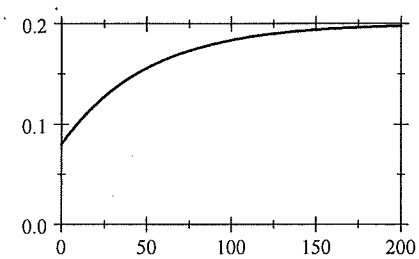
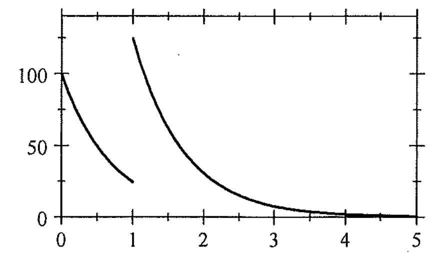
11. a.



b. $0.08\ \text{g/cm}^3$. **c.** $0.12\ \text{g/cm}^3$.

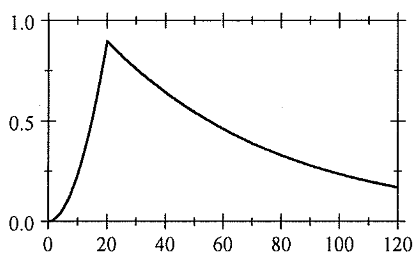
d. $0.2\ \text{g/cm}^3$

12. a.



b. The graph confirms the results of Exercise 48.

13. a.

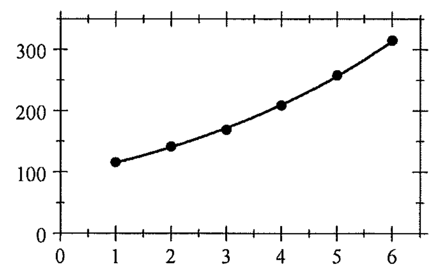


b. 20 seconds. **c.** 35.1 seconds.

14. a. Using **ExpReg** we find

$f(x) = 94.48(1.221^x) = 94.48e^{0.1997x}$.

b.



5.2 Logarithmic Functions

Concept Questions page 351

1. a. $y = \log_b x$ if and only if $x = b^y$.

b. $f(x) = \log_b x$, $b > 0$, $b \neq 1$. Its domain is $(0, \infty)$.

2. a. $\log_b x$ has domain $(0, \infty)$ and range $(-\infty, \infty)$.

b. Its x-intercept is 1.

c. It is continuous on $(0, \infty)$.

d. It is increasing on $(0, \infty)$ if $b > 1$ and decreasing on $(0, \infty)$ if $b < 1$.

3. a. $e^{\ln x} = x$. **b.** $\ln e^x = x$.

4. No. The domain of f is $(-\infty, \infty)$, whereas the domain of g is $(0, \infty)$.

Exercises page 351

1. $\log_2 64 = 6$. **2.** $\log_3 243 = 5$. **3.** $\log_4 \frac{1}{16} = -2$. **4.** $\log_5 \frac{1}{125} = -3$.

5. $\log_{1/3} \frac{1}{3} = 1$. **6.** $\log_{1/2} 16 = -4$. **7.** $\log_{32} 16 = \frac{4}{5}$. **8.** $\log_{81} 27 = \frac{3}{4}$.

9. $\log_{10} 0.001 = -3$. **10.** $\log_{16} 0.5 = -\frac{1}{4}$.

11. $\log 12 = \log 4 \times 3 = \log 4 + \log 3 = 0.6021 + 0.4771 = 1.0792.$

12. $\log \frac{3}{4} = \log 3 - \log 4 = 0.4771 - 0.6021 = -0.125.$

13. $\log 16 = \log 4^2 = 2\log 4 = 2\,(0.6021) = 1.2042.$

14. $\log \sqrt{3} = \log 3^{1/2} = \frac{1}{2}\log 3 = \frac{1}{2}\,(0.4771) = 0.2386.$

15. $\log 48 = \log\left(3 \cdot 4^2\right) = \log 3 + 2\log 4 = 0.4771 + 2\,(0.6021) = 1.6813.$

