A'(2) = -140\left(1 + e^{1.4}\right)e^{-2.8} \approx -43.04$, or decreasing at the rate of 43 mg/day.

b. From the graph of A, we see that the maximum occurs at $t = 1$, that is, at the time when she takes the second dose.

c. The maximum amount is $A(1) = 100\left(1 + e^{1.4}\right)e^{-1.4} \approx 124.66$, or 125 mg.



88. False. $f(x) = 3^x = e^{x \ln 3}$, and so $f'(x) = e^{x \ln 3} \frac{d}{dx}(x \ln 3) = (\ln 3) e^{x \ln 3} = (\ln 3) 3^x$.

89. False. $f(x) = e^{\pi}$ is a constant function and so $f'(x) = 0$.

90. False. $f'(x) = (\ln \pi) \pi^x$. See Exercise 88.

91. False. $f'(x) = \frac{d}{dx}\left(e^{x^2+x}\right) = (2x+1) e^{x^2+x}$.

92. True. Differentiating both sides of the equation with respect to x, we have $\dfrac{d}{dx}\left(x^2 + e^y\right) = \dfrac{d}{dx}(10)$, so $2x + e^y \dfrac{dy}{dx} = 0$ and thus $\dfrac{dy}{dx} = -\dfrac{2x}{e^y}$.

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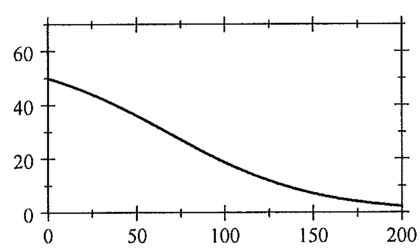
1. 5.4366 **2.** −0.5123 **3.** 12.3929 **4.** 0.0926 **5.** 0.1861 **6.** −1.0311

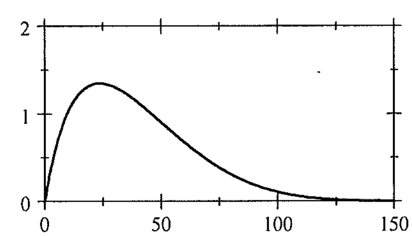
7. a. The initial population of crocodiles is $P(0) = \frac{300}{6} = 50$.

b. $\lim_{t \to \infty} P(t) = \lim_{t \to \infty} \dfrac{300e^{-0.024t}}{5e^{-0.024t} + 1} = \dfrac{0}{0+1} = 0$.

c.

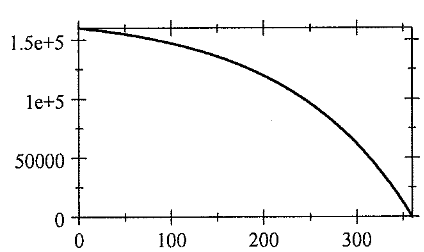


8. a.



b. At $x = 10$, the rate is 57,972/\$1000. If $x = 50$, the rate is −23,418/\$1000. Income distribution is increasing at low income levels and decreasing at higher levels.

9. a.



b. Initially, they owe $B(0) = \$160{,}000$, and their debt is decreasing at the rate of $B'(0) \approx \$87.07$ per month. After 180 payments, they owe $B(180) \approx \$126{,}928.78$ and their debt is decreasing at the rate of $B'(180) \approx \$334.18$ per month.

10. a. At the beginning of June, there are $F(1) \approx 196.20$ or approximately 196 aphids in a typical bean stem. At the beginning of July the number is $F(2) \approx 180.02$, or approximately 180 aphids per bean stem.