16. $\log \frac{1}{300} = \log 1 - \log 300 = -\log 300 = -\log(3 \cdot 100) = -(\log 3 + \log 100) = -(\log 3 + 2\log 10)$

$= -(\log 3 + 2) \approx -2.4771.$

17. $2\ln a + 3\ln b = \ln a^2b^3.$

18. $\frac{1}{2}\ln x + 2\ln y - 3\ln z = \ln \dfrac{x^{1/2}y^2}{3z} = \ln \dfrac{\sqrt{x}y^2}{3z}.$

19. $\ln 3 + \frac{1}{2}\ln x + \ln y - \frac{1}{3}\ln z = \ln \dfrac{3\sqrt{x}y}{\sqrt[3]{z}}.$

20. $\ln 2 + \frac{1}{2}\ln(x+1) - 2\ln\left(1+\sqrt{x}\right) = \ln \dfrac{2\,(x+1)^{1/2}}{\left(1+\sqrt{x}\right)^2}.$

21. $\log x\,(x+1)^4 = \log x + \log(x+1)^4 = \log x + 4\log(x+1).$

22. $\log x\left(x^2+1\right)^{-1/2} = \log x - \frac{1}{2}\log\left(x^2+1\right).$

23. $\log \dfrac{\sqrt{x+1}}{x^2+1} = \log(x+1)^{1/2} - \log\left(x^2+1\right) = \frac{1}{2}\log(x+1) - \log\left(x^2+1\right).$

24. $\ln \dfrac{e^x}{1+e^x} = x - \ln(1+e^x).$

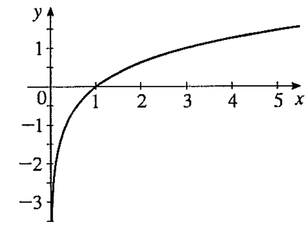
25. $\ln xe^{-x^2} = \ln x - x^2.$

26. $\ln x\,(x+1)(x+2) = \ln x + \ln(x+1) + \ln(x+2).$

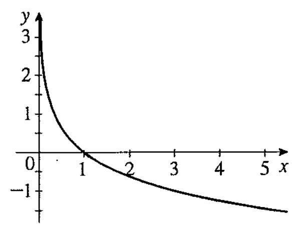
27. $\ln\left(\dfrac{x^{1/2}}{x^2\sqrt{1+x^2}}\right) = \ln x^{1/2} - \ln x^2 - \ln\left(1+x^2\right)^{1/2} = \frac{1}{2}\ln x - 2\ln x - \frac{1}{2}\ln\left(1+x^2\right) = -\frac{3}{2}\ln x - \frac{1}{2}\ln\left(1+x^2\right).$

28. $\ln \dfrac{x^2}{\sqrt{x}\,(1+x)^2} = 2\ln x - \frac{1}{2}\ln x - 2\ln(1+x) = \frac{3}{2}\ln x - 2\ln(1+x).$

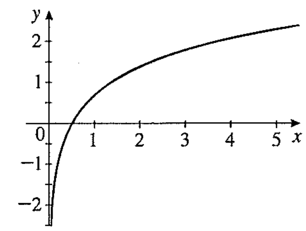
29. $y = \log_3 x$.



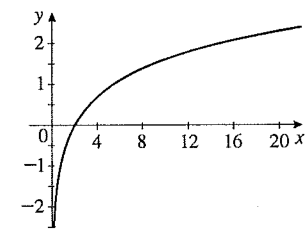
30. $y = \log_{1/3} x$.



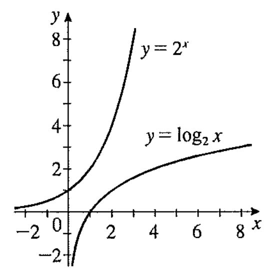
31. $y = \ln 2x$.



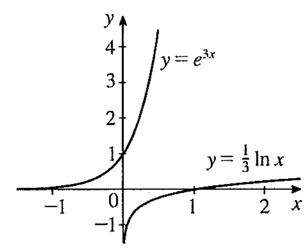
32. $y = \ln \frac{1}{2}x$.



33. $y = 2^x$ and $y = \log_2 x$.



34. $y = e^{3x}$ and $y = \frac{1}{3}\ln x$.



35. $e^{0.4t} = 8$, so $0.4t \ln e = \ln 8$ and thus $0.4t = \ln 8$ because $\ln e = 1$. Therefore, $t = \dfrac{\ln 8}{0.4} \approx 5.1986$.

36. $\frac{1}{3}e^{-3t} = 0.9$, $e^{-3t} = 2.7$. Taking the logarithm, we have $-3t \ln e = \ln 2.7$, so $t = -\dfrac{\ln 2.7}{3} \approx -0.3311$.

37. $5e^{-2t} = 6$, so $e^{-2t} = \frac{6}{5} = 1.2$. Taking the logarithm, we have $-2t \ln e = \ln 1.2$, so $t = -\dfrac{\ln 1.2}{2} \approx -0.0912$.

38. $4e^{t-1} = 4$, so $e^{t-1} = 1$, $\ln e^{t-1} = \ln 1$, $(t-1)\ln e = 0$, and $t = 1$.

39. $2e^{-0.2t} - 4 = 6$, so $2e^{-0.2t} = 10$. Taking the logarithm on both sides of this last equation, we have $\ln e^{-0.2t} = \ln 5$, $-0.2t \ln e = \ln 5$, $-0.2t = \ln 5$, and and $t = -\frac{\ln 5}{0.2} \approx -8.0472$.

40. $12 - e^{0.4t} = 3$, $e^{0.4t} = 9$, $\ln e^{0.4t} = \ln 9$, $0.4t \ln e = \ln 9$, and $0.4t = \ln 9$, so $t = \frac{\ln 9}{0.4} \approx 5.4931$.

41. $\dfrac{50}{1 + 4e^{0.2t}} = 20$, so $1 + 4e^{0.2t} = \dfrac{50}{20} = 2.5$, $4e^{0.2t} = 1.5$, $e^{0.2t} = \dfrac{1.5}{4} = 0.375$, $\ln e^{0.2t} = \ln 0.375$, and $0.2t = \ln 0.375$. Thus, $t = \dfrac{\ln 0.375}{0.2} \approx -4.9041$.

42. $\dfrac{200}{1 + 3e^{-0.3t}} = 100$, so $1 + 3e^{-0.3t} = \frac{200}{100} = 2$, $3e^{-0.3t} = 1$, $e^{-0.3t} = \frac{1}{3}$, and $\ln e^{-0.3t} = \ln \frac{1}{3} = \ln 1 - \ln 3 = -\ln 3$. Thus, $-0.3t \ln e = -\ln 3$, so $0.3t = \ln 3$. Therefore, $t = \dfrac{\ln 3}{0.3} \approx 3.6620$.

43. Taking logarithms of both sides, we obtain $\ln A = \ln Be^{-t/2}$, $\ln A = \ln B + \ln e^{-t/2}$, and $\ln A - \ln B = -\dfrac{t}{2}\ln e$, so $\ln \dfrac{A}{B} = -\dfrac{t}{2}$ and $t = -2\ln \dfrac{A}{B} = 2\ln \dfrac{B}{A}$.

44. $\frac{A}{1+Be^{t/2}} = C$, $A = C + BCe^{t/2}$, $A - C = BCe^{t/2}$, $\frac{A-C}{BC} = e^{t/2}$, and $\frac{t}{2} = \ln \frac{A-C}{BC}$, so $t = 2\ln\left(\frac{A-C}{BC}\right)$.

45. $f(1) = 2$, so $a + b(0) = 2$. Thus, $a = 2$. Therefore, $f(x) = 2 + b\ln x$. We calculate $f(2) = 4$, so $2 + b\ln 2 = 4$. Solving for b, we obtain $b = \frac{2}{\ln 2} \approx 2.8854$, so $f(x) = 2 + 2.8854\ln x$.

46. **a.** The average life expectancy in 1907 was $W(1) = 49.9$ years.

b. The average life expectancy in 2027 will be $W(7) = 49.9 + 17.1\ln 7 \approx 83.2$ years.

47. $p(x) = 19.4\ln x + 18$. For a child weighing 92 lb, we find $p(92) = 19.4\ln 92 + 18 \approx 105.7$, or approximately 106 millimeters of mercury.

48. **a.** $5 = \log\frac{I}{I_0}$, so $\frac{I}{I_0} = 10^5$ and $I = 10^5 I_0 = 100{,}000 I_0$.

b. $8 = \log\frac{I}{I_0}$, from which we find $I = 10^8 I_0$. Thus, it is 1000 times greater.

c. $7.0 = \log\frac{I}{I_0}$ gives $I = 10^{7.0} I_0$. So it is $\frac{10^{7.0}}{10^5} = 10^2$, or 100 times greater than one with magnitude 5.

49. **a.** $30 = 10\log\frac{I}{I_0}$, so $3 = \log\frac{I}{I_0}$, and $\frac{I}{I_0} = 10^3 = 1000$. Thus, $I = 1000 I_0$.

b. When $D = 80$, $I = 10^8 I_0$ and when $D = 30$, $I = 10^3 I_0$. Therefore, an 80-decibel sound is $10^8/10^3 = 10^5 = 100{,}000$ times louder than a 30-decibel sound.

c. It is $10^{15}/10^8 = 10^7 = 10{,}000{,}000$ times louder.

50. We solve the equation $29.92e^{-0.2x} = 20$, obtaining $e^{-0.2x} = \frac{20}{29.92} = 0.6684$, $-0.2x = \ln 0.6684$, and $x = -\frac{\ln 0.6684}{0.2} \approx 2.01$. Thus, the balloonist's altitude is 2.01 miles.

51. **a.** The temperature when it was first poured is given by $T(0) = 70 + 100e^0 = 170$, or 170°F.

b. We solve the equation $70 + 100e^{-0.0446t} = 120$; $100e^{-0.0446t} = 50$, obtaining $e^{-0.0446t} = \frac{50}{100} = \frac{1}{2}$, $\ln e^{-0.0446t} = \ln\frac{1}{2} = \ln 1 - \ln 2 = -\ln 2$, $-0.0446t = -\ln 2$, and so $t = \frac{\ln 2}{0.0446} \approx 15.54$. Thus, it will take approximately 15.54 minutes.

52. We solve the equation $\frac{160}{1+240e^{-0.2t}} = 80$ for t, obtaining $1 + 240e^{-0.2t} = \frac{160}{80}$, $240e^{-0.2t} = 2 - 1 = 1$, $e^{-0.2t} = \frac{1}{240}$, $-0.2t = \ln\frac{1}{240}$, and $t = -\frac{1}{0.2}\ln\frac{1}{240} \approx 27.40$, or approximately 27.4 years old.

53. When $f(t) = 40$, we have $\frac{46.5}{1+2.324e^{-0.05113t}} = 40$, so $1 + 2.324e^{-0.05113t} = \frac{46.5}{40}$, $2.324e^{-0.05113t} = \frac{46.5}{40} - 1 = 0.1625$, $e^{-0.05113t} = \frac{0.1625}{2.324}$, $-0.05113t = \ln\left(\frac{0.1625}{2.324}\right)$, and $t \approx 52.03$. Thus, the percentage of obese adults will reach 40% around 2022.

54. We solve the equation $200\left(1-0.956e^{-0.18t}\right)=140$ for t, obtaining $1-0.956e^{-0.18t}=\frac{140}{200}=0.7$, $-0.956e^{-0.18t}=0.7-1=-0.3$, $e^{-0.18t}=\frac{0.3}{0.956}$, $-0.18t=\ln\left(\frac{0.3}{0.956}\right)$, and finally $t=-\dfrac{\ln\left(\frac{0.3}{0.956}\right)}{0.18}\approx 6.43875$. Thus, it is approximately 6.4 years old.

55. a. We solve the equation $0.08+0.12e^{-0.02t}=0.18$, obtaining $0.12e^{-0.02t}=0.1$, $e^{-0.02t}=\frac{0.1}{0.12}=\frac{1}{1.2}$, $\ln e^{-0.02t}=\ln\frac{1}{1.2}=\ln 1-\ln 1.2=-\ln 1.2$, $-0.02t=-\ln 1.2$, and $t=\frac{\ln 1.2}{0.02}\approx 9.116$, or approximately 9.1 seconds.

b. We solve the equation $0.08+0.12e^{-0.02t}=0.16$, obtaining $0.12e^{-0.02t}=0.08$, $e^{-0.02t}=\frac{0.08}{0.12}=\frac{2}{3}$, $-0.02t=\ln\frac{2}{3}$, and $t=-\frac{1}{0.02}\ln\frac{2}{3}\approx 20.2733$, or approximately 20.3 seconds.

56. a. We solve the equation $0.08\left(1-e^{-0.02t}\right)=0.02$, obtaining $1-e^{-0.02t}=\frac{0.02}{0.08}=\frac{1}{4}$, $-e^{-0.02t}=\frac{1}{4}-1=-\frac{3}{4}$, $e^{-0.02t}=\frac{3}{4}$, $\ln e^{-0.02t}=\ln\frac{3}{4}$, $-0.02t=\ln\frac{3}{4}$, and so $t\approx 14.38$, or 14.38 seconds.

b. $1-e^{-0.02t}=\frac{0.04}{0.08}$, so $-e^{-0.02t}=\frac{1}{2}-1=-\frac{1}{2}$, $e^{-0.02t}=0.5$, and $t=-\frac{\ln 0.5}{0.02}\approx 34.66$, or 34.66 seconds.

57. With $T_0=70$, $T_1=98.6$, and $T=80$, we have $80=70+(98.6-70)(0.97)^t$, so $28.6(0.97)^t=10$ and $(0.97)^t=0.34965$. Taking logarithms, we have $\ln(0.97)^t=\ln 0.34965$, or $t=\dfrac{\ln 0.34965}{\ln 0.97}\approx 34.50$. Thus, he was killed $34\frac{1}{2}$ hours earlier, at 1:30 p.m.

58. a. Solving the given demand equation $p=50\ln\dfrac{50}{x}$ for x in terms of p, we find $\ln\left(\dfrac{50}{x}\right)=\dfrac{p}{50}$, so $\dfrac{50}{x}=e^{p/50}$ and $x=f(p)=50e^{-p/50}=50e^{-0.02p}$ for $p>0$. Next, we find $f'(p)=-e^{-0.02p}$, and so $E(p)=-\dfrac{pf'(p)}{f(p)}=\dfrac{pe^{-0.02p}}{50e^{-0.02p}}=\dfrac{p}{50}$. Now $E(p)<1$ if and only if $\dfrac{p}{50}<1$ or $p<50$, and similarly $E(p)=1$ when $p=50$ and $f(p)>1$ if $p>50$. Thus, demand is inelastic if $0<p<50$, unitary if $p=50$, and elastic if $p>50$.

b. Since demand is unitary at $p=50$, we see that at that price, a slight increase in the unit price will not affect revenue.

59. False. Take $x=e$. Then $(\ln e)^3=1^3=1\neq 3\ln e=3$.

60. False. Take $a=b=1$. Then $\ln(a+b)=\ln(1+1)=\ln 2\neq\ln a+\ln b=\ln 1+\ln 1=0$.

61. True. $e^{\ln b}=b$ and $\ln e^b=b$ as well.

62. False. Take $a=2e$ and $b=e$. Then $\ln a-\ln b=\ln 2e-\ln e=\ln 2+\ln e-\ln e=\ln 2$, But $\ln(a-b)=\ln(2e-e)=\ln e=1$.

63. True. $g(x)=\ln x$ is continuous and greater than zero on $(1,\infty)$. Therefore, $f(x)=\dfrac{1}{\ln x}$ is continuous on $(1,\infty)$.

64. True. If $a=\log_2 3$, then $3=2^a$ and $\ln 3=\ln 2^a=a\ln 2$, so $a=\dfrac{\ln 3}{\ln 2}$. Similarly, if $b=\log_3 2$, then $2=3^b$, $\ln 2=b\ln 3$, and $b=\dfrac{\ln 2}{\ln 3}$. Therefore, $ab=\left(\log_2 3\right)\left(\log_3 2\right)=\dfrac{\ln 3}{\ln 2}\cdot\dfrac{\ln 2}{\ln 3}=1$.